22087214

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 2

Thursday 8 May 2008 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

0114

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer ***all*** *the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Consider the data set $\{k-2, k, k+1, k+4\}$, where $k \in \mathbb{R}$.

(a) Find the mean of this data set in terms of k. *[3 marks]*

Each number in the above data set is now **decreased** by 3.

(b) Find the mean of this **new** data set in terms of k. *[2 marks]*

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2. *[Maximum mark: 6]*

The depth, $h(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right),\ 0 \le t \le 24.$$

(a) Find the maximum depth and the minimum depth of the water. *[3 marks]*

(b) Find the values of t for which $h(t) \ge 8$. *[3 marks]*

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3. *[Maximum mark: 5]*

The curve $y = e^{-x} - x + 1$ intersects the x-axis at P.

(a) Find the x-coordinate of P. *[2 marks]*

(b) Find the area of the region completely enclosed by the curve and the coordinate axes. *[3 marks]*

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4. *[Maximum mark: 6]*

A continuous random variable X has probability density function

$$f(x)=\begin{cases}12x^2(1-x), & \text{for } 0\le x\le 1,\\ 0, & \text{otherwise.}\end{cases}$$

Find the probability that X lies between the mean and the mode.

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5. *[Maximum mark: 7]*

Consider triangle ABC with $B\hat{A}C = 37.8^\circ$, $AB = 8.75$ and $BC = 6$.

Find AC.

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6. *[Maximum mark: 7]*

Consider the curve with equation $f(x) = \mathrm{e}^{-2x^2}$ for $x < 0$.

Find the coordinates of the point of inflexion and justify that it is a point of inflexion.

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7. *[Maximum mark: 6]*

Over a one month period, Ava and Sven play a total of n games of tennis.

The probability that Ava wins any game is 0.4. The result of each game played is independent of any other game played.

Let X denote the number of games won by Ava over a one month period.

(a) Find an expression for $\mathrm{P}(X=2)$ in terms of n. *[3 marks]*

(b) If the probability that Ava wins two games is 0.121 correct to three decimal places, find the value of n. *[3 marks]*

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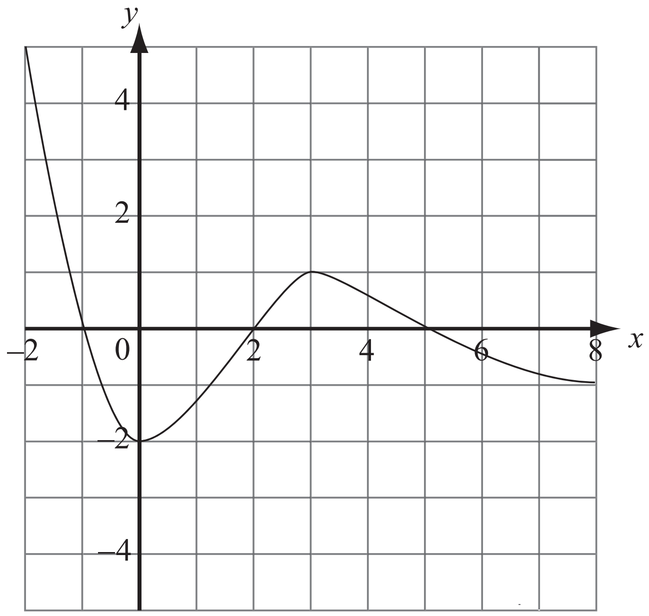
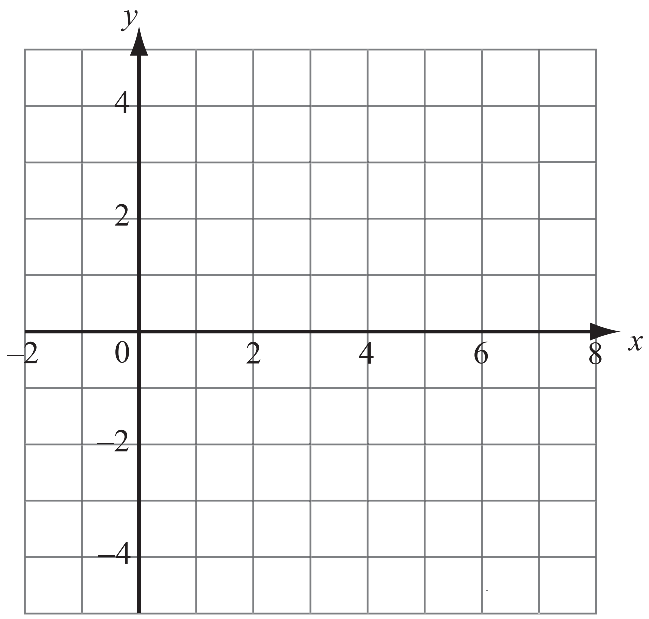
8. *[Maximum mark: 5]*

