

MARKSCHEME

May 2005

MATHEMATICS

Higher Level

Paper 1

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Paper 1 Markscheme

Instructions to Examiners

Note: Where there are 2 marks (e.g. M2, A2) for an answer do NOT split the marks unless otherwise instructed.

1 Method of Marking

- (a) All marking must be done using a **red** pen.
- (b) In this paper, the maximum mark is awarded for a **correct answer**, irrespective of the method used. Thus, if the correct answer appears in the answer box, award the maximum mark and move on to the next question; in this case there is no need to check the method.
- (c) If an **answer is wrong**, then marks should be awarded for the method according to the markscheme. Examiners should record these marks using the abbreviations shown on the markscheme. (A correct answer incorrectly transferred to the answer box is awarded the maximum mark.)

2 Abbreviations

The markscheme may make use of the following abbreviations:

- (C) Marks awarded for **Correct** answers (irrespective of working shown)
- (M) Marks awarded for **Method**
- (A) Marks awarded for an **Answer** or for **Accuracy**
- (R) Marks awarded for clear **Reasoning**

Note: It is not possible to award (M0)(A1).

Follow through (ft) marks should be awarded where a correct method has been attempted but error(s) are made in subsequent working which is essentially correct.

- Penalize the error when it first occurs
- Accept the incorrect result as the appropriate quantity in all relevant subsequent working
- If the question becomes much simpler then use discretion to award fewer marks

Examiners should use **(d)** to indicate where discretion has been used. It should only be used for decisions on follow through and alternative methods. It must be accompanied by a brief note to explain the decision made.

3 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme. Indicate the awarding of these marks by **(d)**.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1, METHOD 2, etc.** Other alternative (part) solutions, are indicated by **EITHER...OR**. Where possible, alignment will also be used to assist examiners to identify where these alternatives start and finish.

It should be noted that **G** marks have been removed, and GDC solutions will not be indicated using the **OR** notation as on previous markschemes.

- (b) Unless the question specifies otherwise, accept **equivalent** forms. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.

On the markscheme, these equivalent numerical or algebraic forms will be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect, *i.e.* once the correct answer is seen, ignore further working, unless it contradicts the answer. This includes more than the required number of solutions, unless otherwise specified in the markscheme.

- (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7, 1·7, 1,7; different forms of vector notation such as \vec{u} , \bar{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

4 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized **once only IN THE PAPER** for an accuracy error (**AP**). Award the marks as usual then write $-1(\text{AP})$ against the answer and also on the **front** cover.

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
OR
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from 2003).

Note: If there is no working shown, and answers are given to the correct two significant figures, apply the **AP**. However, do **not** accept answers to one significant figure without working.

Incorrect answers are wrong, and the accuracy penalty should not be applied to incorrect answers.

5 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

Examples

1. Accuracy

A question leads to the answer 4.6789....

- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy : both should be penalised the first time this type of error occurs.
- 4.67 is incorrectly rounded – penalise on the first occurrence.

Note: All these “incorrect” answers may be assumed to come from 4.6789..., even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalized as being incorrect answers, not as examples of accuracy errors.

2. Alternative solutions

The polynomial $x^2 - 4x + 3$ is a factor of $x^3 + (a - 4)x^2 + (3 - 4a)x + 3$.
Calculate the value of the constant a .

METHOD 1

Using the information given it follows that

$$x^3 + (a - 4)x^2 + (3 - 4a)x + 3 \equiv (x^2 - 4x + 3)(x + 1) \quad (M1)(A1)$$

Comparing coefficients of x^2 (or x) (M1)

$$a - 4 = -3 \text{ (or } 3 - 4a = -1) \quad (A1)(A1)$$

$$\text{giving } a = 1 \quad (A1) \quad (C6)$$

METHOD 2

$$x^2 - 4x + 3 = (x - 3)(x - 1) \quad (M1)(A1)$$

EITHER

$$1 + (a - 4) + (3 - 4a) + 3 = 0 \quad (M1)(A1)$$

$$\text{Solving, } a = 1 \quad (M1)(A1)$$

OR

$$27 + 9(a - 4) + 3(3 - 4a) + 3 = 0 \quad (M1)(A1)$$

$$\text{Solving, } a = 1 \quad (M1)(A1) \quad (C6)$$

Note that the first line of **METHOD 2** applies to both **EITHER** and **OR** alternatives.

