INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

May 2001

MATHEMATICS

Higher Level

Paper 1

14 pages

1. $\int t^{\frac{1}{3}}\left(1-\frac{1}{2t^{\frac{5}{3}}}\right)\mathrm{d}t=\int t^{\frac{1}{3}}\left(1-\frac{t^{-\frac{5}{3}}}{2}\right)\mathrm{d}t$

$=\int\left(t^{\frac{1}{3}}-\frac{t^{-\frac{4}{3}}}{2}\right)\mathrm{d}t$ ***(M1)***

$=\frac{3}{4}t^{\frac{4}{3}}+\frac{3}{2}t^{-\frac{1}{3}}+C$ ***(M1)(A1)*** ***(C3)***

Note: Do not penalise for the absence of $+C$.

[3 marks]

2. $2\sin x=\tan x$
$\Rightarrow 2\sin x\cos x-\sin x=0$
$\Rightarrow \sin x(2\cos x-1)=0$ ***(M1)***
$\Rightarrow \sin x=0,\quad \cos x=\frac{1}{2}$
$\Rightarrow x=0,\quad x=\pm\frac{\pi}{3}$ or ±1.05 (3 s. f.) ***(A1)(A1)*** ***(C3)***

OR

$x=0,\quad x=\pm\frac{\pi}{3}$ (or ±1.05 (3 s. f.)) ***(G1)(G1)(G1)*** ***(C3)***

Note: Award ***(G2)*** for $x=0,\pm60^\circ$.

[3 marks]

3. The matrix is of the form

$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$, which represents reflection in $y = x\tan\theta$ ***(M1)***

therefore $\cos 2\theta = \frac{4}{5}, 2\theta > 0$ ***(M1)***

$$\theta = 18.4^\circ \text{ or } \theta = 0.322 \text{ (radians)}$$

The matrix represents reflection in the line

$y = \frac{1}{3}x$ (or $y = 0.333x$, or $y = x\tan 18.4^\circ$, or $y = x\tan 0.322$) ***(A1)*** ***(C3)***

OR

The matrix is of the form

$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$, which represents reflection in $y = x\tan\theta$ ***(M1)***

therefore $\tan 2\theta = \frac{3}{4}, 2\theta > 0$, $\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{3}{4}$ ***(M1)***

$3\tan^2\theta + 8\tan\theta - 3 = 0$, $\Rightarrow \tan\theta = \frac{1}{3}$

The matrix represents reflection in the line

$y = \frac{1}{3}x$ (or $y = 0.333x$, or $y = x\tan 18.4^\circ$, or $y = x\tan 0.322$) ***(A1)*** ***(C3)***

[3 marks]

4. $3x^2 + 4y^2 = 7$

When $x = 1$, $y = 1$ (since $y > 0$) ***(M1)***

$\frac{\mathrm{d}}{\mathrm{d}x}(3x^2 + 4y^2 = 7) \Rightarrow 6x + 8y\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3x}{4y}$ ***(A1)***

The gradient where $x = 1$ and $y = 1$ is $-\frac{3}{4}$ ***(A1)*** ***(C3)***

OR

$3x^2 + 4y^2 = 7$

$\Rightarrow y = \sqrt{\frac{7 - 3x^2}{4}}$, since $y > 0$ ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3x}{2(7 - 3x)^{\frac{1}{2}}}$ ***(A1)***

$= -\frac{3}{4}$, when $x = 1$ ***(A1)*** ***(C3)***

[3 marks]

5. (a) For the set of values of x for which $f(x)$ is real and finite,

$\frac{1}{x^2} - 2 \geq 0, x \neq 0$ ***(M1)***

$x^2 \leq \frac{1}{2}, x \neq 0$

$-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}, x \neq 0$ ***(A1)*** ***(C2)***

(b) $y \geq 0$ ***(A1)*** ***(C1)***

[3 marks]

6. (a) The unbiased estimate of the population mean is 29.9. ***(G1)*** ***(C1)***

(b) The unbiased estimate of the population variance is 0.0336. ***(G2)*** ***(C2)***

[3 marks]

7. (a) $r = \dfrac{u_2}{u_1} = \dfrac{192}{48} = 4$ *(A1)* *(C1)*

OR

$r = \dfrac{u_{n+1}}{u_n} = \dfrac{3(4)^{n+2}}{3(4)^{n+1}} = 4$ *(A1)* *(C1)*

(b) $S_n = \dfrac{u_1(r^n - 1)}{(r-1)} = \dfrac{48(4^n - 1)}{3}$ *(M1)*

$= 16(4^n - 1)$ *(A1)* *(C2)*

[3 marks]

8. x-intercepts are $= \pi, 2\pi, 3\pi$. *(A1)*

Area required $= \left| \int_{\pi}^{2\pi} \dfrac{\sin x}{x}\, dx \right| + \int_{2\pi}^{3\pi} \dfrac{\sin x}{x}\, dx$ *(M1)*

$= 0.4338 + 0.2566$

$= 0.690 \text{ units}^2$ *(G1)* *(C3)*

[3 marks]

