N09/5/MATHL/HP3/ENG/TZ0/SE

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MATHEMATICS
HIGHER LEVEL
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS

Wednesday 18 November 2009 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 13]*

Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x} + \frac{y^2}{x^2} \text{ (where } x > 0\text{)}$$

given that $y = 2$ when $x = 1$. Give your answer in the form $y = f(x)$.

2. *[Maximum mark: 10]*

The function f is defined by $f(x) = \mathrm{e}^{(\mathrm{e}^x - 1)}$.

(a) Assuming the Maclaurin series for e^x, show that the Maclaurin series for $f(x)$ is $1 + x + x^2 + \frac{5}{6}x^3 + \dots$. *[5 marks]*

(b) Hence or otherwise find the value of $\lim_{x \to 0} \frac{f(x) - 1}{f'(x) - 1}$. *[5 marks]*

3. *[Maximum mark: 9]*

The sequence $\{u_n\}$ is defined for $n \in \mathbb{Z}^+$ by $u_n = \frac{2n^2}{n^2 + 1}$.

(a) Find the value L of $\lim_{n \to \infty} u_n$. *[2 marks]*

(b) Use the formal ε, N definition of convergence to **prove** that $\lim_{n \to \infty} u_n = L$. *[7 marks]*

4. *[Maximum mark: 13]*

Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$.

(a) Using one of the standard tests for convergence, show that the series is convergent. *[3 marks]*

(b) (i) Express $\frac{1}{n(n+3)}$ in partial fractions.

(ii) Hence find the sum of the above infinite series. *[10 marks]*

5. *[Maximum mark: 15]*

(a) Find the radius of convergence of the infinite series

$$\frac{1}{2}x + \frac{1\times 3}{2\times 5}x^2 + \frac{1\times 3\times 5}{2\times 5\times 8}x^3 + \frac{1\times 3\times 5\times 7}{2\times 5\times 8\times 11}x^4 + \dots .$$ *[7 marks]*

(b) Determine whether the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n} + n\pi\right)$ is convergent or divergent. *[8 marks]*