88087206

MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Thursday 13 November 2008 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 12]*

    The hens on a farm lay either white or brown eggs. The eggs are put into boxes of six. The farmer claims that the number of brown eggs in a box can be modelled by the binomial distribution, $B(6, p)$. By inspecting the contents of 150 boxes of eggs she obtains the following data.

Number of brown eggs	0	1	2	3	4	5	6
Number of boxes	7	32	35	50	22	4	0

    (a) Show that this data leads to an estimated value of $p = 0.4$. *[1 mark]*

    (b) Stating null and alternative hypotheses, carry out an appropriate test at the 5 % level to decide whether the farmer's claim can be justified. *[11 marks]*

2. *[Maximum mark: 9]*

    The apple trees in a large orchard have, for several years, suffered from a disease for which the outward sign is a red discolouration on some leaves.

    The fruit grower knows that the mean number of discoloured leaves per tree is 42.3. The fruit grower suspects that the disease is caused by an infection from a nearby group of cedar trees. He cuts down the cedar trees and, the following year, counts the number of discoloured leaves on a random sample of seven apple trees. The results are given in the table below.

Tree	A	B	C	D	E	F	G
Number of red leaves	32	16	57	28	55	12	45

    (a) From these data calculate an unbiased estimate of the population variance. *[3 marks]*

    (b) Stating null and alternative hypotheses, carry out an appropriate test at the 10 % level to justify the cutting down of the cedar trees. *[6 marks]*

3.   *[Maximum mark: 12]*

(a)   The heating in a residential school is to be increased on the third frosty day during the term. If the probability that a day will be frosty is 0.09, what is the probability that the heating is increased on the 25th day of the term?   *[5 marks]*

(b)   On which day is the heating most likely to be increased?   *[7 marks]*

4.   *[Maximum mark: 15]*

(a)   A random variable, X, has probability density function defined by

$$f(x) = \begin{cases} 100, & \text{for } -0.005 \leq x < 0.005 \\ 0, & \text{otherwise.} \end{cases}$$

Determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.   *[5 marks]*

(b)   When a real number is rounded to two decimal places, an error is made. Show that this error can be modelled by the random variable X.   *[2 marks]*

(c)   A list contains 20 real numbers, each of which has been given to two decimal places. The numbers are then added together.

(i)   Write down bounds for the resulting error in this sum.

(ii)   Using the central limit theorem, estimate to two decimal places the probability that the absolute value of the error exceeds 0.01.

(iii)   State clearly any assumptions you have made in your calculation.   *[8 marks]*

5.   *[Maximum mark: 12]*

A drinks machine sells cans of *Ecoglug* and *Glucofizz*. The number of cans of *Ecoglug* and *Glucofizz* sold during a fixed period of time have Poisson distributions with means 8 per hour and 12 per hour, respectively.

(a)   Find the probability that in a given 15 minute period fewer than 3 cans of *Ecoglug* are sold.   *[2 marks]*

(b)   Find the probability that in a given 30 minute period more than 12 cans of drinks are sold.   *[3 marks]*

(c)   In a given 5 minute period, exactly 2 cans are sold. Find the probability that they are both *Glucofizz*.   *[7 marks]*