IBO

IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICS
HIGHER LEVEL
PAPER 1

Monday 5 November 2007 (afternoon)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

1. *[Maximum mark: 6]*

Given that $(x-2)$ and $(x+2)$ are factors of $f(x)=x^3+px^2+qx+4$, find the value of p and of q.

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2. *[Maximum mark: 6]*

Find the coefficient of the x^3 term in the expansion of $\left(2-\frac{3x}{2}\right)^6$.

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3. *[Maximum mark: 6]*

A sample of discrete data is drawn from a population and given as

66, 72, 65, 70, 69, 73, 65, 71, 75.

Find

(a) the interquartile range; *[2 marks]*

(b) an estimate for the mean of the population; *[2 marks]*

(c) an unbiased estimate of the variance of the population. *[2 marks]*

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4. *[Maximum mark: 6]*

The first and fourth terms of a geometric series are 18 and $-\frac{2}{3}$ respectively.

Find

(a) the sum of the first n terms of the series; *[4 marks]*

(b) the sum to infinity of the series. *[2 marks]*

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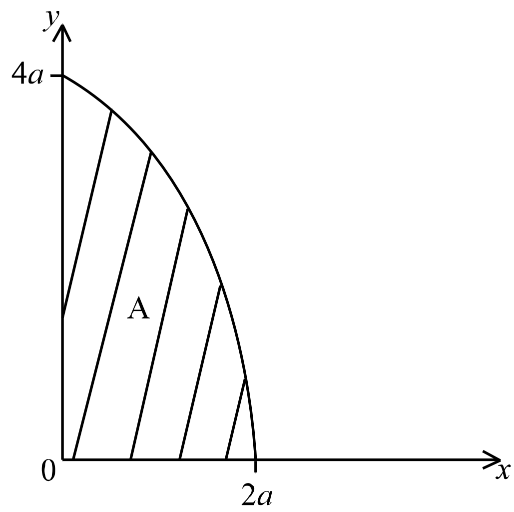
5. *[Maximum mark: 6]*

The diagram below shows the shaded region A which is bounded by the axes and part of the curve $y^2 = 8a(2a - x)$, $a > 0$. Find in terms of a the volume of the solid formed when A is rotated through 360° around the x-axis.



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6. *[Maximum mark: 6]*

Given that $y = \mathrm{e}^{-x^2}$ find

(a) $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$; *[3 marks]*

(b) the exact values of the x-coordinates of the points of inflexion on the graph of $y = \mathrm{e}^{-x^2}$, justifying that they are points of inflexion. *[3 marks]*

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7. *[Maximum mark: 6]*

Find the non-unique solution for the following system of simultaneous equations

$$\begin{aligned} x - y - z &= 3 \\ x - 2y + z &= 2 \\ 2x - y - 4z &= 7 \end{aligned}$$

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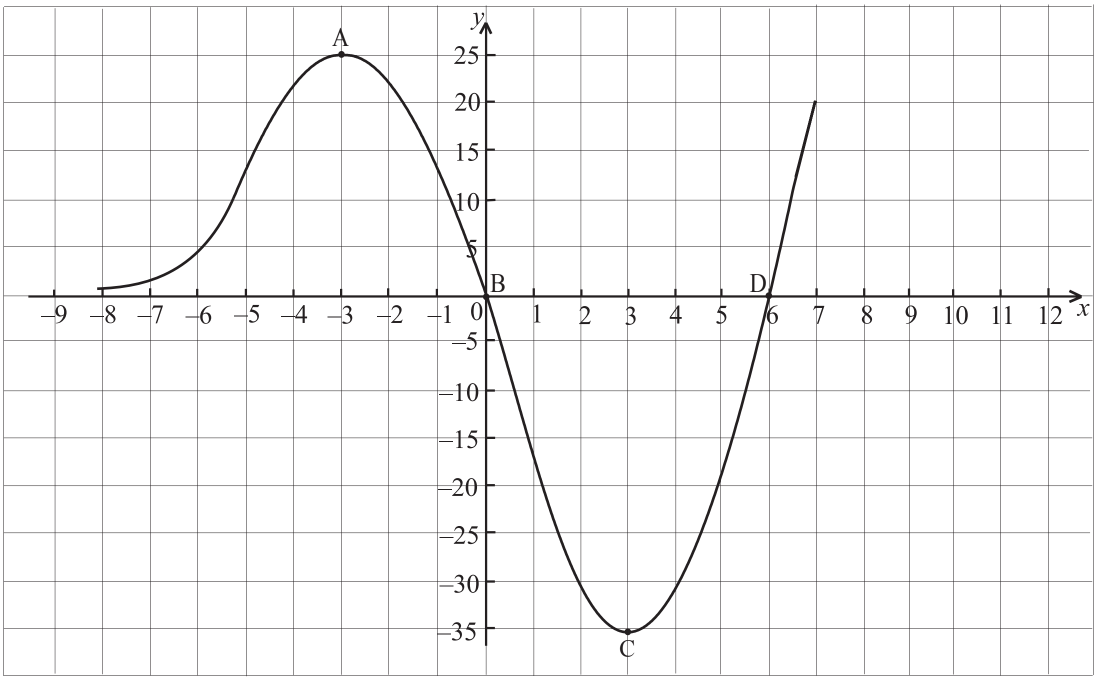
8. *[Maximum mark: 6]*

