IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M07/5/MATHL/HP1/ENG/TZ2/XX+

22077207

MATHEMATICS
HIGHER LEVEL
PAPER 1

Monday 7 May 2007 (afternoon)

2 hours

Candidate session number

0	0							

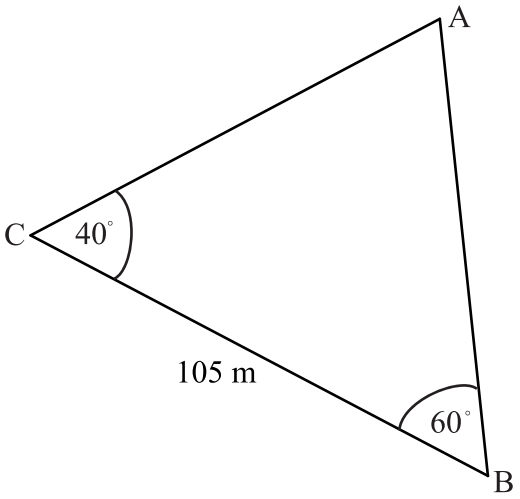
INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

0119

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

1. The following diagram shows ΔABC, where $BC = 105$ m, $A\hat{C}B = 40^\circ$, $A\hat{B}C = 60^\circ$.



Find AB.

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2. In a sample of 50 boxes of light bulbs, the number of defective light bulbs per box is shown below.

Number of defective light bulbs per box	0	1	2	3	4	5	6
Number of boxes	7	3	15	11	6	5	3

(a) Calculate the median number of defective light bulbs per box.

(b) Calculate the mean number of defective light bulbs per box.

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3. Find the cosine of the angle θ between the planes π_1 and π_2, where π_1 has equation $-2x + y - z = 2$ and π_2 has equation $x + 2y - z = 6$.

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4. Solve $2(\ln x)^2 = 3\ln x - 1$ for x. Give your answers in **exact** form.

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5. Solve the differential equation $\frac{dy}{dx} = 2xy^2$ given that $y = 1$ when $x = 0$.
Give your answer in the form $y = f(x)$.

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6. Given that $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{c} = -\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ are the position vectors of the points A, B and C respectively, calculate the area of triangle ABC.

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7. A biology test consists of seven multiple choice questions. Each question has five possible answers, only one of which is correct. At least four correct answers are required to pass the test. Juan does not know the answer to any of the questions so, for each question, he selects the answer at random.

(a) Find the probability that Juan answers exactly four questions correctly.

(b) Find the probability that Juan passes the biology test.

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8. Consider the system of equations $\boldsymbol{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ where $\boldsymbol{A} = \begin{pmatrix} k+1 & -k \\ 2 & k-1 \end{pmatrix}$ and $k \in \mathbb{R}$.

(a) Find $\det \boldsymbol{A}$.

(b) Find the set of values of k for which the system has a unique solution.

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9. A particle moves in a straight line. At time t seconds, its displacement from a fixed point O is s metres, and its velocity, v metres per second, is given by $v = 3t^2 - 4t + 2$, $t \geq 0$. When $t = 0$, $s = -3$. Find the value of t when the particle is at O.

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10. A continuous random variable X has the probability density function f given by

$$f(x) = \begin{cases} \dfrac{8}{\pi(x^2+4)}, & 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) State the mode of X.

(b) Find the **exact** value of $\mathrm{E}(X)$.

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11. The polynomial $P(z) = z^3 + mz^2 + nz - 8$ is divisible by $(z + 1 + \mathrm{i})$, where $z \in \mathbb{C}$ and $m, n \in \mathbb{R}$. Find the value of m and of n.

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12. The function f is defined by $f(x) = x^2 - 2x + k(3k+2)$ where $k \in \mathbb{R}$. Find the set of values of k for which $f(x) = 0$ has two distinct real roots.

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13. Consider the arithmetic series $-6 + 1 + 8 + 15 + \ldots$.
Find the least number of terms so that the sum of the series is greater than $10\,000$.

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14. The graph of $y = \cos x$ is transformed into the graph of $y = 8 - 2\cos\frac{\pi x}{6}$.
Find a sequence of simple geometric transformations that does this.

