22087212

MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Monday 19 May 2008 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 8]*

A coin was tossed 200 times and 115 of these tosses resulted in 'heads'. Use a two-tailed test with significance level 1 % to investigate whether or not the coin is biased.

2. *[Maximum mark: 14]*

The following frequency table summarises the lifetimes, x hours, of 1000 electrical components of a certain type.

Lifetime (hours)	$0 \le x < 100$	$100 \le x < 200$	$200 \le x < 300$	$300 \le x < 400$	$400 \le x < 500$	$x \ge 500$
Frequency	622	225	96	32	13	12

Stating your hypotheses, use an appropriate test to determine, at the 5 % significance level, whether or not these data can be modelled by an exponential distribution with mean 100 hours.

3. *[Maximum mark: 14]*

A shop sells apples and pears. The weights, in grams, of the apples may be assumed to have a $N(200, 15^2)$ distribution and the weights of the pears, in grams, may be assumed to have a $N(120, 10^2)$ distribution.

(a) Find the probability that the weight of a randomly chosen apple is more than double the weight of a randomly chosen pear. *[8 marks]*

(b) A shopper buys 3 apples and 4 pears. Find the probability that the total weight is greater than 1000 grams. *[6 marks]*

4.   *[Maximum mark: 14]*

The random variable X is normally distributed with unknown mean μ and unknown variance σ^2. A random sample of 10 observations on X was taken and the following 95 % confidence interval for μ was correctly calculated as $[4.35, 4.53]$.

(a)   Calculate an unbiased estimate for

(i) μ,

(ii) σ^2.   *[8 marks]*

(b)   The value of μ is thought to be 4.5, so the following hypotheses are defined.

$$H_0 : \mu = 4.5; \; H_1 : \mu < 4.5$$

(i)   Find the *p*-value of the observed sample mean.

(ii)   State your conclusion if the significance level is

(a)   1 %,

(b)   10 %.   *[6 marks]*

5.   *[Maximum mark: 10]*

Anna has a fair cubical die with the numbers 1, 2, 3, 4, 5, 6 respectively on the six faces. When she tosses it, the score is defined as the number on the uppermost face. One day, she decides to toss the die repeatedly until all the possible scores have occurred at least once.

(a)   Having thrown the die once, she lets X_2 denote the number of additional throws required to obtain a different number from the one obtained on the first throw. State the distribution of X_2 and hence find $E(X_2)$.   *[3 marks]*

(b)   She then lets X_3 denote the number of additional throws required to obtain a different number from the two numbers already obtained. State the distribution of X_3 and hence find $E(X_3)$.   *[2 marks]*

(c)   By continuing the process, show that the expected number of tosses needed to obtain all six possible scores is 14.7.   *[5 marks]*