22097208

International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS

Thursday 14 May 2009 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 11]*

Find

(a) $\lim\limits_{x \to 0} \dfrac{\tan x}{x + x^2}$; *[4 marks]*

(b) $\lim\limits_{x \to 1} \dfrac{1 - x^2 + 2x^2 \ln x}{1 - \sin \dfrac{\pi x}{2}}$. *[7 marks]*

2. *[Maximum mark: 17]*

The variables x and y are related by $\dfrac{dy}{dx} - y \tan x = \cos x$.

(a) Find the Maclaurin series for y up to and including the term in x^2 given that $y = -\dfrac{\pi}{2}$ when $x = 0$. *[7 marks]*

(b) Solve the differential equation given that $y = 0$ when $x = \pi$. Give the solution in the form $y = f(x)$. *[10 marks]*

3. *[Maximum mark: 12]*

(a) Determine whether the series $\sum\limits_{n=1}^{\infty} \sin \dfrac{1}{n}$ is convergent or divergent. *[5 marks]*

(b) Show that the series $\sum\limits_{n=2}^{\infty} \dfrac{1}{n(\ln n)^2}$ is convergent. *[7 marks]*

4. *[Maximum mark: 20]*

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 + x^2}{2x^2}$ for which $y = -1$ when $x = 1$.

(a) Use Euler's method with a step length of 0.25 to find an estimate for the value of y when $x = 2$. *[7 marks]*

(b) (i) Solve the differential equation giving your answer in the form $y = f(x)$.

(ii) Find the value of y when $x = 2$. *[13 marks]*