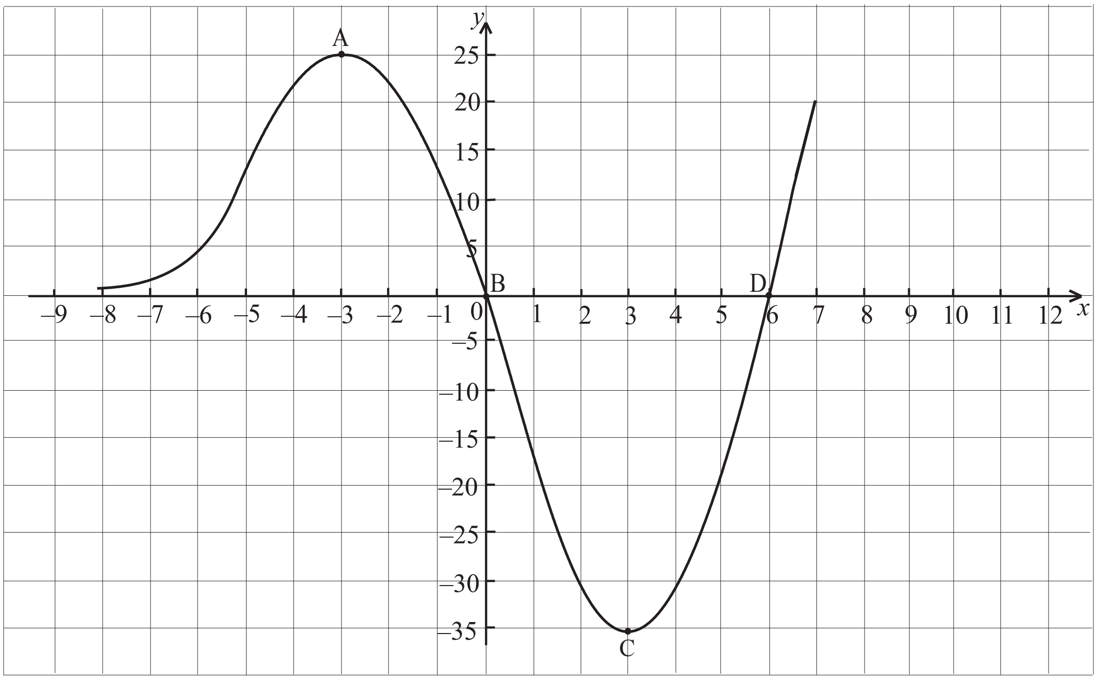
The diagrams below show the graph of $y = f(x)$ which passes through the points A, B, C and D.

Sketch, indicating clearly the images of A, B, C and D, the graphs of

(a) $y = f(x-4)$; *[2 marks]*



(b) $y = f(-3x)$. *[4 marks]*



9. *[Maximum mark: 6]*

A furniture manufacturer makes tables. A table leg is considered to be oversize if its width is greater than 10.5 cm and undersize if its width is less than 9.5 cm. From past experience it is found that 2 % of the table legs that are made are oversize and that 4 % of the table legs are undersize. The widths of the table legs are normally distributed with mean μ cm and standard deviation σ cm. Find the value of μ and of σ.

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10. *[Maximum mark: 6]*

Determine the values of k for which $\begin{pmatrix} k & 1 & 1 \\ 2 & k & -2 \\ 1 & -2 & k \end{pmatrix}$ is singular.

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11. *[Maximum mark: 6]*

The lines l_1 and l_2 have equations

$$\boldsymbol{r_1} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} \text{ and } \boldsymbol{r_2} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$$

respectively, where λ and μ are parameters.

