22107204

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 2

Thursday 6 May 2010 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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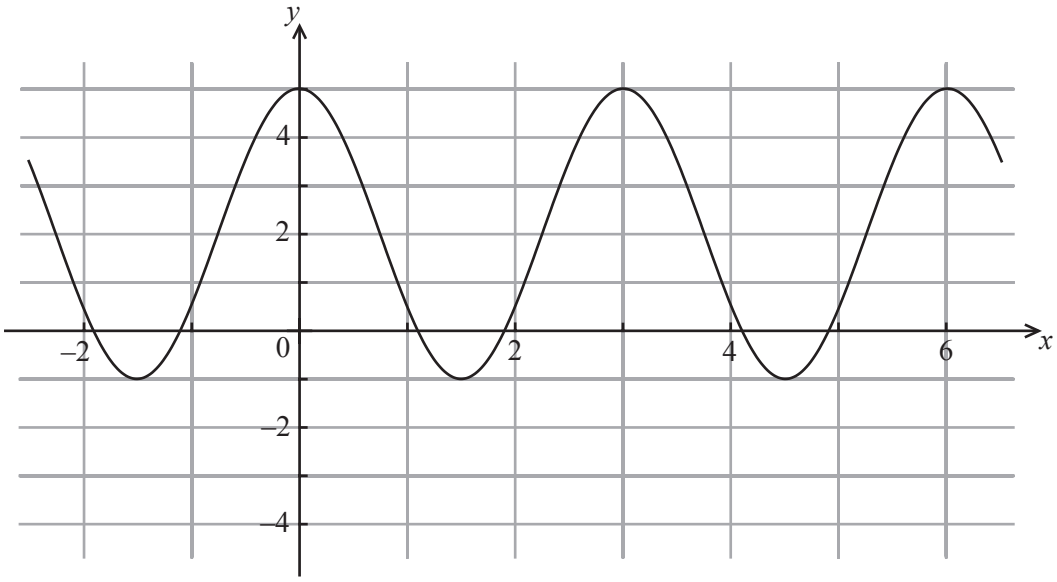
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 4]*

The graph below shows $y = a\cos(bx) + c$.



Find the value of a, the value of b and the value of c.

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2. *[Maximum mark: 5]*

The system of equations

$$\begin{aligned} 2x - y + 3z &= 2 \\ 3x + y + 2z &= -2 \\ -x + 2y + az &= b \end{aligned}$$

is known to have more than one solution. Find the value of a and the value of b.

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3. *[Maximum mark: 6]*

In the right circular cone below, O is the centre of the base which has radius 6 cm. The points B and C are on the circumference of the base of the cone. The height AO of the cone is 8 cm and the angle $\hat{BOC}$ is 60°.

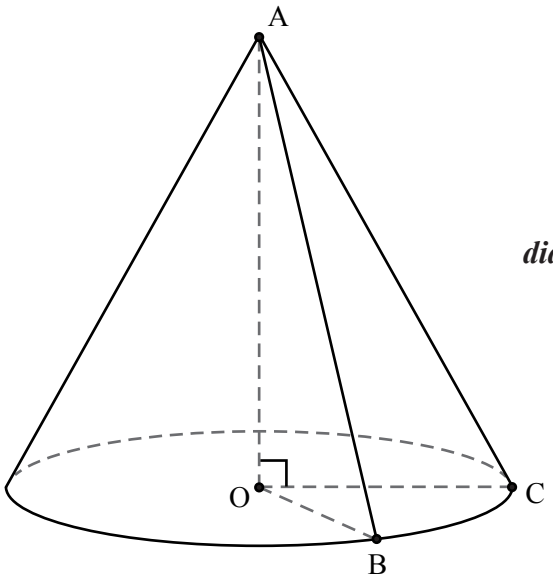

diagram not to scale

Calculate the size of the angle $\hat{BAC}$.

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4. *[Maximum mark: 7]*

(a) Solve the equation $z^3 = -2 + 2\mathrm{i}$, giving your answers in modulus-argument form. *[6 marks]*

(b) **Hence** show that one of the solutions is $1 + \mathrm{i}$ when written in Cartesian form. *[1 mark]*

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5. *[Maximum mark: 6]*

Let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ be non-singular 2×2 matrices, $\boldsymbol{I}$ the 2×2 identity matrix and k a scalar. The following statements are **incorrect**. For each statement, write down the correct version of the right hand side.

