22087217

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 3 – SETS, RELATIONS AND GROUPS

Monday 19 May 2008 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 16]*

(a) Draw the Cayley table for the set of integers $G = \{0, 1, 2, 3, 4, 5\}$ under addition modulo 6, $+_6$. *[3 marks]*

(b) Show that $\{G, +_6\}$ is a group. *[4 marks]*

(c) Find the order of each element. *[3 marks]*

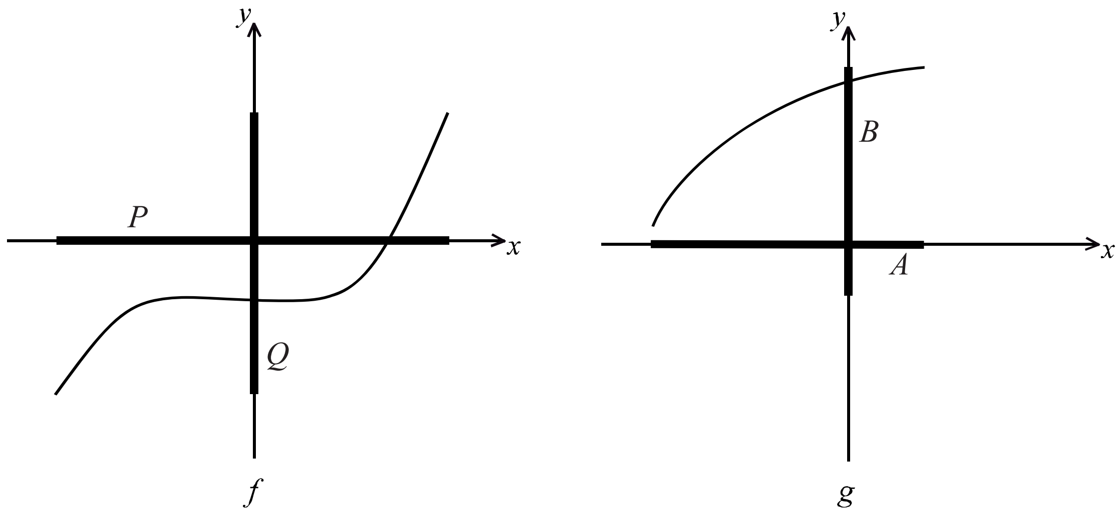
(d) Show that $\{G, +_6\}$ is cyclic and state its generators. *[2 marks]*

(e) Find a subgroup with three elements. *[2 marks]*

(f) Find the other proper subgroups of $\{G, +_6\}$. *[2 marks]*

2. *[Maximum mark: 13]*

(a) Below are the graphs of the two functions $f : P \to Q$ and $g : A \to B$.



Determine, with reference to features of the graphs, whether the functions are injective and/or surjective. *[4 marks]*

(This question continues on the following page)

(Question 2 continued)

(b) Given two functions $h: X \to Y$ and $k: Y \to Z$.

Show that

(i) if both h and k are injective then so is the composite function $k \circ h$;

(ii) if both h and k are surjective then so is the composite function $k \circ h$. *[9 marks]*

3. *[Maximum mark: 6]*

Prove that $(A \cap B) \setminus (A \cap C) = A \cap (B \setminus C)$ where A, B and C are three subsets of the universal set U.

4. *[Maximum mark: 19]*

(a) The relation aRb is defined on $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ if and only if ab is the square of a positive integer.

(i) Show that R is an equivalence relation.

(ii) Find the equivalence classes of R that contain more than one element. *[10 marks]*

(b) Given the group $(G, *)$, a subgroup $(H, *)$ and $a, b \in G$, we define $a \sim b$ if and only if $ab^{-1} \in H$. Show that $\sim$ is an equivalence relation. *[9 marks]*

5. *[Maximum mark: 6]*

(a) Write down why the table below is a Latin square. *[2 marks]*

$$\begin{array}{c} \\ d \\ e \\ b \\ a \\ c \end{array}\begin{array}{c} \begin{array}{ccccc} d & e & b & a & c \end{array} \\ \begin{bmatrix} c & d & e & b & a \\ d & e & b & a & c \\ a & b & d & c & e \\ b & a & c & e & d \\ e & c & a & d & b \end{bmatrix} \end{array}$$

(b) Use Lagrange's theorem to show that the table is not a group table. *[4 marks]*