3. Follow through

Question

Calculate the acute angle between the lines with equations

$$r = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \text{and} \quad r = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Markscheme

Angle between lines = angle between direction vectors (may be implied) **(A1)**

Direction vectors are $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (may be implied) **(A1)**

$$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left| \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right| \left| \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right| \cos \theta \quad \text{span style="float: right;">**(M1)**$$

$$4 \times 1 + 3 \times (-1) = \sqrt{(4^2 + 3^2)} \sqrt{(1^2 + (-1)^2)} \cos \theta \quad \text{span style="float: right;">**(A1)**$$

$$\cos \theta = \frac{1}{5\sqrt{2}} \quad (= 0.1414\dots) \quad \text{span style="float: right;">**(A1)**$$

$$\theta = 81.9^\circ \quad (1.43 \text{ radians}) \quad \text{span style="float: right;">**(A1) (C6)**$$

Examples of solutions and marking

Solutions	Marks allocated	
<p>1. $\begin{pmatrix} 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right \left \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right \cos \theta$</p> <p>$\cos \theta = \frac{7}{5\sqrt{2}}$</p> <p>$\theta = 8.13^\circ$</p>	<p>(A1)(A1) implied (M1)</p> <p>(A0)(A1)</p> <p>(A1)ft</p>	<p>Total 5 marks</p>
<p>2. $\cos \theta = \frac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix}}{\sqrt{17}\sqrt{20}}$</p> <p>$= 0.2169$</p> <p>$\theta = 77.5^\circ$</p>	<p>(A0)(A0) wrong vectors implied (M1) for correct method, (A1)ft</p> <p>(A1)ft</p> <p>(A1)ft</p>	<p>Total 4 marks</p>

END OF EXAMPLES

QUESTION 1

$$2\mathbf{a} - \mathbf{b} = 2 \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \quad (M1)$$

$$= \begin{pmatrix} 5 \\ -1 \\ -4 \end{pmatrix} \quad (A1)$$

$$\begin{pmatrix} 5 \\ -1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ y \\ 3 \end{pmatrix} = 0 \quad (M1)(A1)$$

$$10 - y - 12 = 0 \quad (A1)$$

$$y = -2 \quad (A1) \quad (C6)$$

QUESTION 2

$$\begin{pmatrix} 5 \\ 2 \end{pmatrix} (3x)^3 \left(\frac{-2}{x} \right)^2 \quad (M1)$$

$$= 10(27x^3) \left(\frac{4}{x^2} \right) \quad (A1)(A1)(A1)$$

$$= 1080x \quad (A1)$$

$$\text{Coefficient is } 1080 \quad (A1) \quad (C6)$$

QUESTION 3

$$x = 2\lambda + 3, \quad y = \lambda - 1, \quad z = -3\lambda + 5 \quad (A1)(A1)(A1)$$

$$2(2\lambda + 3) - (\lambda - 1) + 3(-3\lambda + 5) = 10 \quad (M1)$$

$$\lambda = 2 \quad (A1)$$

$$\text{Point of intersection } (7, 1, -1) \quad (A1) \quad (C6)$$

QUESTION 4

$$\det \mathbf{A} = 5 + 3(x + 1) = 8 + 3x \quad (A1)$$

$$\det \mathbf{B} = 1 \begin{vmatrix} 1 & 0 \\ -1 & x \end{vmatrix} - x \begin{vmatrix} -2 & 0 \\ 0 & x \end{vmatrix} - 2 \begin{vmatrix} -2 & 1 \\ 0 & -1 \end{vmatrix} \quad (M1)$$

$$= x + 2x^2 - 4 \quad (A1)$$

$$8 + 3x = x + 2x^2 - 4 \quad (M1)$$

$$0 = 2x^2 - 2x - 12$$

$$0 = 2(x - 3)(x + 2)$$

$$x = 3 \quad \text{or} \quad x = -2 \quad (A1)(A1) \quad (C6)$$

QUESTION 5

$$4 - x^2 = (x + 1) \cos x$$

$$x = -1.92 \text{ or } x = 2.72$$

(M1)(A1)**EITHER**

$$\int_{-1.92}^{2.72} (4 - x^2 - (x + 1) \cos x) dx$$

(M1)(A1)

$$= 9.4053 \dots = 9.41$$

(A2)**(C6)****OR**

$$\int (4 - x^2 - (x + 1) \cos x) dx$$

(M1)

$$= 4x - \frac{x^3}{3} - (x + 1) \sin x - \cos x$$

(A1)(A1)

