M08/5/MATHL/HP1/ENG/TZ2/XX/M

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

May 2008

MATHEMATICS

Higher Level

Paper 1

18 pages

*This markscheme is **confidential** and for the exclusive use of examiners in this examination session.*

*It is the property of the International Baccalaureate and must **not** be reproduced or distributed to any other person without the authorization of IB Cardiff.*

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**: often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent** ***A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc.*
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

  Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

  $$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

  Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

*Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**. Award the marks as usual then write **(AP)** against the answer. On the **front** cover write –1**(AP).** Deduct 1 mark from the total for the paper, not the question*.

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

SECTION A

1. (a) Using $\sum \mathrm{P}(X = x) = 1$ *(M1)*

$4c + 6c + 6c + 4c = 1 \quad (20c = 1)$ ***A1***

$c = \frac{1}{20} \quad (= 0.05)$ ***A1*** ***N1***

(b) Using $\mathrm{E}(X) = \sum x\mathrm{P}(X = x)$ *(M1)*

$= (1\times 0.2) + (2\times 0.3) + (3\times 0.3) + (4\times 0.2)$ *(A1)*

$= 2.5$ ***A1*** ***N1***

Notes:	Only one of the first two marks can be implied. Award ***M1A1A1*** if the *x* values are averaged only if symmetry is explicitly mentioned.

[6 marks]

2. **METHOD 1**

As $(x+1)$ is a factor of $P(x)$, then $P(-1) = 0$ *(M1)*

$\Rightarrow a - b + 1 = 0$ (or equivalent) ***A1***

As $(x-2)$ is a factor of $P(x)$, then $P(2) = 0$ *(M1)*

$\Rightarrow 4a + 2b + 10 = 0$ (or equivalent) ***A1***

Attempting to solve for *a* and *b* ***M1***

$a = -2$ and $b = -1$ ***A1*** ***N1***

[6 marks]

METHOD 2

By inspection third factor must be $x - 1$. *(M1)**A1***

$(x+1)(x-2)(x-1) = x^3 - 2x^2 - x + 2$ *(M1)**A1***

Equating coefficients $a = -2$, $b = -1$ *(M1)**A1*** ***N1***

[6 marks]

METHOD 3

Considering $\frac{P(x)}{x^2 - x - 2}$ or equivalent *(M1)*

$$\frac{P(x)}{x^2 - x - 2} = (x + a + 1) + \frac{(a + b + 3)x + 2(a + 2)}{x^2 - x - 2}$$ ***A1A1***

Recognising that $(a + b + 3)x + 2(a + 2) = 0$ *(M1)*

Attempting to solve for *a* and *b* ***M1***

$a = -2$ and $b = -1$ ***A1*** ***N1***

[6 marks]

3. METHOD 1

$\text{AC} = 5$ and $\text{AB} = \sqrt{13}$ (may be seen on diagram) ***(A1)***

$\cos\alpha = \frac{3}{5}$ **and** $\sin\alpha = \frac{4}{5}$ ***(A1)***

$\cos\beta = \frac{3}{\sqrt{13}}$ **and** $\sin\beta = \frac{2}{\sqrt{13}}$ ***(A1)***

Note: If only the two cosines are correctly given award ***(A1)(A1)(A0)***.

Use of $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ ***(M1)***

$$= \frac{3}{5} \times \frac{3}{\sqrt{13}} + \frac{4}{5} \times \frac{2}{\sqrt{13}} \quad \text{(substituting)}$$ ***M1***

$$= \frac{17}{5\sqrt{13}} \quad \left(= \frac{17\sqrt{13}}{65}\right)$$ ***A1 N1***

[6 marks]

METHOD 2

$\text{AC} = 5$ and $\text{AB} = \sqrt{13}$ (may be seen on diagram) ***(A1)***

Use of $\cos(\alpha + \beta) = \frac{\text{AC}^2 + \text{AB}^2 - \text{BC}^2}{2(\text{AC})(\text{AB})}$ ***(M1)***

$$= \frac{25 + 13 - 36}{2 \times 5 \times \sqrt{13}} \quad \left(= \frac{1}{5\sqrt{13}}\right)$$ ***A1***