The graph of $y = f(x)$ for $-2 \le x \le 8$ is shown.



On the set of axes provided, sketch the graph of $y = \frac{1}{f(x)}$, clearly showing any asymptotes and indicating the coordinates of any local maxima or minima.



9. *[Maximum mark: 7]*

Consider $w = \dfrac{z}{z^2 + 1}$ where $z = x + \mathrm{i}y$, $y \neq 0$ and $z^2 + 1 \neq 0$.

Given that $\operatorname{Im} w = 0$, show that $|z| = 1$.

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10. *[Maximum mark: 6]*

Find the set of values of x for which $\left|0.1x^2 - 2x + 3\right| < \log_{10} x$.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 21]*

The distance travelled by students to attend Gauss College is modelled by a normal distribution with mean 6 km and standard deviation 1.5 km.

(a) (i) Find the probability that the distance travelled to Gauss College by a randomly selected student is between 4.8 km and 7.5 km.

(ii) 15 % of students travel less than d km to attend Gauss College. Find the value of d. *[7 marks]*

At Euler College, the distance travelled by students to attend their school is modelled by a normal distribution with mean μ km and standard deviation σ km.

(b) If 10 % of students travel more than 8 km and 5 % of students travel less than 2 km, find the value of μ and of σ. *[6 marks]*

The number of telephone calls, T, received by Euler College each minute can be modelled by a Poisson distribution with a mean of 3.5.

(c) (i) Find the probability that at least three telephone calls are received by Euler College in **each** of two successive one-minute intervals.

(ii) Find the probability that Euler College receives 15 telephone calls during a randomly selected five-minute interval. *[8 marks]*

12. *[Maximum mark: 20]*

Let $\boldsymbol{M}^2 = \boldsymbol{M}$ where $\boldsymbol{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $bc \neq 0$.

(a) (i) Show that $a + d = 1$.

(ii) Find an expression for bc in terms of a. *[5 marks]*

(b) **Hence** show that $\boldsymbol{M}$ is a singular matrix. *[3 marks]*

(c) If all of the elements of $\boldsymbol{M}$ are positive, find the range of possible values for a. *[3 marks]*

(d) Show that $(\boldsymbol{I} - \boldsymbol{M})^2 = \boldsymbol{I} - \boldsymbol{M}$ where $\boldsymbol{I}$ is the identity matrix. *[3 marks]*

(e) Prove by mathematical induction that $(\boldsymbol{I} - \boldsymbol{M})^n = \boldsymbol{I} - \boldsymbol{M}$ for $n \in \mathbb{Z}^+$. *[6 marks]*

13. *[Maximum mark: 19]*

A particle moves in a straight line in a positive direction from a fixed point O.

The velocity $v\ \text{m s}^{-1}$, at time t seconds, where $t \geq 0$, satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{-v(1+v^2)}{50}.$$

The particle starts from O with an initial velocity of $10\ \text{m s}^{-1}$.

(a) (i) Express as a definite integral, the time taken for the particle's velocity to decrease from $10\ \text{m s}^{-1}$ to $5\ \text{m s}^{-1}$.

(ii) **Hence** calculate the time taken for the particle's velocity to decrease from $10\ \text{m s}^{-1}$ to $5\ \text{m s}^{-1}$. *[5 marks]*

(b) (i) Show that, when $v > 0$, the motion of this particle can also be described by the differential equation $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{-(1+v^2)}{50}$ where x metres is the displacement from O.

(ii) Given that $v = 10$ when $x = 0$, solve the differential equation expressing x in terms of v.

(iii) **Hence** show that $v = \dfrac{10 - \tan\frac{x}{50}}{1 + 10\tan\frac{x}{50}}$. *[14 marks]*