International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 3 – DISCRETE MATHEMATICS

Monday 19 May 2008 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 7]*

Use the Euclidean Algorithm to find the greatest common divisor of 7854 and 3315.

Hence state the number of solutions to the diophantine equation $7854x + 3315y = 41$ and justify your answer.

2. *[Maximum mark: 9]*

(a) Define what is meant by the statement $x \equiv y \pmod{n}$ where x, y, $n \in \mathbb{Z}^+$. *[1 mark]*

(b) Hence prove that if $x \equiv y \pmod{n}$ then $x^2 \equiv y^2 \pmod{n}$. *[4 marks]*

(c) Determine whether or not $x^2 \equiv y^2 \pmod{n}$ implies that $x \equiv y \pmod{n}$. *[4 marks]*

3. *[Maximum mark: 11]*

The positive integer N is expressed in base p as $(a_n a_{n-1} \ldots a_1 a_0)_p$.

(a) Show that when $p = 2$, N is even if and only if its least significant digit, a_0, is 0. *[5 marks]*

(b) Show that when $p = 3$, N is even if and only if the sum of its digits is even. *[6 marks]*

4. *[Maximum mark: 16]*

The graph G has the following cost adjacency matrix.

	A	B	C	D	E
A	-	9	-	8	4
B	9	-	7	-	2
C	-	7	-	7	3
D	8	-	7	-	5
E	4	2	3	5	-

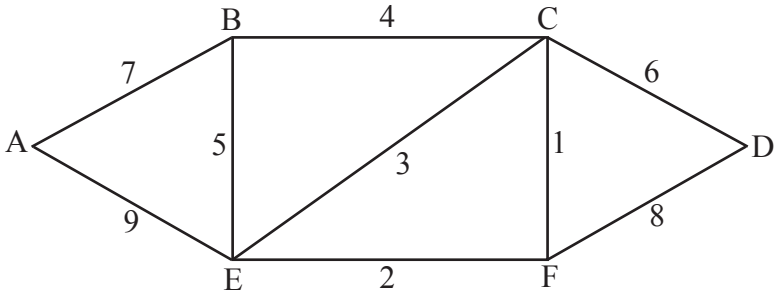
(a) Draw G in a planar form. *[2 marks]*

(b) Giving a reason, determine the maximum number of edges that could be added to G while keeping the graph both simple and planar. *[4 marks]*

(c) List all the distinct Hamiltonian cycles in G beginning and ending at A, noting that two cycles each of which is the reverse of the other are to be regarded as identical. Hence determine the Hamiltonian cycle of least weight. *[10 marks]*

5. *[Maximum mark: 17]*

(a) The weighted graph H is shown below.

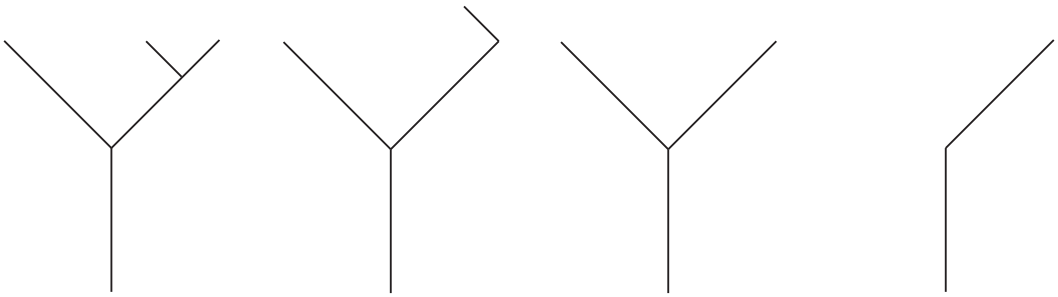


Use Kruskal's Algorithm, indicating the order in which the edges are added, to find and draw the minimum spanning tree for H. *[6 marks]*

(b) (i) A tree has v vertices. State the number of edges in the tree, justifying your answer.

(ii) We will call a graph with v vertices a "forest" if it consists of c components each of which is a tree.

Here is an example of a forest with 4 components.



How many edges will a forest with v vertices and c components have? *[5 marks]*

(c) A graph has an odd number of vertices. Prove that the degree of at least one of the vertices must be even. *[6 marks]*