Use of $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$ ***(M1)***

$\cos\alpha = \frac{3}{5}$ **and** $\cos\beta = \frac{3}{\sqrt{13}}$ ***(A1)***

$$\cos(\alpha - \beta) = \frac{17}{5\sqrt{13}} \quad \left(= 2 \times \frac{3}{5} \times \frac{3}{\sqrt{13}} - \frac{1}{5\sqrt{13}}\right) \left(= \frac{17\sqrt{13}}{65}\right)$$ ***A1 N1***

[6 marks]

4. (a) $h(x) = g\left(\dfrac{4}{x+2}\right)$ ***(M1)***

$= \dfrac{4}{x+2} - 1 \quad \left(= \dfrac{2-x}{2+x}\right)$ ***A1***

(b) **METHOD 1**

$x = \dfrac{4}{y+2} - 1$ (interchanging x and y) ***M1***

Attempting to solve for y ***M1***

$(y+2)(x+1) = 4 \quad \left(y+2 = \dfrac{4}{x+1}\right)$ ***(A1)***

$h^{-1}(x) = \dfrac{4}{x+1} - 2 \quad (x \neq -1)$ ***A1*** ***N1***

METHOD 2

$x = \dfrac{2-y}{2+y}$ (interchanging x and y) ***M1***

Attempting to solve for y ***M1***

$xy + y = 2 - 2x \quad \left(y(x+1) = 2(1-x)\right)$ ***(A1)***

$h^{-1}(x) = \dfrac{2(1-x)}{x+1} \quad (x \neq -1)$ ***A1*** ***N1***

Note:	In either **METHOD 1** or **METHOD 2** rearranging first and interchanging afterwards is equally acceptable.

[6 marks]

5. (a) Attempting implicit differentiation ***M1***

$$2x+y+x\frac{\mathrm{d}y}{\mathrm{d}x}+2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$$ ***A1***

EITHER

Substituting $x=-1,\ y=k$ *e.g.* $-2+k-\frac{\mathrm{d}y}{\mathrm{d}x}+2k\frac{\mathrm{d}y}{\mathrm{d}x}=0$ ***M1***

Attempting to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject ***M1***

OR

Attempting to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject *e.g.* $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-(2x+y)}{x+2y}$ ***M1***

Substituting $x=-1,\ y=k$ into $\frac{\mathrm{d}y}{\mathrm{d}x}$ ***M1***

THEN

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2-k}{2k-1}$$ ***A1*** ***N1***

(b) Solving $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ for k gives $k=2$ ***A1***

[6 marks]

6. Using integration by parts ***(M1)***

$u=x,\ \frac{\mathrm{d}u}{\mathrm{d}x}=1,\ \frac{\mathrm{d}v}{\mathrm{d}x}=\sin 2x$ and $v=-\frac{1}{2}\cos 2x$ ***(A1)***

$$\left[x\left(-\frac{1}{2}\cos 2x\right)\right]_0^{\frac{\pi}{6}}-\int_0^{\frac{\pi}{6}}\left(-\frac{1}{2}\cos 2x\right)\mathrm{d}x$$ ***A1***

$$=\left[x\left(-\frac{1}{2}\cos 2x\right)\right]_0^{\frac{\pi}{6}}+\left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$$ ***A1***

Note:	Award the ***A1A1*** above if the limits are not included.

$$\left[x\left(-\frac{1}{2}\cos 2x\right)\right]_0^{\frac{\pi}{6}}=-\frac{\pi}{24}$$ ***A1***

$$\left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}=\frac{\sqrt{3}}{8}$$ ***A1***

$$\int_0^{\frac{\pi}{6}}x\sin 2x\,\mathrm{d}x=\frac{\sqrt{3}}{8}-\frac{\pi}{24}$$ ***AG*** ***N0***

Note:	Allow ***FT*** on the last two ***A1*** marks if the expressions are the negative of the correct ones.

