M10/5/MATHL/HP2/ENG/TZ2/XX/M+

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

May 2010

MATHEMATICS

Higher Level

Paper 2

Samples to team leaders	June 8 2010
Everything (marks, scripts etc) to IB Cardiff	June 15 2010

13 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

  Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

  $f'(x) = \left(2\cos(5x-3)\right) \; 5 \quad \left(= 10\cos(5x-3)\right)$ ***A1***

  Award ***A1*** for $\left(2\cos(5x-3)\right) \; 5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Candidates should be penalized ***once only IN THE PAPER*** *for an accuracy error* ***(AP)****. Award the marks as usual then write* ***(AP)*** *against the answer. On the* ***front*** *cover write –1****(AP)****. Deduct 1 mark from the total for the paper, not the question*.

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

SECTION A

1. (a) $18n-10$ (or equivalent) ***A1***

(b) $\sum_{1}^{n}(18r-10)$ (or equivalent) ***A1***

(c) by use of GDC or algebraic summation or sum of an AP ***(M1)***

$\sum_{1}^{15}(18r-10)=2010$ ***A1***

[4 marks]

2. (a) $p+q=0.44$ ***A1***

$2.5p+3.5q=1.25$ ***(M1)A1***

$p=0.29,\ q=0.15$ ***A1***

(b) use of $\mathrm{Var}(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2$ ***(M1)***

$\mathrm{Var}(X)=2.10$ ***A1***

[6 marks]

3. (a) required to solve $\mathrm{P}\left(Z<\frac{21-15}{\sigma}\right)=0.8$ ***(M1)***

$\frac{6}{\sigma}=0.842...$ (or equivalent) ***(M1)***

$\Rightarrow \sigma=7.13$ (days) ***A1*** ***N1***

(b) $\mathrm{P}(\text{survival after } 21 \text{ days})=0.337$ ***(M1)A1***

[5 marks]

4. (a) rewrite the equation as $(4x-1)\ln 2=(x+5)\ln 8+(1-2x)\log_2 16$ ***(M1)***

$(4x-1)\ln 2=(3x+15)\ln 2+4-8x$ ***(M1)(A1)***

$x=\frac{4+16\ln 2}{8+\ln 2}$ ***A1***

(b) $x=a^2$ ***(M1)***

$a=1.318$ ***A1***

Note:	Treat 1.32 as an ***AP***.
	Award ***A0*** for $\pm$.

[6 marks]

5. use of cosine rule: $\text{BC} = \sqrt{(8^2 + 7^2 - 2 \times 7 \times 8 \cos 70)} = 8.6426...$ ***(M1)A1***

Note: Accept an expression for BC^2.

$\text{BD} = 5.7617\ldots \quad (\text{CD} = 2.88085\ldots)$ ***A1***

use of sine rule: $\hat{B} = \arcsin\left(\frac{7 \sin 70}{\text{BC}}\right) = 49.561...^\circ \quad (\hat{C} = 60.4387...^\circ)$ ***(M1)A1***

use of cosine rule: $\text{AD} = \sqrt{8^2 + \text{BD}^2 - 2 \times \text{BD} \times 8 \cos B} = 6.12\,(\text{cm})$ ***A1***

Note: Scale drawing method not acceptable.

[6 marks]

6. (a) required to solve $\mathrm{e}^{-\lambda} + \lambda \mathrm{e}^{-\lambda} = 0.123$ ***M1A1***
solving to obtain $\lambda = 3.63$ ***A2*** ***N2***

Note: Award ***A2*** if an additional negative solution is seen but ***A0*** if only a negative solution is seen.

(b) $\mathrm{P}(0 < X < 9)$
$= \mathrm{P}(X \le 8) - \mathrm{P}(X = 0)$ (or equivalent) ***(M1)***
$= 0.961$ ***A1***

[6 marks]

7. (a) use GDC or manual method to find a, b and c ***(M1)***
obtain $a = 2$, $b = -1$, $c = 3$ (in any identifiable form) ***A1***

(b) use GDC or manual method to solve second set of equations ***(M1)***
obtain $x = \frac{4 - 11t}{2}$; $y = \frac{-7t}{2}$; $z = t$ (or equivalent) ***(A1)***

$\boldsymbol{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5.5 \\ -3.5 \\ 1 \end{pmatrix}$ (accept equivalent vector forms) ***M1A1***

Note: Final ***A1*** requires $\boldsymbol{r} =$ or equivalent.

