22097209

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 3 – SETS, RELATIONS AND GROUPS

Thursday 14 May 2009 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 17]*

(a) Show that $\{1, -1, \mathrm{i}, -\mathrm{i}\}$ forms a group of complex numbers G under multiplication. *[4 marks]*

(b) Consider $S = \{e, a, b, a*b\}$ under an associative operation $*$ where e is the identity element. If $a*a = b*b = e$ and $a*b = b*a$, show that

(i) $a*b*a = b$,

(ii) $a*b*a*b = e$. *[2 marks]*

(c) (i) Write down the Cayley table for $H = \{S, *\}$.

(ii) Show that H is a group.

(iii) Show that H is an Abelian group. *[6 marks]*

(d) For the above groups, G and H, show that one is cyclic and write down why the other is not. Write down all the generators of the cyclic group. *[4 marks]*

(e) Give a reason why G and H are not isomorphic. *[1 mark]*

2. *[Maximum mark: 11]*

The binary operation $*$ is defined on $\mathbb{R}$ as follows. For any elements $a, b \in \mathbb{R}$

$$a * b = a + b + 1.$$

(a) (i) Show that $*$ is commutative.

(ii) Find the identity element.

(iii) Find the inverse of the element a. *[5 marks]*

(b) The binary operation $\bullet$ is defined on $\mathbb{R}$ as follows. For any elements $a, b \in \mathbb{R}$ $a \bullet b = 3ab$. The set S is the set of all ordered pairs (x, y) of real numbers and the binary operation $\odot$ is defined on the set S as

$$(x_1, y_1) \odot (x_2, y_2) = (x_1 * x_2, y_1 \bullet y_2).$$

Determine whether or not $\odot$ is associative. *[6 marks]*

3. *[Maximum mark: 14]*

The relation R is defined on $\mathbb{Z} \times \mathbb{Z}$ such that $(a, b) R (c, d)$ if and only if $a - c$ is divisible by 3 and $b - d$ is divisible by 2.

(a) Prove that R is an equivalence relation. *[7 marks]*

(b) Find the equivalence class for $(2, 1)$. *[2 marks]*

(c) Write down the five remaining equivalence classes. *[5 marks]*

4. *[Maximum mark: 11]*

(a) Show that $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(x, y) = (2x + y, x - y)$ is a bijection. *[10 marks]*

(b) Find the inverse of f. *[1 mark]*

5. *[Maximum mark: 7]*

Prove that set difference is not associative.