[6 marks]

7. **EITHER**

Using $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ ***(M1)***

$0.6\mathrm{P}(B) = \mathrm{P}(A \cap B)$ ***A1***

Using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ to obtain $0.8 = 0.6 + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***A1***

Substituting $0.6\mathrm{P}(B) = \mathrm{P}(A \cap B)$ into above equation ***M1***

OR

As $\mathrm{P}(A|B) = \mathrm{P}(A)$ then *A* and *B* are independent events ***M1R1***

Using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A) \times \mathrm{P}(B)$ ***A1***

to obtain $0.8 = 0.6 + \mathrm{P}(B) - 0.6 \times \mathrm{P}(B)$ ***A1***

THEN

$0.8 = 0.6 + 0.4\mathrm{P}(B)$ ***A1***

$\mathrm{P}(B) = 0.5$ ***A1*** ***N1***

[6 marks]

8. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\arctan(x-1)\right) = \dfrac{1}{1+(x-1)^2}$ (or equivalent) ***A1***

$m_N = -2$ and so $m_T = \dfrac{1}{2}$ ***(R1)***

Attempting to solve $\dfrac{1}{1+(x-1)^2} = \dfrac{1}{2}$ (or equivalent) for *x* ***M1***

$x = 2$ (as $x > 0$) ***A1***

Substituting $x = 2$ **and** $y = \dfrac{\pi}{4}$ to find *c* ***M1***

$c = 4 + \dfrac{\pi}{4}$ ***A1*** ***N1***

[6 marks]

9. 10 cm water depth corresponds to $16\sec\left(\frac{\pi x}{36}\right)-32=-6$ ***(A1)***

Rearranging to obtain an equation of the form $\sec\left(\frac{\pi x}{36}\right)=k$ or equivalent

i.e. making a trignometrical function the subject of the equation. ***M1***

$\cos\left(\frac{\pi x}{36}\right)=\frac{8}{13}$ ***(A1)***

$\frac{\pi x}{36}=\pm\arccos\frac{8}{13}$ ***M1***

$x=\pm\frac{36}{\pi}\arccos\frac{8}{13}$ ***A1***

Note: Do not penalise the omission of $\pm$.

Width of water surface is $\frac{72}{\pi}\arccos\frac{8}{13}$ (cm) ***R1*** ***N1***

Note: Candidate who starts with 10 instead of -6 has the potential to gain the two ***M1*** marks and the ***R1*** mark.

[6 marks]

10. **METHOD 1**

Use of $|\boldsymbol{a}\times\boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}|\sin\theta$ ***(M1)***

$|\boldsymbol{a}\times\boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2\sin^2\theta$ ***(A1)***

Note: Only one of the first two marks can be implied.

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2(1-\cos^2\theta)$ ***A1***

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2|\boldsymbol{b}|^2\cos^2\theta$ ***(A1)***

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - \left(|\boldsymbol{a}||\boldsymbol{b}|\cos\theta\right)^2$ ***(A1)***

Note: Only one of the above two ***A1*** marks can be implied.

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\bullet\boldsymbol{b})^2$ ***A1***

Hence LHS = RHS ***AG*** ***N0***

[6 marks]

METHOD 2

Use of $\boldsymbol{a}\bullet\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$ ***(M1)***

$|\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\bullet\boldsymbol{b})^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - \left(|\boldsymbol{a}||\boldsymbol{b}|\cos\theta\right)^2$ ***(A1)***

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2|\boldsymbol{b}|^2\cos^2\theta$ ***(A1)***

Note: Only one of the above two ***A1*** marks can be implied.

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2(1-\cos^2\theta)$ ***A1***

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2\sin^2\theta$ ***A1***

$= |\boldsymbol{a}\times\boldsymbol{b}|^2$ ***A1***

Hence $\text{LHS} = \text{RHS}$ ***AG*** ***N0***

Notes: Candidates who independently correctly simplify both sides and show that $\text{LHS} = \text{RHS}$ should be awarded full marks.

If the candidate starts off with expression that they are trying to prove and concludes that $\sin^2\theta = (1-\cos^2\theta)$ award ***M1A1A1A1A0A0***.