Note: Award **(A1)** for $4x - \frac{x^3}{3}$, and **(A1)** for $-(x + 1) \sin x - \cos x$.

$$\text{Area} = 9.4053 \dots = 9.41$$

(A1)**(C6)****QUESTION 6**

$$P(|X - 25| < 3) = P(22 < X < 28)$$

(M1)(A1)(A1)

standard deviation = 2 (need not be seen)

(A1)

$$P(|X - 25| < 3) = 0.866$$

(A2)**(C6)****QUESTION 7**

$$(a) \quad X \sim B\left(20, \frac{1}{4}\right)$$

$$E(X) = np$$

$$= 5$$

(M1)**(A1)****(C2)**

$$(b) \quad P(X = 5) = \binom{20}{5} (0.25)^5 (0.75)^{15}$$

(M1)(A1)

$$= 0.202$$

(A2)**(C4)****QUESTION 8**

$$r\theta = 2r$$

(M1)

$$\theta = 2$$

(A1)

$$\text{Area} = \frac{\theta}{2} r^2 - \frac{1}{2} r^2 \sin \theta$$

(M1)

$$\theta = 2 \Rightarrow \text{area} = r^2 - \frac{1}{2} r^2 (\sin 2) \left(= r^2 \left(1 - \frac{1}{2} \sin 2\right) \right)$$

(M1)(A1)

$$k = 1 - \frac{1}{2} \sin 2 \quad (= 1 - \sin 1 \cos 1) \quad (= 0.545)$$

(A1)**(C6)**

QUESTION 9

(a) **METHOD 1**

$$f(x) = \frac{1}{2} \ln(x^2 + 4) \quad (M1)$$

$$f'(x) = \frac{1}{2} \left(\frac{2x}{x^2 + 4} \right) \quad (M1)$$

$$f'(x) = \frac{x}{x^2 + 4} \quad (A1) \quad (C3)$$

METHOD 2

$$f'(x) = \frac{1}{\sqrt{x^2 + 4}} \times \frac{1}{2} (x^2 + 4)^{-\frac{1}{2}} (2x) \quad (M1)(A1)$$

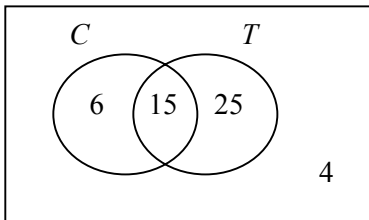
$$= \frac{x}{x^2 + 4} \quad (A1) \quad (C3)$$

(b) $f''(x) = \frac{x^2 + 4 - x(2x)}{(x^2 + 4)^2} \quad (M1)(A1)$

$$= \frac{4 - x^2}{(x^2 + 4)^2} \quad (A1) \quad (C3)$$

QUESTION 10

(a) **METHOD 1**



(M1)(A1)

$$P(T \cap C) = 0.3 \quad (A1) \quad (C3)$$

METHOD 2

$$n(T \cup C) = n(T) + n(C) - n(T \cap C) \quad (M1)$$

$$46 = 40 + 21 - n(T \cap C)$$

$$n(T \cap C) = 15 \quad (A1)$$

$$P(T \cap C) = 0.3 \quad (A1) \quad (C3)$$

(b) $P(T | C) = \frac{P(T \cap C)}{P(C)} \quad (M1)$

$$= \frac{0.3}{0.42} \left(= \frac{5}{7}, 0.714 \text{ to 3 s.f.} \right) \quad (A2) \quad (C3)$$

QUESTION 11

METHOD 1

$$\frac{z(3-4i)}{(3+4i)(3-4i)} - \frac{5i(z-1)}{-5 \times 5i \times i} = \frac{5(3+4i)}{(3-4i)(3+4i)}$$

(M1)(A1)

$$z(3-4i) - 5i(z-1) = 15 + 20i$$

(A1)

$$z(3-4i-5i) = 15 + 20i - 5i$$

(A1)

$$z = \frac{15+15i}{3-9i}$$

(A1)

$$z = -1 + 2i$$

(A1)

(C6)

METHOD 2

$$z\left(\frac{1}{3+4i} + \frac{1}{5i}\right) = \left(\frac{5}{3-4i} + \frac{1}{5i}\right)$$

