

22067204

**MATHEMATICS
HIGHER LEVEL
PAPER 1**

Wednesday 3 May 2006 (afternoon)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

- 1. In an arithmetic sequence the second term is 7 and the sum of the first five terms is 50. Find the common difference of this arithmetic sequence.

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2. Let $z_1 = r \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$ and $z_2 = 1 + \sqrt{3}i$.

(a) Write z_2 in modulus-argument form.

(b) Find the value of r if $|z_1 z_2^3| = 2$.

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3. The graph of $y = 2x^2 + 4x + 7$ is translated using the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find the equation of the translated graph, giving your answer in the form $y = ax^2 + bx + c$.

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4. Let $f(x) = 3x^2 - x + 4$. Find the values of m for which the line $y = mx + 1$ is a tangent to the graph of f .

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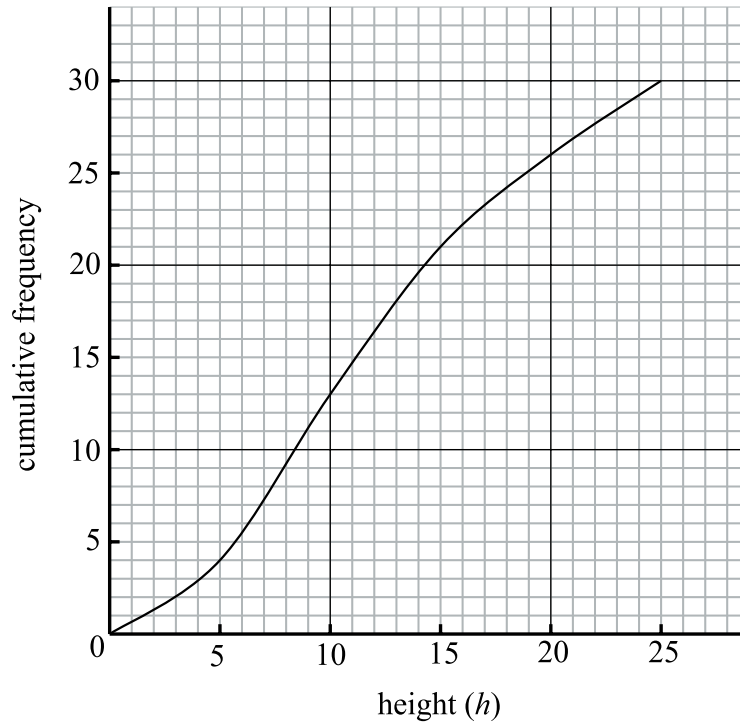


5. The polynomial $P(x) = 2x^3 + ax^2 - 4x + b$ is divisible by $(x-1)$ and by $(x+3)$. Find the value of a and of b .

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6. The following is the cumulative frequency diagram for the heights of 30 plants given in centimetres.



- (a) Use the diagram to estimate the median height.

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- (b) Complete the following frequency table.

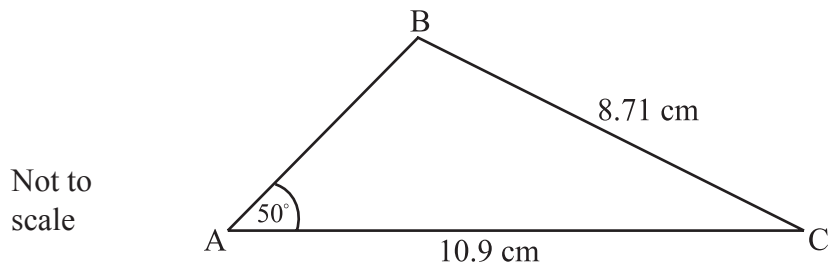
Height (h)	Frequency
$0 \leq h < 5$	4
$5 \leq h < 10$	9
$10 \leq h < 15$	
$15 \leq h < 20$	
$20 \leq h < 25$	

- (c) Hence estimate the mean height.

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7. In the **obtuse-angled** triangle ABC, $AC = 10.9$ cm, $BC = 8.71$ cm and $\hat{BAC} = 50^\circ$.



Find the area of triangle ABC.

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8. The weights in grams of bread loaves sold at a supermarket are normally distributed with mean 200 g. The weights of 88 % of the loaves are less than 220 g. Find the standard deviation.

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9. Solve $|\ln(x+3)|=1$. Give your answers in **exact** form.

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10. Let $f(x) = 2^{0.5x}$ and $g(x) = 3^{-0.5x} + \frac{5}{3}$. Let R be the region completely enclosed by the graphs of f and g , and the y -axis. Find the area of R .