If the candidate uses two general 3D vectors and explicitly finds the expressions correctly award full marks. Use of 2D vectors gains a maximum of 2 marks.

If two specific vectors are used no marks are gained.

[6 marks]

SECTION B

11. (a) Use of $\cos\theta = \dfrac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{AB}\right|}$ ***(M1)***

$\overrightarrow{AB} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ ***A1***

$\left|\overrightarrow{AB}\right| = \sqrt{3}$ **and** $\left|\overrightarrow{OA}\right| = 3\sqrt{2}$ ***A1***

$\overrightarrow{OA} \bullet \overrightarrow{AB} = 6$ ***A1***

substituting gives $\cos\theta = \dfrac{2}{\sqrt{6}}$ $\left(= \dfrac{\sqrt{6}}{3}\right)$ or equivalent ***M1*** ***N1***

[5 marks]

(b) $L_1 : \boldsymbol{r} = \overrightarrow{OA} + s\overrightarrow{AB}$ or equivalent ***(M1)***

$L_1 : \boldsymbol{r} = \boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k} + s(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$ or equivalent ***A1***

Note: Award ***(M1)A0*** for omitting "$\boldsymbol{r} =$" in the final answer.

[2 marks]

(c) Equating components and forming equations involving s and t ***(M1)***

$1 + s = 2 + 2t$, $-1 - s = 4 + t$, $4 + s = 7 + 3t$

Having two of the above three equations ***A1A1***

Attempting to solve for s or t ***(M1)***

Finding either $s = -3$ or $t = -2$ ***A1***

Explicitly showing that these values satisfy the third equation ***R1***

Point of intersection is $(-2, 2, 1)$ ***A1*** ***N1***

Note: Position vector is not acceptable for final ***A1***.

[7 marks]

continued …

Question 11 continued

(d) **METHOD 1**

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 3 \\ -3 \end{pmatrix}$$ ***(A1)***

$x = 1 + 2\lambda - 3\mu$, $y = -1 + \lambda + 3\mu$ and $z = 4 + 3\lambda - 3\mu$ ***M1A1***

Elimination of the parameters ***M1***

$x + y = 3\lambda$ so $4(x + y) = 12\lambda$ and $y + z = 4\lambda + 3$ so $3(y + z) = 12\lambda + 9$

$3(y + z) = 4(x + y) + 9$ ***A1***

Cartesian equation of plane is $4x + y - 3z = -9$ (or equivalent) ***A1*** ***N1***

[6 marks]

METHOD 2

EITHER

The point $(2, 4, 7)$ lies on the plane.

The vector joining $(2, 4, 7)$ and $(1, -1, 4)$ and $2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ are parallel to the plane. So they are perpendicular to the normal to the plane.

$(\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) - (2\boldsymbol{i} + 4\boldsymbol{j} + 7\boldsymbol{k}) = -\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{k}$ ***(A1)***

$$\boldsymbol{n} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -1 & -5 & -3 \\ 2 & 1 & 3 \end{vmatrix}$$ ***M1***

$= -12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k}$ or equivalent parallel vector ***A1***

OR

L_1 and L_2 intersect at D$(-2, 2, 1)$

$\overrightarrow{\text{AD}} = (-2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}) - (\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) = -3\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}$ ***(A1)***

$$\boldsymbol{n} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 1 & 3 \\ -3 & 3 & -3 \end{vmatrix}$$ ***M1***

$= -12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k}$ or equivalent parallel vector ***A1***

THEN

$\boldsymbol{r} \bullet \boldsymbol{n} = (\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) \bullet (-12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k})$ ***M1***

$= 27$ ***A1***

Cartesian equation of plane is $4x + y - 3z = -9$ (or equivalent) ***A1*** ***N1***

[6 marks]

Total [20 marks]

12. (a) $r = -\frac{1}{3}$ ***(A1)***

$S_\infty = \frac{27}{1+\frac{1}{3}}$ ***M1***

$S_\infty = \frac{81}{4} \quad (= 20.25)$ ***A1*** ***N1***

[3 marks]

