MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate number

Monday 5 May 2003 (afternoon)

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your candidate number in the box above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. A geometric sequence has all positive terms. The sum of the first two terms is 15 and the sum to infinity is 27. Find the value of

(a) the common ratio;

(b) the first term.

Working:

Answers:

(a) ______________________

(b) ______________________

2. Find all the values of θ in the interval $[0, \pi]$ which satisfy the equation

$$\cos 2\theta = \sin^2 \theta .$$

Working:

Answers:

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3. Given that $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{c} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$, find $(\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c}$.

Working:

Answer:

4. The polynomial $x^3 + ax^2 - 3x + b$ is divisible by $(x-2)$ and has a remainder 6 when divided by $(x+1)$. Find the value of a and of b.

Working:

Answers:

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5. Given that $\boldsymbol{A} = \begin{pmatrix} 3 & -2 \\ -3 & 4 \end{pmatrix}$ and $\boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the values of λ for which $(\boldsymbol{A} - \lambda \boldsymbol{I})$ is a singular matrix.

Working:

Answers:

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6. When a boy plays a game at a fair, the probability that he wins a prize is 0.25. He plays the game 10 times. Let X denote the total number of prizes that he wins. Assuming that the games are independent, find

(a) $\mathrm{E}(X)$;

(b) $\mathrm{P}(X \le 2)$.

Working:

Answers:

(a) ______________________

(b) ______________________

7. The function f is given by $f(x) = 2 - x^2 - e^x$.

Write down

(a) the maximum value of $f(x)$;

(b) the two roots of the equation $f(x) = 0$.

Working:

Answers:

(a) ______________________

(b) ______________________

8. In the triangle ABC, $\hat{A} = 30^\circ$, $BC = 3$ and $AB = 5$. Find the two possible values of $\hat{B}$.

Working:

Answers:

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9. The independent events A, B are such that $\mathrm{P}(A) = 0.4$ and $\mathrm{P}(A \cup B) = 0.88$. Find

(a) $\mathrm{P}(B)$;

(b) the probability that either A occurs or B occurs, but **not** both.

Working:

Answers:

(a) ______________________________

(b) ______________________________

10. A curve has equation $x^3y^2 = 8$. Find the equation of the normal to the curve at the point $(2, 1)$.

Working:

Answer:

11. The complex number z satisfies the equation

$$\sqrt{z} = \frac{2}{1-\mathrm{i}} + 1 - 4\mathrm{i}\,.$$

Express z in the form $x + \mathrm{i}y$ where $x, y \in \mathbb{Z}$.

Working:

Answer:

12. Find the **exact** value of x satisfying the equation

$$(3^x)(4^{2x+1}) = 6^{x+2}\,.$$

Give your answer in the form $\dfrac{\ln a}{\ln b}$ where $a, b \in \mathbb{Z}$.

Working:

Answer:

13. Solve the inequality $|x-2| \geq |2x+1|$.

Working:

Answer:

14. The random variable X is normally distributed and

$$P(X \leq 10) = 0.670$$
$$P(X \leq 12) = 0.937.$$

Find $E(X)$.

Working:

Answer:

15. The point A is the foot of the perpendicular from the point $(1, 1, 9)$ to the plane $2x + y - z = 6$. Find the coordinates of A.

Working:

Answer:

16. A particle moves in a straight line. Its velocity v ms^{-1} after t seconds is given by $v = \mathrm{e}^{-\sqrt{t}} \sin t$. Find the total distance travelled in the time interval $[0, 2\pi]$.

Working:

Answer:

17. The function f is defined for $x \le 0$ by $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$.

Find an expression for $f^{-1}(x)$.

Working:

Answer:

18. Using the substitution $y = 2 - x$, or otherwise, find $\int \left(\dfrac{x}{2 - x}\right)^2 \mathrm{d}x$.

Working:

Answer:

19. A teacher drives to school. She records the time taken on each of 20 randomly chosen days. She finds that

$$\sum_{i=1}^{20} x_i = 626 \text{ and } \sum_{i=1}^{20} x_i^2 = 19780.8 \text{, where } x_i \text{ denotes the time, in minutes, taken on the } i^{\text{th}} \text{ day.}$$

Calculate an unbiased estimate of

(a) the mean time taken to drive to school;

(b) the variance of the time taken to drive to school.

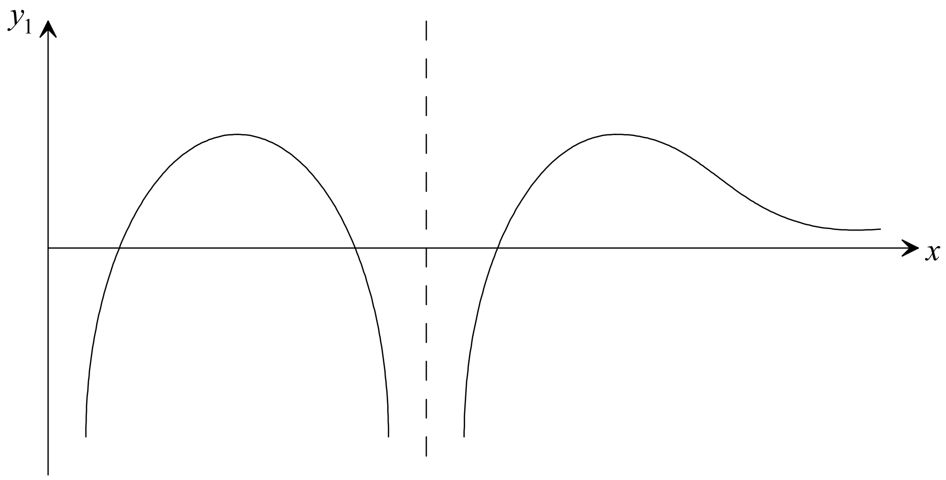
Working:

Answers:

(a) ____________________

(b) ____________________

20. The diagram below shows the graph of $y_1 = f(x)$.



On the axes below, sketch the graph of $y_2 = |f'(x)|$.