(a) $(\boldsymbol{A}+\boldsymbol{B})^2 = \boldsymbol{A}^2 + 2\boldsymbol{AB} + \boldsymbol{B}^2$ *[2 marks]*

(b) $(\boldsymbol{A}-k\boldsymbol{I})^3 = \boldsymbol{A}^3 - 3k\boldsymbol{A}^2 + 3k^2\boldsymbol{A} - k^3$ *[2 marks]*

(c) $\boldsymbol{CA} = \boldsymbol{B} \Rightarrow \boldsymbol{C} = \dfrac{\boldsymbol{B}}{\boldsymbol{A}}$ *[2 marks]*

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6. *[Maximum mark: 5]*

Find the sum of all three-digit natural numbers that are not exactly divisible by 3.

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7. *[Maximum mark: 7]*

Three Mathematics books, five English books, four Science books and a dictionary are to be placed on a student's shelf so that the books of each subject remain together.

(a) In how many different ways can the books be arranged? *[4 marks]*

(b) In how many of these will the dictionary be next to the Mathematics books? *[3 marks]*

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8. *[Maximum mark: 6]*

In a factory producing glasses, the weights of glasses are known to have a mean of 160 grams. It is also known that the interquartile range of the weights of glasses is 28 grams. Assuming the weights of glasses to be normally distributed, find the standard deviation of the weights of glasses.

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9. *[Maximum mark: 6]*

Let $f(x) = \frac{4 - x^2}{4 - \sqrt{x}}$.

(a) State the largest possible domain for f. *[2 marks]*

(b) Solve the inequality $f(x) \geq 1$. *[4 marks]*

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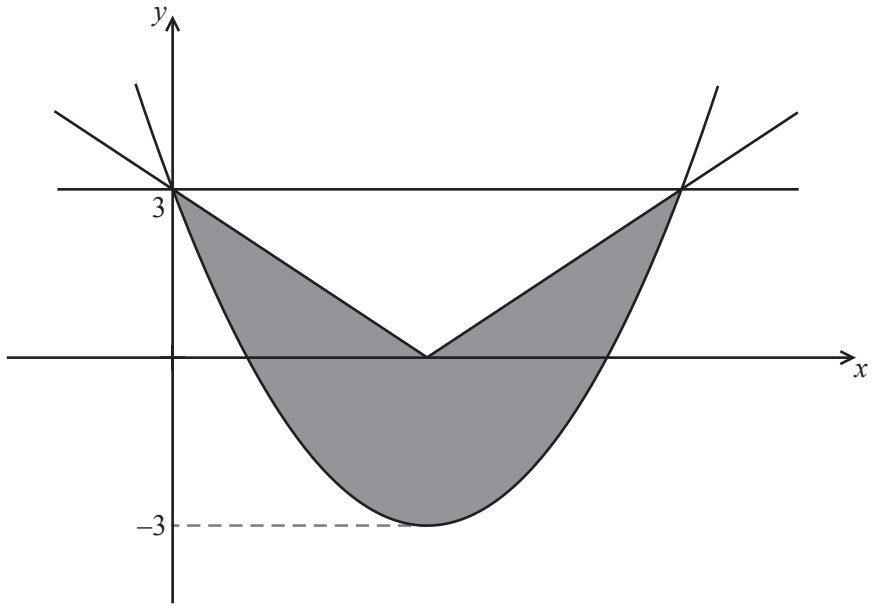
10. *[Maximum mark: 8]*

The diagram below shows the graphs of $y = \left| \frac{3}{2}x - 3 \right|$, $y = 3$ and a quadratic function, that all intersect in the same two points.



Given that the minimum value of the quadratic function is -3, find an expression for the area of the shaded region in the form $\int_0^t (ax^2 + bx + c)\,\mathrm{d}x$, where the constants a, b, c and t are to be determined. (Note: The integral does not need to be evaluated.)

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 20]*

A plane π has vector equation $\boldsymbol{r} = (-2\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}) + \lambda(2\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}) + \mu(6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k})$.

(a) Show that the Cartesian equation of the plane π is $3x + 2y - 6z = 12$. *[6 marks]*

(b) The plane π meets the x, y and z axes at A, B and C respectively. Find the coordinates of A, B and C. *[3 marks]*

(c) Find the volume of the pyramid OABC. *[3 marks]*

(d) Find the angle between the plane π and the x-axis. *[4 marks]*

(e) **Hence**, or otherwise, find the distance from the origin to the plane π. *[2 marks]*

(f) Using your answers from (c) and (e), find the area of the triangle ABC. *[2 marks]*

12. *[Maximum mark: 15]*

Casualties arrive at an accident unit with a mean rate of one every 10 minutes. Assume that the number of arrivals can be modelled by a Poisson distribution.