(b) Attempting to show that the result is true for $n = 1$ ***M1***

$\text{LHS} = a$ **and** $\text{RHS} = \frac{a(1-r)}{1-r} = a$ ***A1***

Hence the result is true for $n = 1$
Assume it is true for $n = k$

$a + ar + ar^2 + \ldots + ar^{k-1} = \frac{a(1-r^k)}{1-r}$ ***M1***

Consider $n = k + 1$:

$a + ar + ar^2 + \ldots + ar^{k-1} + ar^k = \frac{a(1-r^k)}{1-r} + ar^k$ ***M1***

$= \frac{a(1-r^k) + ar^k(1-r)}{1-r}$

$= \frac{a - ar^k + ar^k - ar^{k+1}}{1-r}$ ***A1***

Note:	Award ***A1*** for an equivalent correct intermediate step.

$= \frac{a - ar^{k+1}}{1-r}$

$= \frac{a(1-r^{k+1})}{1-r}$ ***A1***

Note:	Illogical attempted proofs that use the result to be proved would gain ***M1A0A0*** for the last three above marks.

The result is true for $n = k \Rightarrow$ it is true for $n = k + 1$ **and** as it is true for $n = 1$, the result is proved by mathematical induction. ***R1*** ***N0***

Note:	To obtain the final ***R1*** mark a reasonable attempt must have been made to prove the $k + 1$ step.

[7 marks]

Total [10 marks]

13. (a) $\text{AQ} = \sqrt{x^2+4}$ (km) ***(A1)***

$\text{QY} = (2-x)$ (km) ***(A1)***

$T = 5\sqrt{5}\text{AQ} + 5\text{QY}$ ***(M1)***

$= 5\sqrt{5}\sqrt{(x^2+4)} + 5(2-x)$ (mins) ***A1***

[4 marks]

(b) Attempting to use the chain rule on $5\sqrt{5}\sqrt{(x^2+4)}$ ***(M1)***

$\frac{\mathrm{d}}{\mathrm{d}x}\left(5\sqrt{5}\sqrt{(x^2+4)}\right) = 5\sqrt{5} \times \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \times 2x$ ***A1***

$\left(= \frac{5\sqrt{5}x}{\sqrt{x^2+4}}\right)$

$\frac{\mathrm{d}}{\mathrm{d}x}\left(5(2-x)\right) = -5$ ***A1***

$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{5\sqrt{5}x}{\sqrt{x^2+4}} - 5$ ***AG*** ***N0***

[3 marks]

(c) (i) $\sqrt{5}\,x = \sqrt{x^2+4}$ or equivalent ***A1***

Squaring both sides and rearranging to obtain $5x^2 = x^2 + 4$ ***M1***

$x = 1$ ***A1*** ***N1***

Note: Do not award the final ***A1*** for stating a negative solution in final answer.

(ii) $T = 5\sqrt{5}\sqrt{1+4} + 5(2-1)$ ***M1***

$= 30$ (mins) ***A1*** ***N1***

Note: Allow ***FT*** on incorrect x value.

continued …

Question 13 (c) continued

(iii) **METHOD 1**

Attempting to use the quotient rule **M1**

$u = x$, $v = \sqrt{x^2+4}$, $\dfrac{\mathrm{d}u}{\mathrm{d}x} = 1$ and $\dfrac{\mathrm{d}v}{\mathrm{d}x} = x(x^2+4)^{-1/2}$ ***(A1)***

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} = 5\sqrt{5}\left[\frac{\sqrt{x^2+4} - \frac{1}{2}(x^2+4)^{-1/2} \times 2x^2}{(x^2+4)}\right]$$ ***A1***

Attempt to simplify ***(M1)***

$$= \frac{5\sqrt{5}}{(x^2+4)^{3/2}}\left[x^2+4-x^2\right] \text{ or equivalent}$$ ***A1***

$$= \frac{20\sqrt{5}}{(x^2+4)^{3/2}}$$ ***AG***

When $x = 1$, $\dfrac{20\sqrt{5}}{(x^2+4)^{3/2}} > 0$ and hence $T = 30$ is a minimum ***R1*** ***N0***

Note: Allow ***FT*** on incorrect x value, $0 \le x \le 2$.