9. For the line of intersection:

$$\begin{array}{l} -4x + y + z = -2 \\ \underline{3x - y + 2z = -1} \\ -x \quad\; + 3z = -3 \end{array}$$ *(M1)*

$$\begin{array}{l} -8x + 2y + 2z = -4 \\ \underline{\;\;3x - \;y + 2z = -1} \\ 11x - 3y \qquad = \;\;3 \end{array}$$ *(M1)*

The equation of the line of intersection is $x = \dfrac{3y+3}{11} = 3z + 3$ (or equivalent) *(A1)* *(C3)*

OR

Let $x = 0$ $\Rightarrow \begin{cases} y + \;z = -2 \\ -y + 2z = -1 \end{cases}$

$\Rightarrow 3z = -3,\ z = -1,\ y = -1$

$\Rightarrow (0, -1, -1)$ *(M1)*

Let $z = 0$ $\Rightarrow \begin{cases} -4x + y = -2 \\ 3x - y = -1 \end{cases}$

$\Rightarrow -x = -3,\ x = 3,\ y = 10$

$\Rightarrow (3, 10, 0)$ *(M1)*

The equation of the line of intersection is $\boldsymbol{r} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 11 \\ 1 \end{pmatrix}$ (or equivalent) *(A1)* *(C3)*

[3 marks]

10. If $(z+2\text{i})$ is a factor then $(z-2\text{i})$ is also a factor. ***(A1)***

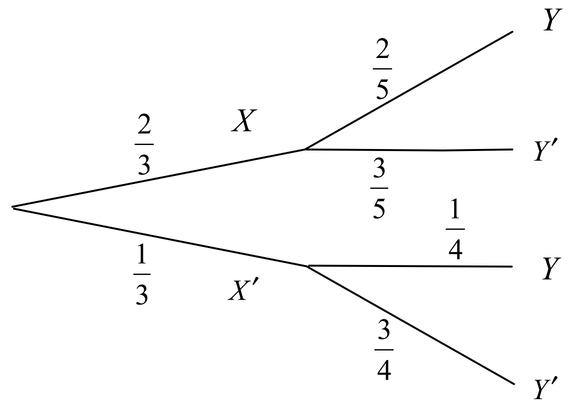
$$(z+2\text{i})(z-2\text{i})=(z^2+4)$$

The other factor is $(2z^3-3z^2+8z-12)\div(z^2+4)=(2z-3)$ ***(M1)(A1)***

The other two factors are $(z-2\text{i})$ and $(2z-3)$. ***(C1)(C2)***

[3 marks]

11.



(a) $\text{P}(Y')=\frac{2}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{3}{4}$ ***(M1)***

$=\frac{13}{20}$ ***(A1)*** ***(C2)***

(b) $\text{P}(X'\cup Y')=1-\text{P}(X\cap Y)=1-\frac{4}{15}$

$=\frac{11}{15}$ ***(A1)*** ***(C1)***

[3 marks]

12. Let $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ be the direction vectors of the two lines. Then the normal to the plane is

$$\boldsymbol{d}_1\times\boldsymbol{d}_2=\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ 1 & -2 & 1\\ 3 & -3 & 5\end{vmatrix}$$ ***(M1)***

$=-7\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k}$ (or equivalent) ***(A1)***

Then equation of the plane is for the form $-7x-2y+3z=c$ or $\boldsymbol{r}.(-7\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k})=c$

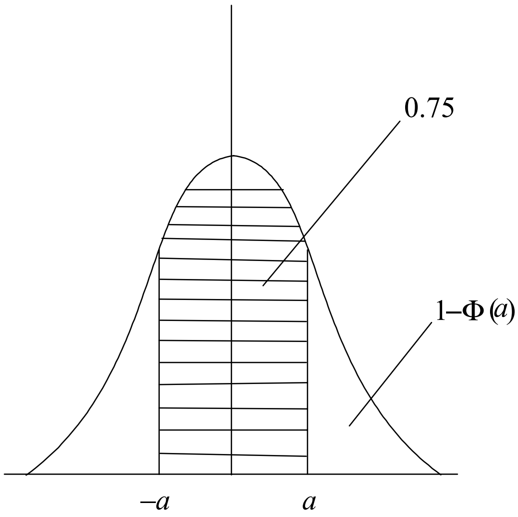
Using the point (1, 1, 2) which is in the plane gives the equation of the plane

$-7x-2y+3z=-3$ or $\boldsymbol{r}.(-7\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k})=-3$ or $\boldsymbol{r}=\begin{pmatrix}1\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\1\end{pmatrix}+\mu\begin{pmatrix}3\\-3\\5\end{pmatrix}$ (or equivalent)

(A1) ***(C3)***

[3 marks]

13.