[6 marks]

8. (a) the expression is

$\frac{n!}{(n-3)!\,3!} - \frac{(2n)!}{(2n-2)!\,2!}$ ***(A1)***

$\frac{n(n-1)(n-2)}{6} - \frac{2n(2n-1)}{2}$ ***M1A1***

$= \frac{n(n^2 - 15n + 8)}{6} \quad \left(= \frac{n^3 - 15n^2 + 8n}{6}\right)$ ***A1***

(b) the inequality is

$\frac{n^3 - 15n^2 + 8n}{6} > 32n$

attempt to solve cubic inequality or equation ***(M1)***

$n^3 - 15n^2 - 184n > 0 \quad n(n-23)(n+8) > 0$

$n > 23 \quad (n \ge 24)$ ***A1***

[6 marks]

9. (a) using de Moivre's theorem

$z^n + \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta \quad (= 2\cos n\theta)$, imaginary part of which is 0

M1A1

so $\mathrm{Im}\left(z^n + \frac{1}{z^n}\right) = 0$ ***AG***

(b) $\frac{z-1}{z+1} = \frac{\cos\theta + \mathrm{i}\sin\theta - 1}{\cos\theta + \mathrm{i}\sin\theta + 1}$

$= \frac{(\cos\theta - 1 + \mathrm{i}\sin\theta)(\cos\theta + 1 - \mathrm{i}\sin\theta)}{(\cos\theta + 1 + \mathrm{i}\sin\theta)(\cos\theta + 1 - \mathrm{i}\sin\theta)}$ ***M1A1***

Note: Award ***M1*** for an attempt to multiply numerator and denominator by the complex conjugate of their denominator.

$\Rightarrow \mathrm{Re}\left(\frac{z-1}{z+1}\right) = \frac{(\cos\theta - 1)(\cos\theta + 1) + \sin^2\theta}{\text{real denominator}}$ ***M1A1***

Note: Award ***M1*** for multiplying out the numerator.

$= \frac{\cos^2\theta + \sin^2\theta - 1}{\text{real denominator}}$ ***A1***

$= 0$ ***AG***

[7 marks]

10. (a) the distance of the spot from P is $x = 500\tan\theta$ ***A1***

the speed of the spot is

$\frac{\mathrm{d}x}{\mathrm{d}t} = 500\sec^2\theta\frac{\mathrm{d}\theta}{\mathrm{d}t} \quad (= 4000\pi\sec^2\theta)$ ***M1A1***

when $x = 2000$, $\sec^2\theta = 17$ $(\theta = 1.32581...)\left(\frac{\mathrm{d}\theta}{\mathrm{d}t} = 8\pi\right)$

$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 500 \times 17 \times 8\pi$ ***M1A1***

speed is 214000 (metres per minute) ***AG***

Note: If their displayed answer does not round to 214 000, they lose the final ***A1***.

(b) $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 8000\pi\sec^2\theta\tan\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}$ **or** $500 \times 2\sec^2\theta\tan\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2$ ***M1A1***

$\left(\text{since } \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = 0\right)$

$= 43000000 \quad (= 4.30 \times 10^7)$ (metres per minute^2) ***A1***

[8 marks]

SECTION B

11. (a) solving to obtain one root: $1, -2$ or -5 *A1*
obtain other roots *A1*

[2 marks]

(b) $D = x \in [-5, -2] \cup [1, \infty)$ (or equivalent) *M1A1*

Note: ***M1*** is for 1 finite and 1 infinite interval.