(a) Find the probability that there are no arrivals in a given half hour period. *[3 marks]*

(b) A nurse works for a two hour period. Find the probability that there are fewer than ten casualties during this period. *[3 marks]*

(c) Six nurses work consecutive two hour periods between 8am and 8pm. Find the probability that no more than three nurses have to attend to less than ten casualties during their working period. *[4 marks]*

(d) Calculate the time interval during which there is a 95 % chance of there being at least two casualties. *[5 marks]*

13. *[Maximum mark: 11]*

Points A, B and C are on the circumference of a circle, centre O and radius r. A trapezium OABC is formed such that AB is parallel to OC, and the angle $\hat{AOC}$ is θ, $\frac{\pi}{2} \leq \theta < \pi$.

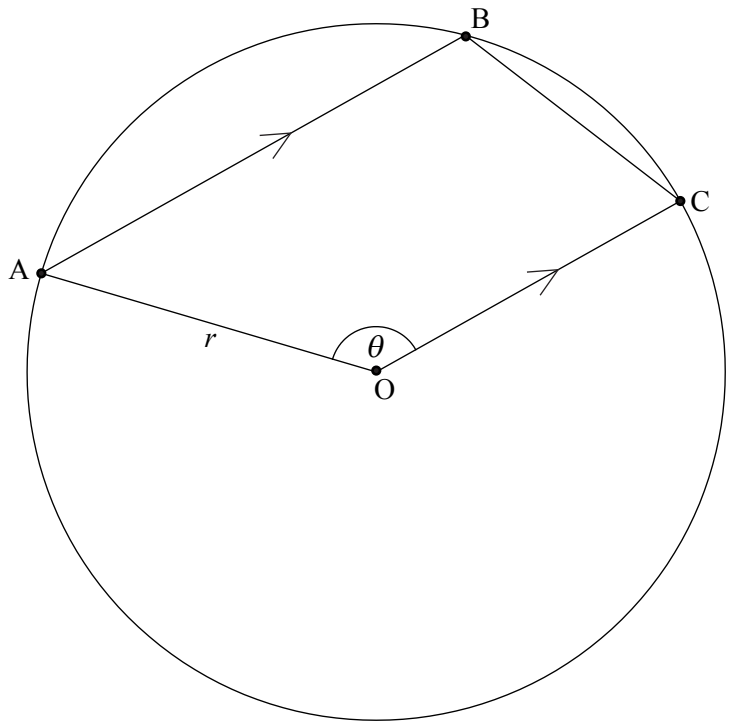

diagram not to scale

(a) Show that angle $\hat{BOC}$ is $\pi - \theta$. *[3 marks]*

(b) Show that the area, T, of the trapezium can be expressed as

$$T = \frac{1}{2}r^2 \sin\theta - \frac{1}{2}r^2 \sin 2\theta .$$ *[3 marks]*

(c) (i) Show that when the area is maximum, the value of θ satisfies

$$\cos\theta = 2\cos 2\theta .$$

(ii) **Hence** determine the maximum area of the trapezium when $r = 1$.
(Note: It is not required to prove that it is a maximum.) *[5 marks]*

14. *[Maximum mark: 14]*

A body is moving through a liquid so that its acceleration can be expressed as

$$\left(-\frac{v^2}{200}-32\right)\text{m s}^{-2},$$

where $v\ \text{m s}^{-1}$ is the velocity of the body at time t seconds.

The initial velocity of the body was known to be $40\ \text{m s}^{-1}$.

(a) Show that the time taken, T seconds, for the body to slow to $V\ \text{m s}^{-1}$ is given by

$$T = 200\int_V^{40}\frac{1}{v^2+80^2}\,\text{d}v.$$ *[4 marks]*

(b) (i) Explain why acceleration can be expressed as $v\frac{\text{d}v}{\text{d}s}$, where s is displacement, in metres, of the body at time t seconds.

(ii) **Hence** find a similar integral to that shown in part (a) for the distance, S metres, travelled as the body slows to $V\ \text{m s}^{-1}$. *[7 marks]*

(c) **Hence**, using parts (a) and (b), find the distance travelled and the time taken until the body momentarily comes to rest. *[3 marks]*