(a) Show that l_1 passes through the point $(2, -7, 4)$. *[2 marks]*

(b) Determine whether the lines l_1 and l_2 intersect. *[4 marks]*

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12. *[Maximum mark: 6]*

In a promotion to try to increase the sales of a particular brand of breakfast cereal, a picture of a soccer player is put in each packet. There are ten different pictures available. Each picture is equally likely to be found in any packet of breakfast cereal.

Charlotte buys four packets of breakfast cereal.

(a) Find the probability that the four pictures in these packets are all different. *[2 marks]*

(b) Of the ten players whose pictures are in the packets, her favourites are Alan and Bob. Find the probability that she finds at least one picture of a favourite player in these four packets. *[4 marks]*

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13. *[Maximum mark: 6]*

Determine the values of x that satisfy the following inequalities

(a) $\frac{|x|+2}{|x|-3} < 4$; *[3 marks]*

(b) $\frac{xe^x}{(x^2-1)} \geq 1$. *[3 marks]*

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14. *[Maximum mark: 6]*

A plane Π has equation $\boldsymbol{r}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix}=16$ and a line l has equations $\frac{x-4}{-1}=\frac{y+2}{2}=\frac{z-6}{4}$.

Show that the line l lies in the plane Π .

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15. *[Maximum mark: 6]*

(a) Solve the equation $2(4^x)+4^{-x}=3$. *[3 marks]*

(b) (i) Solve the equation $a^x = e^{2x+1}$ where $a>0$, giving your answer for x in terms of a.

(ii) For what value of a does the equation have no solution? *[3 marks]*

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16. *[Maximum mark: 6]*

Twelve people travel in three cars, with four people in each car. Each car is driven by its owner. Find the number of ways in which the remaining nine people may be allocated to the cars. (The arrangement of people within a particular car is not relevant).

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17. *[Maximum mark: 6]*

Find $\int_0^a \arcsin x \, dx$, $0 < a < 1$.

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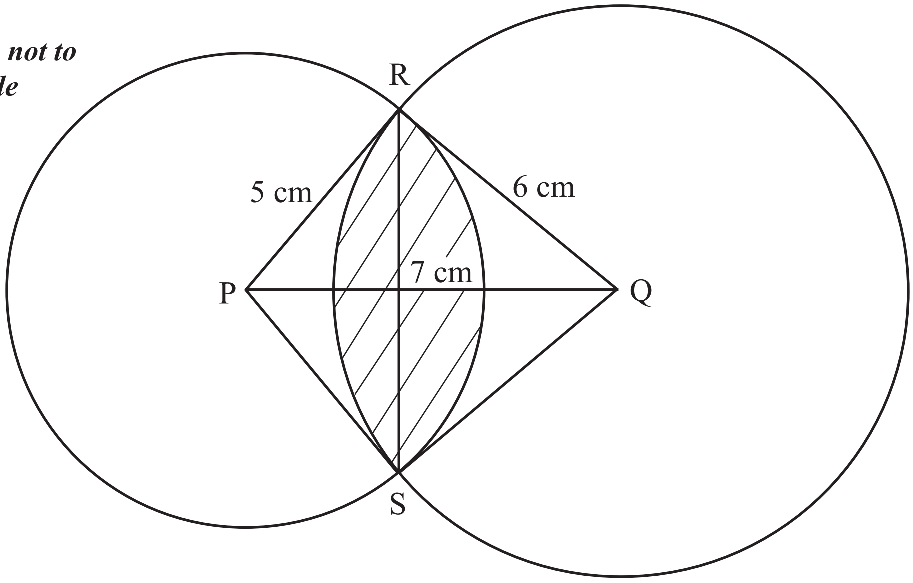
18. *[Maximum mark: 6]*

The diagram below shows a pair of intersecting circles with centres at P and Q with radii of 5 cm and 6 cm respectively. RS is the common chord of both circles and PQ is 7 cm.

diagram not to scale



Find the area of the shaded region.

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19. *[Maximum mark: 6]*

Prove that $\left(\sqrt{3}+\mathrm{i}\right)^n+\left(\sqrt{3}-\mathrm{i}\right)^n$ is real, where $n\in\mathbb{Z}^+$.

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20. *[Maximum mark: 6]*

Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1+y^2}{1+x^2}$, given that $y = \sqrt{3}$ when $x = \frac{\sqrt{3}}{3}$.

Give your answer in the form $y = \frac{ax + \sqrt{a}}{a - x\sqrt{a}}$ where $a \in \mathbb{Z}^+$.

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