M06/5/MATHL/HP2/ENG/TZ0/XX

22067205

MATHEMATICS
HIGHER LEVEL
PAPER 2

Thursday 4 May 2006 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 21]*

Let A be the point $(2, -1, 0)$, B the point $(3, 0, 1)$ and C the point $(1, m, 2)$, where $m \in \mathbb{Z}$, $m < 0$.

(a) (i) Find the scalar product $\overrightarrow{BA} \cdot \overrightarrow{BC}$.

(ii) Hence, given that $A\hat{B}C = \arccos\frac{\sqrt{2}}{3}$, show that $m = -1$. *[6 marks]*

(b) Determine the Cartesian equation of the plane ABC. *[4 marks]*

(c) Find the area of triangle ABC. *[3 marks]*

(d) (i) The line L is perpendicular to plane ABC and passes through A. Find a vector equation of L.

(ii) The point $D(6, -7, 2)$ lies on L. Find the volume of the pyramid ABCD. *[8 marks]*

2. *[Maximum mark: 21]*

Let $z = \cos\theta + \mathrm{i}\sin\theta$, for $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$.

(a) (i) Find z^3 using the binomial theorem.

(ii) Use de Moivre´s theorem to show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta \text{ and } \sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ *[10 marks]*

(b) **Hence** prove that $\frac{\sin 3\theta - \sin\theta}{\cos 3\theta + \cos\theta} = \tan\theta$. *[6 marks]*

(c) Given that $\sin\theta = \frac{1}{3}$, find the **exact** value of $\tan 3\theta$. *[5 marks]*

3. *[Maximum mark: 23]*

Particle A moves in a straight line, starting from O_A, such that its velocity in metres per second for $0 \le t \le 9$ is given by

$$v_A = -\frac{1}{2}t^2 + 3t + \frac{3}{2}.$$

Particle B moves in a straight line, starting from O_B, such that its velocity in metres per second for $0 \le t \le 9$ is given by

$$v_B = e^{0.2t}.$$

(a) Find the maximum value of v_A, justifying that it is a maximum. *[5 marks]*

(b) Find the acceleration of B when $t = 4$. *[3 marks]*

The displacements of A and B from O_A and O_B respectively, at time *t* are s_A metres and s_B metres. When $t = 0$, $s_A = 0$, and $s_B = 5$.

(c) Find an expression for s_A and for s_B, giving your answers in terms of *t*. *[7 marks]*

(d) (i) Sketch the curves of s_A and s_B on the same diagram.

(ii) Find the values of *t* at which $s_A = s_B$. *[8 marks]*

4. *[Total mark: 31]*

Part A *[Maximum mark: 12]*

The time, T minutes, required by candidates to answer a question in a mathematics examination has probability density function

$$f(t) = \begin{cases} \frac{1}{72}(12t - t^2 - 20), & \text{for } 4 \le t \le 10 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find

(i) μ, the expected value of T;

(ii) σ^2, the variance of T. *[7 marks]*

(b) A candidate is chosen at random. Find the probability that the time taken by this candidate to answer the question lies in the interval $[\mu - \sigma, \mu]$. *[5 marks]*

Part B *[Maximum mark: 19]*

Andrew shoots 20 arrows at a target. He has a probability of 0.3 of hitting the target. All shots are independent of each other. Let X denote the number of arrows hitting the target.

(a) Find the mean and standard deviation of X. *[5 marks]*

(b) Find

(i) $\mathrm{P}(X = 5)$;

(ii) $\mathrm{P}(4 \le X \le 8)$. *[6 marks]*

Bill also shoots arrows at a target, with probability of 0.3 of hitting the target. All shots are independent of each other.

(c) Calculate the probability that Bill hits the target for the first time on his third shot. *[3 marks]*

(d) Calculate the minimum number of shots required for the probability of at least one shot hitting the target to exceed 0.99. *[5 marks]*

5. *[Maximum mark: 24]*

Consider the system of equations $\boldsymbol{T}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -42 \end{pmatrix}$, where $\boldsymbol{T} = \begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & r \\ 3r & 0 & s \end{pmatrix}$.

(a) Find the solution of the system when $r = 0$ and $s = 3$. *[4 marks]*

(b) The solution of the system is not unique.

(i) Show that $s = \frac{9}{2}r^2$.

(ii) When $r = 2$ and $s = 18$, show that the system can be solved, and find the general solution. *[11 marks]*

(c) Use mathematical induction to prove that, when $r = 0$,

$$\boldsymbol{T}^n = \begin{pmatrix} (-1)^n & 2^n - (-1)^n & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & s^n \end{pmatrix}, n \in \mathbb{Z}^+.$$ *[9 marks]*