(M1)(A1)

$$z(0.12 - 0.36i) = 0.6 + 0.6i$$

(A1)(A1)

$$z = -1 + 2i$$

(A2)

(C6)

QUESTION 12

$$y = \frac{1}{2} \Rightarrow 2 \sin \frac{x}{2} = 1$$

(M1)

$$\frac{x}{2} = \frac{\pi}{6} \text{ or } \frac{x}{2} = \frac{5\pi}{6}$$

(A1)

$$x = \frac{5\pi}{3}$$

(A1)

$$2 \cos(xy) \left(y + x \frac{dy}{dx} \right) = 0$$

(M1)

$$\frac{dy}{dx} = -\frac{y}{x}$$

(A1)

$$\frac{dy}{dx} = -\frac{1}{2} \div \frac{5\pi}{3} = -\frac{3}{10\pi} (= -0.0955)$$

(A1)

(C6)

QUESTION 13

For re-arranging the information (e.g. the following table)

(M1)

Profit	-7500	-2500	2500	7500	12500	17500
Frequency	3	4	15	17	5	6

(A1)(A1)

Note: Award (A1) for correct mid-class values, (A1) for correct frequencies.

EITHER

$$\sigma = 6422.61\dots$$

$$\sigma = 6420$$

(A3)

(C6)

OR

$$\mu = 6000$$

(A1)

Substituting into an appropriate formula for σ

(M1)

$$\sigma = 6422.61\dots$$

$$\sigma = 6420 \quad (\text{Do not accept } 6490)$$

(A1)

(C6)

QUESTION 14

(a) $f \circ g(x) = e^{2\sin\left(\frac{\pi x}{2}\right)}$

(A1)

Period = period of $\sin\left(\frac{\pi x}{2}\right)$

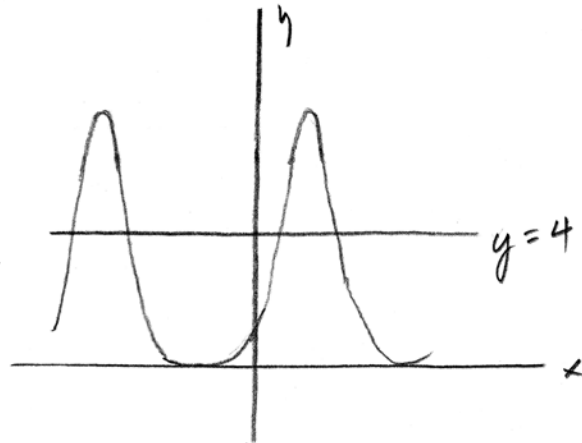
(M1)

$$= 4 \quad (\text{Accept } \frac{720^\circ}{\pi} = 229^\circ)$$

(A1)

(C3)

(b)



(M1)

$$x \in (0.488 + 4k, 1.51 + 4k) \quad (k \in \mathbb{Z}) \quad (\text{Accept } x \in (28.0^\circ + \frac{720^\circ}{\pi}k, 86.5^\circ + \frac{720^\circ}{\pi}k)) \quad (A1)(A1) \quad (C3)$$

Note: Award (A1) for any appropriate values and (A1) for “+ period $\times k$ ”.

QUESTION 15

METHOD 1

$2\log_3(x-3) - \log_3(x+1) = 2$ (A1)

$\log_3 \frac{(x-3)^2}{x+1} = 2$ (A1)

$3^2 = \frac{(x-3)^2}{x+1}$ (A1)

$9x+9 = x^2 - 6x + 9$ (A1)

$0 = x^2 - 15x$

$x = 15$ (A2) (C6)

Note: Award (A1) if $x = 0$ is also given.

METHOD 2

$2\log_3(x-3) + \log_{\frac{1}{3}}(x+1) = 2$

Using change of base formula (M1)