METHOD 2

Attempting to use the product rule ***M1***

$u = x$, $v = \sqrt{x^2+4}$, $\dfrac{\mathrm{d}u}{\mathrm{d}x} = 1$ and $\dfrac{\mathrm{d}v}{\mathrm{d}x} = x(x^2+4)^{-1/2}$ ***(A1)***

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} = 5\sqrt{5}(x^2+4)^{-1/2} - \frac{5\sqrt{5}\,x}{2}(x^2+4)^{-3/2} \times 2x$$ ***A1***

$$\left(= \frac{5\sqrt{5}}{(x^2+4)^{1/2}} - \frac{5\sqrt{5}\,x^2}{(x^2+4)^{3/2}}\right)$$

Attempt to simplify ***(M1)***

$$= \frac{5\sqrt{5}(x^2+4) - 5\sqrt{5}\,x^2}{(x^2+4)^{3/2}} \qquad \left(= \frac{5\sqrt{5}(x^2+4-x^2)}{(x^2+4)^{3/2}}\right)$$ ***A1***

$$= \frac{20\sqrt{5}}{(x^2+4)^{3/2}}$$ ***AG***

When $x = 1$, $\dfrac{20\sqrt{5}}{(x^2+4)^{3/2}} > 0$ and hence $T = 30$ is a minimum ***R1*** ***N0***

Note: Allow ***FT*** on incorrect x value, $0 \le x \le 2$.

[11 marks]

Total [18 marks]

14. (a) **EITHER**

$$w^5 = \left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)^5$$ ***(M1)***

$= \cos 2\pi + \mathrm{i}\sin 2\pi$ ***A1***

$= 1$ ***A1***

Hence w is a root of $z^5 - 1 = 0$ ***AG***

OR

Solving $z^5 = 1$ ***(M1)***

$z = \cos\frac{2\pi}{5}n + \mathrm{i}\sin\frac{2\pi}{5}n, \quad n = 0, 1, 2, 3, 4.$ ***A1***

$n = 1$ gives $\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}$ which is w ***A1***

[3 marks]

(b) $(w-1)(1+w+w^2+w^3+w^4) = w + w^2 + w^3 + w^4 + w^5 - 1 - w - w^2 - w^3 - w^4$ ***M1***

$= w^5 - 1$ ***A1***

Since $w^5 - 1 = 0$ **and** $w \neq 1$, $w^4 + w^3 + w^2 + w + 1 = 0$. ***R1***

[3 marks]

(c) $1 + w + w^2 + w^3 + w^4 =$

$$1 + \cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5} + \left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)^2 + \left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)^3 + \left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)^4$$

(M1)

$$= 1 + \cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5} + \cos\frac{4\pi}{5} + \mathrm{i}\sin\frac{4\pi}{5} + \cos\frac{6\pi}{5} + \mathrm{i}\sin\frac{6\pi}{5} + \cos\frac{8\pi}{5} + \mathrm{i}\sin\frac{8\pi}{5}$$ ***M1***

$$= 1 + \cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5} + \cos\frac{4\pi}{5} + \mathrm{i}\sin\frac{4\pi}{5} + \cos\frac{4\pi}{5} - \mathrm{i}\sin\frac{4\pi}{5} + \cos\frac{2\pi}{5} - \mathrm{i}\sin\frac{2\pi}{5}$$ ***M1A1A1***

Note:	Award ***M1*** for attempting to replace 6π and 8π by 4π and 2π. Award ***A1*** for correct cosine terms and ***A1*** for correct sine terms.

$$= 1 + 2\cos\frac{4\pi}{5} + 2\cos\frac{2\pi}{5} = 0$$ ***A1***

Note:	Correct methods involving equating real parts, use of conjugates or reciprocals are also accepted.

$$\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$$ ***AG***

[6 marks]

Note:	Use of cis notation is acceptable throughout this question.

Total [12 marks]