From the diagram

$$1-2(1-\Phi(a))=0.75 \qquad \textit{(M1)}$$

$$2\Phi(a)=1.75 \qquad \textit{(A1)}$$

$$a=1.15 \qquad \textit{(A1)} \qquad \textit{(C3)}$$

[3 marks]

14.

$$\arg\ (b+\mathrm{i})^2 = 60^\circ\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \quad \arg\ (b+\mathrm{i}) \ = 30^\circ, \left(\frac{\pi}{6}\right) \text{since } b>0 \qquad \textit{(M1)}$$

$$\frac{1}{b}=\tan 30^\circ \text{ or } \tan\frac{\pi}{6} \qquad \textit{(A1)}$$

$$b=\sqrt{3} \qquad \textit{(A1)} \qquad \textit{(C3)}$$

OR

$$\arg\ (b+\mathrm{i})^2 = 60^\circ\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \quad \arg\ (b^2-1+2b\mathrm{i}) = 60^\circ\left(\frac{\pi}{3}\right) \qquad \textit{(M1)}$$

$$\frac{2b}{(b^2-1)}=\sqrt{3} \qquad \textit{(A1)}$$

$$\sqrt{3}b^2-2b-\sqrt{3}=0$$

$$\left(\sqrt{3}b+1\right)\left(b-\sqrt{3}\right)=0$$

$$b=\sqrt{3}, \text{ since} >0 \qquad \textit{(A1)} \qquad \textit{(C3)}$$

OR

$$b=1.73 \text{ (3 s.f.)} \qquad \textit{(M0)(G2)} \qquad \textit{(C2)}$$

[3 marks]

15. If X ~ Bin $(5, p)$ and $P(X = 4) = 0.12$ then

$$\binom{5}{4} p^4 (1-p) = 0.12$$ ***(M1)***

$$5p^5 - 5p^4 + 0.12 = 0$$ ***(A1)***

$$p = 0.459 \text{ (3 s.f) or } 0.973 \text{ (3 s.f)}$$ ***(G1)*** ***(C3)***

[3 marks]

16. Given $$\frac{dx}{dt} = kx(5-x)$$

then $$\frac{1}{x(5-x)} \frac{dx}{dt} = k$$ ***(M1)***

$$\int \frac{1}{5x} + \frac{1}{5(5-x)} dx = \int k dt$$ ***(A1)***

$$\frac{1}{5}\ln x - \frac{1}{5}\ln(5-x) = kt + C \textbf{ or } \left(\frac{x}{5-x}\right)^{\frac{1}{5}} = Ae^{kt} \textbf{ or } \left(\frac{x}{5-x}\right) = Ae^{5kt}$$ ***(A1)*** ***(C3)***

[3 marks]

17. Given $s = 40t + 0.5at^2$, then the maximum height is reached when $\frac{ds}{dt} = 0$ ***(M1)***

$$at + 40 = 0$$ ***(M1)***

$$a = \frac{-40}{25} = -1.6 \text{ (units not required)}$$ ***(A1)*** ***(C3)***

[3 marks]

18. For $kx^2 - 3x + (k+2) = 0$ to have two distinct real roots then

$k \neq 0$ ***(A1)***

and $9 - 4k(k+2) > 0$ ***(M1)***

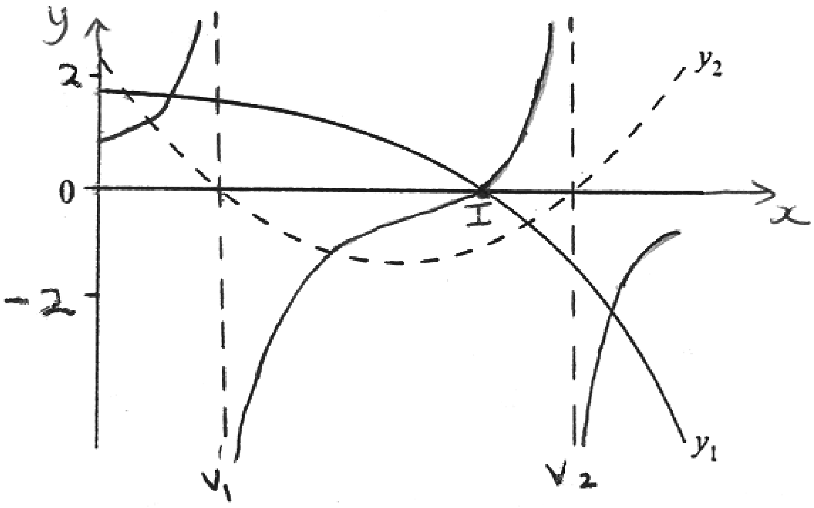
$4k^2 + 8k - 9 < 0$

$-2.803 < k < 0.803$ ***(A1)***

Set of values of k is $-2.80 < k < 0.803, k \neq 0$ ***(C2)(C1)***

[3 marks]

19.



(A1)(A1)(A1) ***(C3)***

Note: Award ***(A1)*** for the shape of the graph (all 3 sections), ***(A1)*** for both asymptotes $(v_1 \text{ and } v_2)$, ***(A1)*** for the x-intercept I.

[3 marks]

20. (a) $f'(x) = \pi\cos(\pi x)e^{(1+\sin \pi x)}$ ***(A1)*** ***(C1)***

(b) For maximum or minimum points, $f'(x) = 0$

$$\cos \pi x = 0$$ ***(M1)***

$$\pi x = \frac{2k+1}{2}\pi$$

then $$x_n = \frac{2n+1}{2}$$ ***(A1)*** ***(C2)***

[3 marks]