[2 marks]

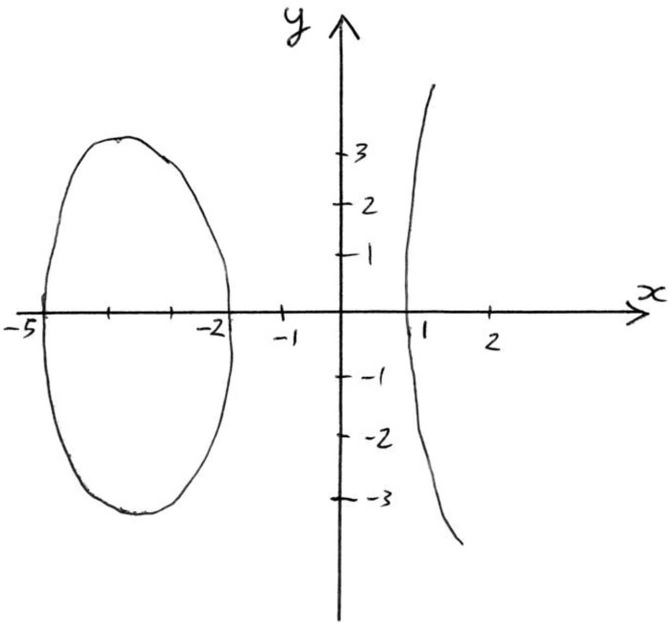
(c) coordinates of local maximum -3.73 $\left(-2-\sqrt{3}\right)$, 3.22 $\left(\sqrt{6\sqrt{3}}\right)$ *A1A1*

[2 marks]

(d) use GDC to obtain one root: $1.41, -3.18$ or -4.23 *A1*
obtain other roots *A1*

[2 marks]

(e)



A1A1A1

Note: Award ***A1*** for shape, ***A1*** for max and for min clearly in correct places, ***A1*** for all intercepts. Award ***A1A0A0*** if only the complete top half is shown.

[3 marks]

(f) required area is twice that of $y = f(x)$ between -5 and -2 *M1A1*
answer 14.9 *A1* ***N3***

Note: Award ***M1A0A0*** for $\int_{-5}^{-2} f(x)\,dx = 7.47...$ or ***N1*** for 7.47.

[3 marks]

Total [14 marks]

12. (a) (i) the median height is 1.18 ***A1***

(ii) the interquartile range is $\text{UQ} - \text{LQ}$
$= 1.22 - 1.13 = 0.09$ (accept answers that round to 0.09) ***A1A1***

Note: Award ***A1*** for the quartiles, ***A1*** for final answer.

[3 marks]

(b) (i)

$1.00 < h \le 1.05$	$1.05 < h \le 1.10$	$1.10 < h \le 1.15$	$1.15 < h \le 1.20$	$1.20 < h \le 1.25$	$1.25 < h \le 1.30$
5	9	13	24	19	10

A1A1

Note: Award ***A1*** for entries within ± 1 of the above values and ***A1*** for a total of 80.

(ii) unbiased estimate of the population mean

$$\left(\frac{5\times 1.025 + 9\times 1.075 + 13\times 1.125 + 24\times 1.175 + 19\times 1.225 + 10\times 1.275}{80}\right) = 1.17$$ ***A1***

unbiased estimate of the population variance

use of $s^2{}_{n-1} = \left(\frac{n}{n-1}\right) s^2{}_n$ or GDC ***(M1)***

obtain 0.00470 ***A1***

[5 marks]

(c) (i) $\text{P}(h \le 1.15 \text{ m}) = \frac{27}{80}$ (0.3375 or 0.338) $\left(\text{allow } \frac{26}{80} \text{ (0.325)}\right)$ ***A1***

(ii) use of the conditional probability formula $\text{P}(A \mid B) = \text{P}(A \cap B) / \text{P}(B)$ ***(M1)***

obtain $\frac{18}{80} \div \frac{27}{80}$ ***(A1)(A1)***

$= \frac{2}{3}$ (0.667) $\left(\text{allow } \frac{18}{26} \text{ (0.692)}\right)$ ***A1***

[5 marks]

Total [13 marks]

13. (a) the area of the first sector is $\frac{1}{2}2^2\theta$ ***(A1)***

the sequence of areas is $2\theta,\ 2k\theta,\ 2k^2\theta\ldots$ ***(A1)***

the sum of these areas is $2\theta(1+k+k^2+\ldots)$ ***(M1)***

$=\frac{2\theta}{1-k}=4\pi$ ***M1A1***

hence $\theta=2\pi(1-k)$ ***AG***

Note: Accept solutions where candidates deal with angles instead of area.

