88097202

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 6 November 2009 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Find the values of k such that the equation $x^3 + x^2 - x + 2 = k$ has three distinct real solutions.

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2. *[Maximum mark: 6]*

The vector equation of line l is given as $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$.

Find the Cartesian equation of the plane containing the line l and the point $\mathrm{A}(4, -2, 5)$.

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3. *[Maximum mark: 7]*

The diagram below shows two concentric circles with centre O and radii 2 cm and 4 cm. The points P and Q lie on the larger circle and $P\hat{O}Q = x$, where $0 < x < \frac{\pi}{2}$.

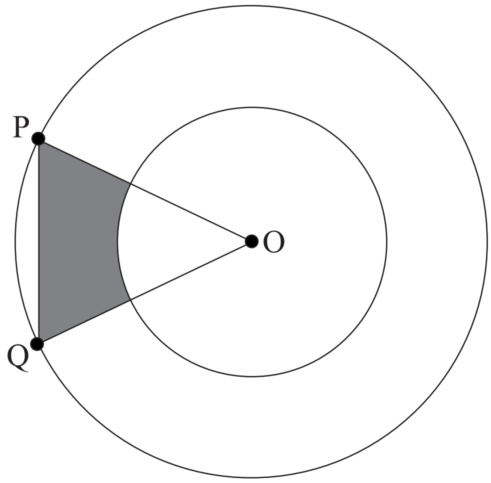

diagram not to scale

(a) Show that the area of the shaded region is $8\sin x - 2x$. *[3 marks]*

(b) Find the maximum area of the shaded region. *[4 marks]*

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4. *[Maximum mark: 6]*

When $\left(1+\dfrac{x}{2}\right)^n$, $n \in \mathbb{N}$, is expanded in ascending powers of x, the coefficient of x^3 is 70.

(a) Find the value of n. *[5 marks]*

(b) Hence, find the coefficient of x^2. *[1 mark]*

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5. *[Maximum mark: 8]*

The annual weather-related loss of an insurance company is modelled by a random variable X with probability density function

$$f(x)=\begin{cases}\dfrac{2.5(200)^{2.5}}{x^{3.5}}, & x\geq 200\\ 0, & \text{otherwise.}\end{cases}$$

Find the median.

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6. *[Maximum mark: 6]*

Tim goes to a popular restaurant that does not take any reservations for tables. It has been determined that the waiting times for a table are normally distributed with a mean of 18 minutes and standard deviation of 4 minutes.

(a) Tim says he will leave if he is not seated at a table within 25 minutes of arriving at the restaurant. Find the probability that Tim will leave without being seated. *[2 marks]*

(b) Tim has been waiting for 15 minutes. Find the probability that he will be seated within the next five minutes. *[4 marks]*

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7. *[Maximum mark: 7]*

Consider the equation $z^3 + az^2 + bz + c = 0$, where $a, b, c \in \mathbb{R}$. The points in the Argand diagram representing the three roots of the equation form the vertices of a triangle whose area is 9. Given that one root is $-1 + 3\mathrm{i}$, find

(a) the other two roots; *[4 marks]*

(b) a, b and c. *[3 marks]*

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8. *[Maximum mark: 7]*

Find the gradient of the curve $e^{xy} + \ln(y^2) + e^y = 1 + e$ at the point $(0, 1)$.

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9. *[Maximum mark: 8]*

Consider the function g, where $g(x) = \dfrac{3x}{5+x^2}$.

(a) Given that the domain of g is $x \geq a$, find the least value of a such that g has an inverse function. *[1 mark]*

(b) On the same set of axes, sketch

(i) the graph of g for this value of a;

(ii) the corresponding inverse, g^{-1}. *[4 marks]*

(c) Find an expression for $g^{-1}(x)$. *[3 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 11]*

Consider the function f, defined by $f(x) = x - a\sqrt{x}$, where $x \geq 0$, $a \in \mathbb{R}^+$.

(a) Find in terms of a

(i) the zeros of f;

(ii) the values of x for which f is decreasing;

(iii) the values of x for which f is increasing;

(iv) the range of f. *[10 marks]*

(b) State the concavity of the graph of f. *[1 mark]*

11. *[Maximum mark: 19]*

Two lines are defined by

$$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \text{ and } l_2 : \frac{x-4}{-3} = \frac{y+7}{4} = -(z+3).$$

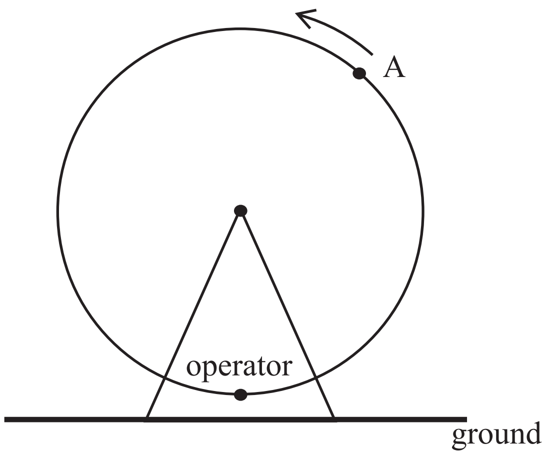
(a) Find the coordinates of the point A on l_1 and the point B on l_2 such that $\overrightarrow{AB}$ is perpendicular to both l_1 and l_2. *[13 marks]*

(b) Find $|AB|$. *[3 marks]*

(c) Find the Cartesian equation of the plane Π which contains l_1 and does not intersect l_2. *[3 marks]*

12. *[Maximum mark: 10]*

Below is a sketch of a Ferris wheel, an amusement park device carrying passengers around the rim of the wheel.



(a) The circular Ferris wheel has a radius of 10 metres and is revolving at a rate of 3 radians per minute. Determine how fast a passenger on the wheel is going vertically upwards when the passenger is at point A, 6 metres higher than the centre of the wheel, and is rising. *[7 marks]*

(b) The operator of the Ferris wheel stands directly below the centre such that the bottom of the Ferris wheel is level with his eyeline. As he watches the passenger his line of sight makes an angle α with the horizontal. Find the rate of change of α at point A. *[3 marks]*

13. *[Maximum mark: 20]*

In each round of two different games Ying tosses three fair coins and Mario tosses two fair coins.

(a) The first game consists of one round. If Ying obtains more heads than Mario, she receives \$5 from Mario. If Mario obtains more heads than Ying, he receives \$10 from Ying. If they obtain the same number of heads, then Mario receives \$2 from Ying. Determine Ying's expected winnings. *[12 marks]*

(b) They now play the second game, where the winner will be the player who obtains the larger number of heads in a round. If they obtain the same number of heads, they play another round until there is a winner. Calculate the probability that Ying wins the game. *[8 marks]*