IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M07/5/MATHL/HP1/ENG/TZ1/XX

22077204

MATHEMATICS
HIGHER LEVEL
PAPER 1

Monday 7 May 2007 (afternoon)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

0121

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

1. Triangle ABC has $\hat{C} = 42^\circ$, $\text{BC} = 1.74 \text{ cm}$, and area 1.19 cm^2.

(a) Find AC.

(b) Find AB.

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2. The events A and B are such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.3$, $\mathrm{P}(A \cup B) = 0.6$.

(a) (i) Find the value of $\mathrm{P}(A \cap B)$.

(ii) Hence show that A and B are not independent.

(b) Find the value of $\mathrm{P}(B \mid A)$.

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3. In an experiment, a trial is repeated n times. The trials are independent and the probability p of success in each trial is constant. Let X be the number of successes in the n trials. The mean of X is 0.4 and the standard deviation is 0.6.

(a) Find p.

(b) Find n.

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4. The function f is defined as $f(x) = \frac{3x-4}{x+2}$, $x \neq -2$.

(a) Find an expression for $f^{-1}(x)$.

(b) Write down the domain of f^{-1}.

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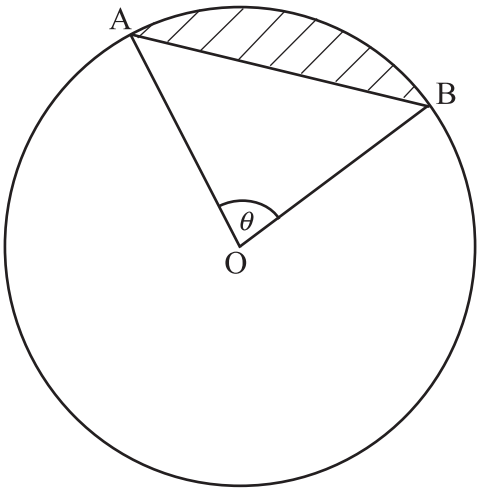
5. The diagram shows a circle centre O and radius 1, with $\text{A}\hat{\text{O}}\text{B} = \theta$, $\theta \neq 0$. The area of $\Delta\,\text{AOB}$ is three times the shaded area.



Find the value of θ.

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6. Let $f(x) = x\ln x - x$, $x > 0$.

(a) Find $f'(x)$.

(b) Using integration by parts find $\int (\ln x)^2 \, dx$.

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7. The complex number z is defined by

$$z = 4\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right) + 4\sqrt{3}\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right).$$

(a) Express z in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where r and θ have exact values.

(b) Find the cube roots of z, expressing in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where r and θ have exact values.

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8. The function f is defined as $f(x) = \sin x \ln x$ for $x \in [0.5, 3.5]$.

(a) Write down the x-intercepts.

(b) The area above the x-axis is A and the **total** area below the x-axis is B. If $A = kB$, find k.

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9. A continuous random variable X has probability density function f defined by

$$f(x) = \begin{cases} e^x, & \text{for } 0 \le x \le \ln 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find the **exact** value of $\mathrm{E}(X)$.

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10. Consider the vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$

$$\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \ \boldsymbol{c} = \begin{pmatrix} 3 \\ 1 \\ \lambda \end{pmatrix}, \ \boldsymbol{d} = \begin{pmatrix} \mu \\ -2 \\ 1 \end{pmatrix}.$$

Let $\boldsymbol{s} = (\boldsymbol{a} \bullet \boldsymbol{b})\boldsymbol{c} + \boldsymbol{d}$, where $\boldsymbol{s}$ is perpendicular to $\boldsymbol{a}$.

Find an expression for λ in terms of μ.

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11. Let $y = x \arcsin x$, $x \in]-1, 1[$. Show that $\frac{d^2y}{dx^2} = \frac{2-x^2}{(1-x^2)^{\frac{3}{2}}}$.

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12. The quadratic function Q is defined by $Q(x) = kx^2 - (k-3)x + (k-8)$, $k \in \mathbb{R}$.
Determine the values of k for which $Q(x) = 0$ has no real roots.

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13. Let $\boldsymbol{A} = \begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$. Given that $\boldsymbol{AX} = k\boldsymbol{X}$, where $k \in \mathbb{R}$, find the values of k for which there is an infinity of solutions for $\boldsymbol{X}$.

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14. An infinite geometric series is given by $\sum_{k=1}^{\infty} 2(4-3x)^k$.

(a) Find the values of x for which the series has a finite sum.

(b) When $x = 1.2$, find the minimum number of terms needed to give a sum which is greater than 1.328.

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15. Given that $e^{xy} - y^2 \ln x = e$ for $x \geq 1$, find $\frac{dy}{dx}$ at the point $(1, 1)$.

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16. Solve the differential equation $(x^2+1)\frac{dy}{dx} - xy = 0$ where $x > 0$, $y > 0$, given that $y = 1$ when $x = 1$.

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17. The functions f and g are defined by

$$f(x) = 2x - 1,$$
$$g(x) = \frac{x}{x+1},\ x \neq -1.$$

Find the values of x for which $(f \circ g)(x) \leq (g \circ f)(x)$.

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18. The following table shows the values of two functions f and g and their first derivatives when $x = 1$ and $x = 0$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	4	1	–4	5
1	–2	3	–1	2

(a) Find the derivative of $\frac{3f(x)}{g(x)-1}$ when $x = 0$.

(b) Find the derivative of $f\left(g(x)+2x\right)$ when $x = 1$.

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19. The solution of $2^{2x+3} = 2^{x+1} + 3$ can be expressed in the form $a + \log_2 b$ where $a, b \in \mathbb{Z}$. Find the value of a and of b.

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20. The volume of a solid is given by

$$V = \frac{4}{3}\pi r^3 + \pi r^2 h .$$

At the time when the radius is 3 cm, the volume is 81π cm^3, the radius is changing at a rate of 2 cm/min and the volume is changing at a rate of 204π cm^3/min. Find the rate of change of the height at this time.

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