[5 marks]

(b) the perimeter of the first sector is $4+2\theta$ ***(A1)***

the perimeter of the third sector is $4+2k^2\theta$ ***(A1)***

the given condition is $4+2k^2\theta=2+\theta$ ***M1***

which simplifies to $2=\theta(1-2k^2)$ ***A1***

eliminating θ, obtain cubic in k: $\pi(1-k)(1-2k^2)-1=0$ ***A1***

or equivalent

solve for $k=0.456$ and then $\theta=3.42$ ***A1A1***

[7 marks]

Total [12 marks]

14. (a) $g \circ f(x) = \dfrac{1}{1+\mathrm{e}^x}$ ***A1***

$1 < 1+\mathrm{e}^x < \infty$ ***(M1)***

range $g \circ f$ is $]0, 1[$ ***A1*** ***N3***

[3 marks]

(b)

Note: Interchange of variables and rearranging can be done in either order.

attempt at solving $y = \dfrac{1}{1+\mathrm{e}^x}$ ***M1***

rearranging

$\mathrm{e}^x = \dfrac{1-y}{y}$ ***M1***

$(g \circ f)^{-1}(x) = \ln\left(\dfrac{1-x}{x}\right)$ ***A1***

Note: The ***A1*** is for RHS.

domain is $]0, 1[$ ***A1***

Note: Final ***A1*** is independent of the ***M*** marks.

[4 marks]

(c) (i) $y = f \circ g \circ h = 1 + \mathrm{e}^{\cos x}$ ***M1A1***

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\sin x\,\mathrm{e}^{\cos x}$ ***M1A1***

$= (1-y)\sin x$ ***AG***

Note: Second ***M1A1*** could also be obtained by solving the differential equation.

(ii) **EITHER**

rearranging

$y\sin x = \sin x - \dfrac{\mathrm{d}y}{\mathrm{d}x}$ ***A1***

$\int y\sin x\,\mathrm{d}x = \int \sin x\,\mathrm{d}x - \int \dfrac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x$ ***M1***

$= -\cos x - y(+c)$ ***A1***

$= -\cos x - \mathrm{e}^{\cos x}(+d)$ ***A1***

OR

$\int y\sin x\,\mathrm{d}x = \int (1+\mathrm{e}^{\cos x})\sin x\,\mathrm{d}x$ ***A1***

$= \int \sin x\,\mathrm{d}x + \int \sin x \times \mathrm{e}^{\cos x}\,\mathrm{d}x$

Note: Either the first or second line gains the ***A1***.

$= -\cos x - \mathrm{e}^{\cos x}(+d)$ ***A1M1A1***

continued …

Question 14 continued

(iii) use of definition of y and the differential equation or GDC to identify first minimum at $x = \pi$ (3.14...) ***(M1)A1***

EITHER

the required integral is

$\pi \int_{y_{\min}}^{y_{\max}} x^2 \mathrm{d}y$ ***M1A1***

Note: $y_{\max} = 1 + \mathrm{e}$ and $y_{\min} = 1 + \mathrm{e}^{-1}$ but these do not need to be specified.

$= \pi \int_{\pi}^{0} -x^2 \sin x \, \mathrm{e}^{\cos x} \mathrm{d}x = \pi \times 4.32... = 13.6$ ***(M1)A1***

OR

the required integral is

$\pi \int_{1+\mathrm{e}^{-1}}^{1+\mathrm{e}} x^2 \, \mathrm{d}y$ ***M1A1***

$= \pi \int_{1+\mathrm{e}^{-1}}^{1+\mathrm{e}} \left(\arccos\left(\ln(y-1)\right)\right)^2 \mathrm{d}y = \pi \times 4.32... = 13.6$ ***M1A1***

Note: $1 + \mathrm{e} = 3.7182...$ and $1 + \mathrm{e}^{-1} = 1.3678...$

[14 marks]

Total [21 marks]