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15. The graph of $y = \sin(3x)$ for $0 \leq x \leq \frac{\pi}{4}$ is rotated through 2π radians about the x-axis. Find the **exact** volume of the solid of revolution formed.

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16. The lengths of a particular species of lizard are normally distributed with a mean length of 50 cm and a standard deviation of 4 cm. A lizard is chosen at random.

(a) Find the probability that its length is greater than 45 cm.

(b) Given that its length is greater than 45 cm, find the probability that its length is greater than 55 cm.

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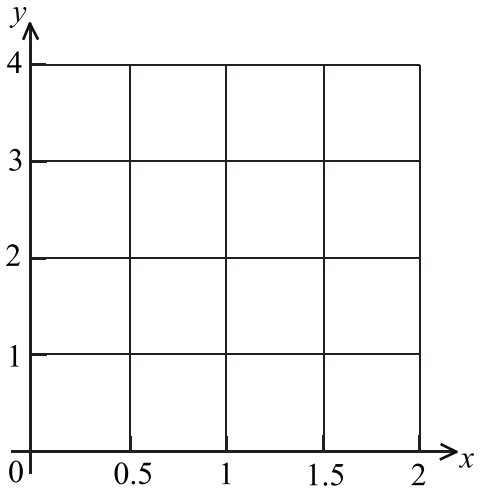
17. For $x \geq \frac{1}{2}$, let $f(x) = x^2 \ln(x+1)$ and $g(x) = \sqrt{2x-1}$.

(a) Sketch the graphs of f and g on the grid below.



(b) Let A be the region completely enclosed by the graphs of f and g. Find the area of A.

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18. The function f is defined by $f(x) = \frac{2x}{x^2+6}$ for $x \geq b$ where $b \in \mathbb{R}$.

(a) Show that $f'(x) = \frac{12-2x^2}{(x^2+6)^2}$.

(b) Hence find the smallest **exact** value of b for which the inverse function f^{-1} exists. Justify your answer.

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19. Find $\int_0^{\ln 3} \frac{e^x}{e^{2x}+9}\,dx$, expressing your answer in **exact** form.

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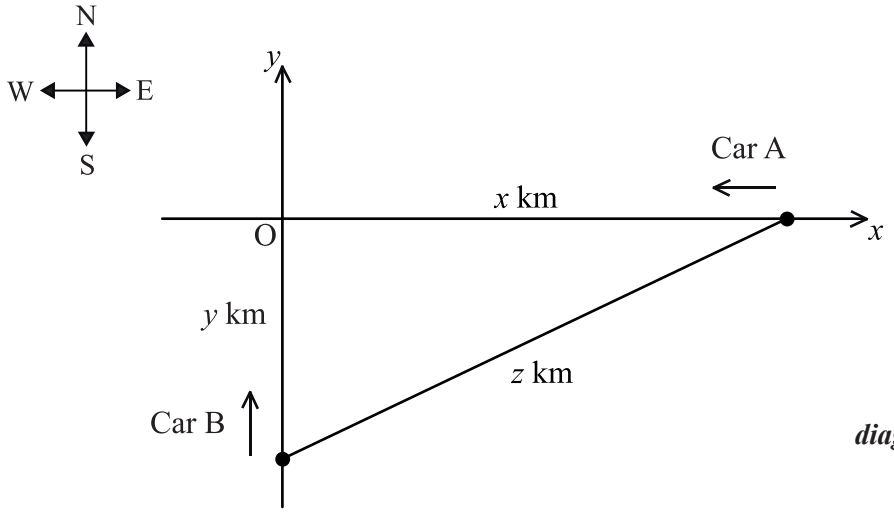
20. Car A is travelling on a straight east-west road in a westerly direction at $60\ \text{km h}^{-1}$. Car B is travelling on a straight north-south road in a northerly direction at $70\ \text{km h}^{-1}$. The roads intersect at the point O. When Car A is x km east of O, and Car B is y km south of O, the distance between the cars is z km.



Find the rate of change of z when Car A is 0.8 km east of O and Car B is 0.6 km south of O.

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