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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Monday 19 May 2008 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 12]*

(a) The random variable Y is such that $\mathrm{E}(2Y+3)=6$ and $\mathrm{Var}(2-3Y)=11$.

Calculate

(i) $\mathrm{E}(Y)$;

(ii) $\mathrm{Var}(Y)$;

(iii) $\mathrm{E}(Y^2)$. *[6 marks]*

(b) Independent random variables R and S are such that

$$R \sim \mathrm{N}(5, 1) \text{ and } S \sim \mathrm{N}(8, 2).$$

The random variable V is defined by $V = 3S - 4R$.

Calculate $\mathrm{P}(V > 5)$. *[6 marks]*

2. *[Maximum mark: 7]*

A factory makes wine glasses. The manager claims that on average 2 % of the glasses are imperfect. A random sample of 200 glasses is taken and 8 of these are found to be imperfect.

Test the manager's claim at a 1 % level of significance using a one-tailed test.

3. *[Maximum mark: 11]*

A teacher wants to determine whether practice sessions improve the ability to memorize digits.

He tests a group of 12 children to discover how many digits of a twelve-digit number could be repeated from memory after hearing them once. He gives them test 1, and following a series of practice sessions, he gives them test 2 one week later. The results are shown in the table below.

Child	A	B	C	D	E	F	G	H	I	J	K	L
Number of digits remembered on test 1	4	6	4	7	8	5	6	7	6	8	4	7
Number of digits remembered on test 2	7	8	5	5	10	7	7	10	8	6	3	9

(a) State appropriate null and alternative hypotheses. *[2 marks]*

(b) Test at the 5 % significance level whether or not practice sessions improve ability to memorize digits, justifying your choice of test. *[9 marks]*

4. *[Maximum mark: 14]*

The number of telephone calls received by a helpline over 80 one-minute periods are summarized in the table below.

Number of calls	0	1	2	3	4	5	6
Frequency	9	12	22	10	11	8	8

(a) Find the exact value of the mean of this distribution. *[2 marks]*

(b) Test, at the 5 % level of significance, whether or not the data can be modelled by a Poisson distribution. *[12 marks]*

5. *[Maximum mark: 16]*

A population is known to have a normal distribution with a variance of 3 and an unknown mean μ. It is proposed to test the hypotheses $H_0 : \mu = 13$, $H_1 : \mu > 13$ using the mean of a sample of size 2.

(a) Find the appropriate critical regions corresponding to a significance level of

(i) 0.05;

(ii) 0.01. *[8 marks]*

(b) Given that the true population mean is 15.2, calculate the probability of making a Type II error when the level of significance is

(i) 0.05;

(ii) 0.01. *[6 marks]*

(c) How is the change in the probability of a Type I error related to the change in the probability of a Type II error? *[2 marks]*