N09/5/MATHL/HP3/ENG/TZ0/SP

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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Wednesday 18 November 2009 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 13]*

The mean weight of a certain breed of bird is believed to be 2.5 kg. In order to test this belief, it is planned to determine the weights $x_1, x_2, x_3, \ldots, x_{16}$ (in kg) of sixteen of these birds and then to calculate the sample mean $\bar{x}$. You may assume that these weights are a random sample from a normal distribution with standard deviation 0.1 kg.

(a) State suitable hypotheses for a two-tailed test. *[2 marks]*

(b) Find the critical region for $\bar{x}$ having a significance level of 5 %. *[6 marks]*

(c) Given that the mean weight of birds of this breed is actually 2.6 kg, find the probability of making a Type II error. *[5 marks]*

2. *[Maximum mark: 19]*

(a) Alan and Brian are athletes specializing in the long jump. When Alan jumps, the length of his jump is a normally distributed random variable with mean 5.2 metres and standard deviation 0.1 metres. When Brian jumps, the length of his jump is a normally distributed random variable with mean 5.1 metres and standard deviation 0.12 metres. For both athletes, the length of a jump is independent of the lengths of all other jumps. During a training session, Alan makes four jumps and Brian makes three jumps. Calculate the probability that the mean length of Alan's four jumps is less than the mean length of Brian's three jumps. *[9 marks]*

(b) Colin joins the squad and the coach wants to know the mean length, μ metres, of his jumps. Colin makes six jumps resulting in the following lengths in metres.

5.21, 5.30, 5.22, 5.19, 5.28, 5.18

(i) Calculate an unbiased estimate of both the mean μ and the variance of the lengths of his jumps.

(ii) Assuming that the lengths of these jumps are independent and normally distributed, calculate a 90 % confidence interval for μ. *[10 marks]*

3. *[Maximum mark: 15]*

The following table shows the result of 100 independent observations on the discrete random variable X.

Value of X	3	4	5	6	7	8 or more
Frequency	25	21	20	15	12	7

Charles believes that X has a negative binomial distribution with parameters $r=3$ and $p=0.6$ and he asks you to carry out a χ^2 test to investigate his belief.

(a) State suitable hypotheses. *[1 mark]*

(b) Calculate the expected frequencies, giving your answers correct to two decimal places. *[8 marks]*

(c) Calculate the value of the χ^2 statistic and determine its p-value. *[5 marks]*

(d) State your conclusion. *[1 mark]*

4. *[Maximum mark: 13]*

The random variable X has the distribution $\mathrm{B}(n,\ p)$.

(a) (i) Show that $\dfrac{\mathrm{P}(X=x)}{\mathrm{P}(X=x-1)} = \dfrac{(n-x+1)\,p}{x(1-p)}$.

(ii) Deduce that if $\mathrm{P}(X=x) > \mathrm{P}(X=x-1)$ then $x < (n+1)\,p$.

(iii) Hence, determine the value of x which maximizes $\mathrm{P}(X=x)$ when $(n+1)\,p$ is not an integer. *[9 marks]*

(b) Given that $n=19$, find the set of values of p for which X has a unique mode of 13. *[4 marks]*