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11. Let $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -1 \\ p \\ 6 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$.

(a) Find $\mathbf{a} \times \mathbf{b}$.

(b) Find the value of p , given that $\mathbf{a} \times \mathbf{b}$ is parallel to \mathbf{c} .

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12. Find $\int e^{2x} \sin x \, dx$.

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13. Let A and B be events such that $P(A) = \frac{1}{5}$, $P(B|A) = \frac{1}{4}$ and $P(A \cup B) = \frac{7}{10}$.

- (a) Find $P(A \cap B)$.
- (b) Find $P(B)$.
- (c) Show that A and B are **not** independent.

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14. Let $f(x) = \cos^3(4x+1)$, $0 \leq x \leq 1$.

(a) Find $f'(x)$.

(b) Find the **exact** values of the three roots of $f'(x) = 0$.

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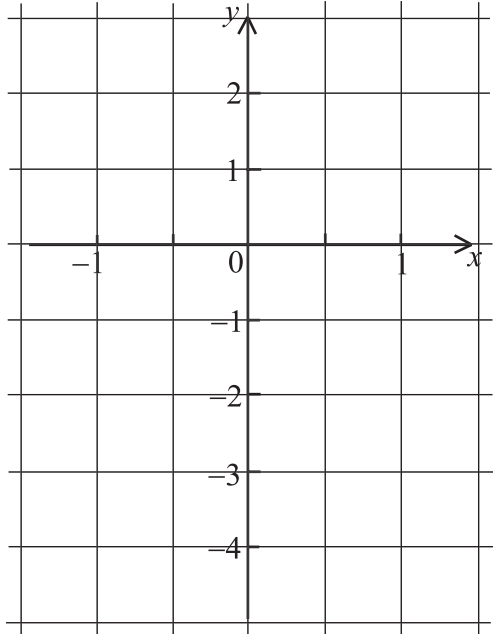
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15. Let f be the function $f(x) = x \arccos x + \frac{1}{2}x$ for $-1 \leq x \leq 1$ and g the function $g(x) = \cos 2x$ for $-1 \leq x \leq 1$.

(a) On the grid below, sketch the graph of f and of g .



(b) Write down the solution of the equation $f(x) = g(x)$.

(c) Write down the range of g .

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16. The number of car accidents occurring per day on a highway follows a Poisson distribution with mean 1.5.
- (a) Find the probability that more than two accidents will occur on a given Monday.
 - (b) Given that at least one accident occurs on another day, find the probability that more than two accidents occur on that day.

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17. Let $A = \begin{pmatrix} 2 & 6 \\ k & -1 \end{pmatrix}$ and $B = \begin{pmatrix} h & 3 \\ -3 & 7 \end{pmatrix}$, where h and k are integers. Given that $\det A = \det B$ and that $\det AB = 256h$,

- (a) show that h satisfies the equation $49h^2 - 130h + 81 = 0$;
- (b) hence find the value of k .

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18. Given that $3^{x+y} = x^3 + 3y$, find $\frac{dy}{dx}$.

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19. There are 10 seats in a row in a waiting room. There are six people in the room.

(a) In how many different ways can they be seated?

(b) In the group of six people, there are three sisters who must sit next to each other. In how many different ways can the group be seated?

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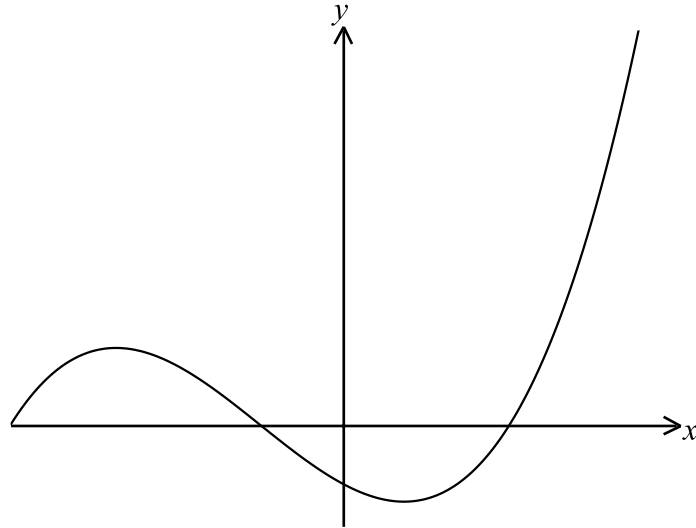
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20. Each of the diagrams below shows the graph of a function f . Sketch on the given axes the graph of

(a) $|f(-x)|$;



(b) $\frac{1}{f(x)}$.