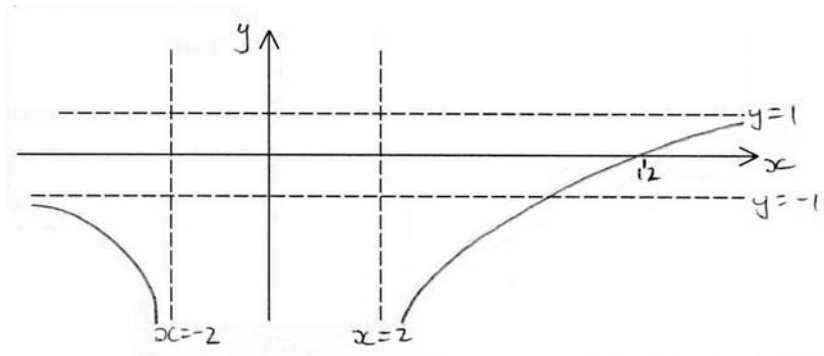
$\frac{2\log(x-3)}{\log 3} + \frac{\log(x+1)}{\log \frac{1}{3}} = 2$ (A1)(A1)

$x = 15$ (A3) (C6)

Note: Award (A2) for $x = 0, x = 15$.

QUESTION 16

(a)



(A1) (C1)

Note: Award (A1) for both branches **and** the x-intercept.

(b) (i) x-intercept is 12 ($x = 12$) (A1) (C1)

(ii) Asymptotes are $x = -2, x = 2$ (A1)(A1) (C2)
 $y = -1, y = 1$ (A1)(A1) (C2)

QUESTION 17

$$\frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{x^2 + 3x + 12}{x(x+2)^2} \quad (M1)$$

$$A = 3 \quad B = -2 \quad C = -5 \quad (A1)(A1)(A1)$$

$$\int f(x) dx = \int \frac{3}{x} - \frac{2}{x+2} - \frac{5}{(x+2)^2} dx$$

$$= 3 \ln|x| - 2 \ln|x+2| + \frac{5}{(x+2)} + C \quad (A1)(A1) \quad (C6)$$

Note: Award (A1) for the first two terms and (A1) for the third term.

QUESTION 18

$$f(x) = ax^3 + bx^2 + 30x + c$$

$$f'(x) = 3ax^2 + 2bx + 30, \quad f'(1) = 0 \Rightarrow 3a + 2b + 30 = 0 \quad (M1)$$

$$f''(x) = 6ax + 2b, \quad f''(3) = 0 \Rightarrow 18a + 2b = 0 \quad (M1)$$

$$a = 2 \quad (A1)$$

$$b = -18 \quad (A1)$$

$$f(1) = 7 \Rightarrow 2 - 18 + 30 + c = 7 \quad (M1)$$

$$c = -7 \quad (A1)$$

$$a = 2, b = -18, c = -7 \quad (\text{accept } 2x^3 - 18x^2 + 30x - 7) \quad (C6)$$

QUESTION 19

Let $\hat{D}AC = \theta \Rightarrow \hat{B}AC = 2\theta$, $t = \tan\theta$, $x = AC$

METHOD 1

$$\tan \theta = \left(\frac{2}{x}\right), \quad \tan 2\theta = \frac{5}{x} \quad (A1)(A1)$$

$$\tan 2\theta = \frac{2t}{1-t^2} \left(= \frac{5}{x}\right) \quad (M1)$$

$$\frac{2t}{1-t^2} = \frac{5t}{2} \quad (A1)$$

$$4t = 5t - 5t^3$$

$$0 = t(1 - 5t^2)$$

$$t = 0, \pm \frac{1}{\sqrt{5}}$$

$$t = \frac{1}{\sqrt{5}} \quad (A1)$$

$$\theta = 24.1^\circ \text{ (0.421 radians)} \quad (A1) \quad (C6)$$

METHOD 2

$$\tan \theta = \left(\frac{2}{x}\right), \quad \tan 2\theta = \frac{5}{x} \quad (A1)(A1)$$

$$x = \frac{2}{\tan \theta}, \quad x = \frac{5}{\tan 2\theta} \quad (A1)$$

$$\frac{2}{\tan \theta} = \frac{5}{\tan 2\theta} \quad (M1)$$

$$\theta = 24.1^\circ \text{ (0.421 radians)} \quad (A2) \quad (C6)$$

QUESTION 20

$$\frac{r}{h} = \frac{4}{12} \Rightarrow r = \frac{h}{3} \quad (A1)$$

$$V = \frac{\pi}{3} \left(\frac{h}{3}\right)^2 (h) \quad (M1)$$

$$V = \frac{\pi}{27} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt} \quad (M1)(A1)$$

$$10 = \frac{\pi}{9} (6)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{90}{36\pi} \left(= \frac{5}{2\pi} = 0.796\right) \text{ metres per min} \quad (A1)(A1) \quad (C6)$$