22087207

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 1

Wednesday 7 May 2008 (afternoon)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Express $\dfrac{1}{(1-\mathrm{i}\sqrt{3})^3}$ in the form $\dfrac{a}{b}$ where $a, b \in \mathbb{Z}$.

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2. *[Maximum mark: 6]*

Let $\boldsymbol{M}$ be the matrix $\begin{pmatrix} \alpha & 2\alpha & 0 \\ 0 & \alpha & 1 \\ -1 & -1 & \alpha \end{pmatrix}$.

Find all the values of α for which $\boldsymbol{M}$ is singular.

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3. *[Maximum mark: 5]*

A circular disc is cut into twelve sectors whose areas are in an arithmetic sequence. The angle of the largest sector is twice the angle of the smallest sector.

Find the size of the angle of the smallest sector.

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4. *[Maximum mark: 5]*

In triangle ABC, $\text{AB} = 9\,\text{cm}$, $\text{AC} = 12\,\text{cm}$, and $\hat{B}$ is twice the size of $\hat{C}$.

Find the cosine of $\hat{C}$.

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5. *[Maximum mark: 5]*

If $f(x) = x - 3x^{\frac{2}{3}}$, $x > 0$,

(a) find the x-coordinate of the point P where $f'(x) = 0$; *[2 marks]*

(b) determine whether P is a maximum or minimum point. *[3 marks]*

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6. *[Maximum mark: 7]*

Find the area between the curves $y = 2 + x - x^2$ and $y = 2 - 3x + x^2$.

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7. *[Maximum mark: 6]*

The common ratio of the terms in a geometric series is 2^x.

(a) State the set of values of x for which the sum to infinity of the series exists. *[2 marks]*

(b) If the first term of the series is 35, find the value of x for which the sum to infinity is 40. *[4 marks]*

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8. *[Maximum mark: 8]*

The functions f and g are defined as:

$$f(x) = \mathrm{e}^{x^2}, \ x \geq 0$$
$$g(x) = \frac{1}{x+3}, \ x \neq -3.$$

(a) Find $h(x)$ where $h(x) = g \circ f(x)$. *[2 marks]*

(b) State the domain of $h^{-1}(x)$. *[2 marks]*

(c) Find $h^{-1}(x)$. *[4 marks]*

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9. *[Maximum mark: 7]*

The random variable T has the probability density function

$$f(t) = \frac{\pi}{4}\cos\left(\frac{\pi t}{2}\right), -1 \le t \le 1 .$$

Find

(a) $\mathrm{P}(T = 0)$; *[2 marks]*

(b) the interquartile range. *[5 marks]*

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10. *[Maximum mark: 6]*

The region bounded by the curve $y = \frac{\ln(x)}{x}$ and the lines $x = 1$, $x = \mathrm{e}$, $y = 0$ is rotated through 2π radians about the x-axis.

Find the volume of the solid generated.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 20]*

The points A, B, C have position vectors $\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k}$, $\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k}$, $3\boldsymbol{i}+\boldsymbol{k}$ respectively and lie in the plane π.

(a) Find

(i) the area of the triangle ABC;

(ii) the shortest distance from C to the line AB;

(iii) the cartesian equation of the plane π. *[14 marks]*

The line L passes through the origin and is normal to the plane π, it intersects π at the point D.

(b) Find

(i) the coordinates of the point D;

(ii) the distance of π from the origin. *[6 marks]*

12. *[Maximum mark: 27]*

The function f is defined by $f(x) = x\,\mathrm{e}^{2x}$.

It can be shown that $f^{(n)}(x) = (2^n x + n\,2^{n-1})\,\mathrm{e}^{2x}$ for all $n \in \mathbb{Z}^+$, where $f^{(n)}(x)$ represents the n^{th} derivative of $f(x)$.

(a) By considering $f^{(n)}(x)$ for $n = 1$ and $n = 2$, show that there is one minimum point P on the graph of f, and find the coordinates of P. *[7 marks]*

(b) Show that f has a point of inflexion Q at $x = -1$. *[5 marks]*

(c) Determine the intervals on the domain of f where f is

(i) concave up;

(ii) concave down. *[2 marks]*

(d) Sketch f, clearly showing any intercepts, asymptotes and the points P and Q. *[4 marks]*

(e) Use mathematical induction to prove that $f^{(n)}(x) = (2^n x + n\,2^{n-1})\,\mathrm{e}^{2x}$ for all $n \in \mathbb{Z}^+$, where $f^{(n)}(x)$ represents the n^{th} derivative of $f(x)$. *[9 marks]*

13. *[Maximum mark: 13]*

A gourmet chef is renowned for her spherical shaped soufflé. Once it is put in the oven, its volume increases at a rate proportional to its radius.

(a) Show that the radius r cm of the soufflé, at time t minutes after it has been put in the oven, satisfies the differential equation $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r}$, where k is a constant. *[5 marks]*

(b) Given that the radius of the soufflé is 8 cm when it goes in the oven, and 12 cm when it's cooked 30 minutes later, find, to the nearest cm, its radius after 15 minutes in the oven. *[8 marks]*