



**MATHEMATICS  
 HIGHER LEVEL PAPER 1**

Wednesday 3 May 2000 (afternoon)

2 hours

Name

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Number

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**INSTRUCTIONS TO CANDIDATES**

- Write your name and candidate number in the boxes above.
- Do not open this paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /60	TOTAL /60	TOTAL /60

*Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

1. (a) Sketch the graph of  $f(x) = \sin 3x + \sin 6x$ ,  $0 \leq x \leq 2\pi$ .
- (b) Write down the exact period of the function  $f$ .

*Working:*

*Answer:*

(b) \_\_\_\_\_

2. The transformation  $T_1$  is represented by the matrix  $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$  and the transformation  $T_2$  is represented by the matrix  $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ .

(a) Calculate the matrix  $(T_1 T_2)^{-1}$ .

(b) Describe the transformation represented by the matrix  $(T_1 T_2)^{-1}$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

3. Let  $z_1 = a \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$  and  $z_2 = b \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$ .

Express  $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}^3$  in the form  $z = x + yi$ .

*Working:*

*Answer:*

4. A sample of 70 batteries was tested to see how long they last. The results were:

Time (hours)	Number of batteries (frequency)
$0 \leq t < 10$	2
$10 \leq t < 20$	4
$20 \leq t < 30$	8
$30 \leq t < 40$	9
$40 \leq t < 50$	12
$50 \leq t < 60$	13
$60 \leq t < 70$	8
$70 \leq t < 80$	7
$80 \leq t < 90$	6
$90 \leq t \leq 100$	1
<b>Total</b>	<b>70</b>

Find

- (a) the sample standard deviation;
- (b) an unbiased estimate of the standard deviation of the population from which this sample is taken.

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_

5. Find the coefficient of  $x^7$  in the expansion of  $(2 + 3x)^{10}$ , giving your answer as a whole number.

*Working:*

*Answer:*

6. The system of equations represented by the following matrix equation has an infinite number of solutions.

$$\begin{pmatrix} 2 & -1 & -9 \\ 1 & 2 & 3 \\ 2 & 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ k \end{pmatrix}$$

Find the value of  $k$ .

*Working:*

*Answer:*

7. In a game a player rolls a biased tetrahedral (four-faced) die. The probability of each possible score is shown below.

<b>Score</b>	1	2	3	4
<b>Probability</b>	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	$x$

Find the probability of a total score of six after two rolls.

*Working:*

*Answer:*

8. Find a vector that is normal to the plane containing the lines  $L_1$  and  $L_2$ , whose equations are:

$$L_1: \mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$$

$$L_2: \mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{j} + 3\mathbf{k})$$

*Working:*

*Answer:*

9. The sum of the first  $n$  terms of an arithmetic sequence is  $S_n = 3n^2 - 2n$ . Find the  $n$ th term  $u_n$ .

*Working:*

*Answer:*

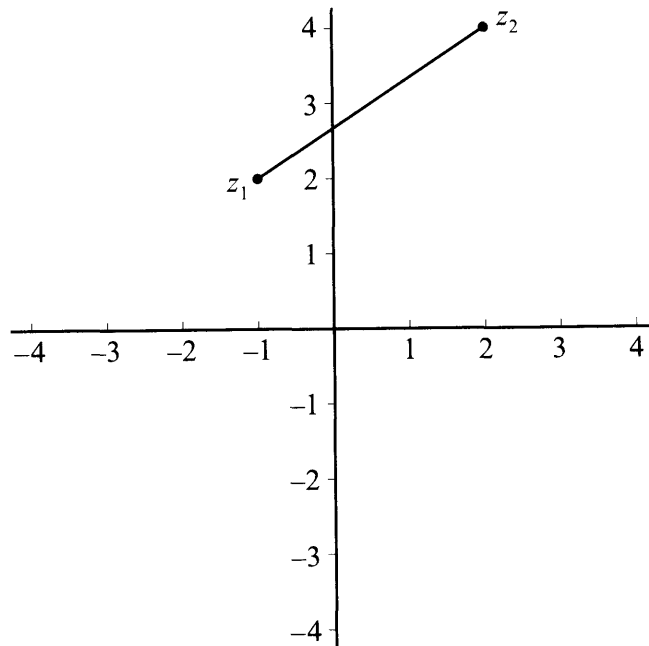


10. The plane  $6x - 2y + z = 11$  contains the line  $x - 1 = \frac{y + 1}{2} = \frac{z - 3}{l}$ . Find  $l$ .

*Working:*

*Answer:*

11. The points  $z_1 = -1 + 2i$  and  $z_2 = 2 + 4i$  and the line segment  $[z_1 z_2]$  are shown in the complex plane below.



Let  $z_3 = -i z_1$  and  $z_4 = -i z_2$ .

- (a) Plot  $z_3$  and  $z_4$  on the complex plane and draw the line segment  $[z_3 z_4]$ .
- (b) Write down the transformation that maps the line segment  $[z_1 z_2]$  onto the line segment  $[z_3 z_4]$ .

*Working:*

*Answer:*

(b) \_\_\_\_\_

12. The probability distribution of a discrete random variable  $X$  is given by

$$P(X = x) = k\left(\frac{2}{3}\right)^x, \text{ for } x = 0, 1, 2, \dots$$

Find the value of  $k$ .

*Working:*

*Answer:*

13. The velocity,  $v$ , of an object, at a time  $t$ , is given by  $v = ke^{-\frac{t}{2}}$ , where  $t$  is in seconds and  $v$  is in  $\text{m s}^{-1}$ . Find the distance travelled between  $t = 0$  and  $t = a$ .

*Working:*

*Answer:*

14. Mr Blue, Mr Black, Mr Green, Mrs White, Mrs Yellow and Mrs Red sit around a circular table for a meeting. Mr Black and Mrs White must not sit together.

Calculate the number of different ways these six people can sit at the table without Mr Black and Mrs White sitting together.

*Working:*

*Answer:*

15. Find the coordinates of the point which is nearest to the origin on the line

$$L: x = 1 - \lambda, y = 2 - 3\lambda, z = 2.$$

*Working:*

*Answer:*

16. Given that  $x > 0$ , find the solution of the following system of equations:

$$\frac{8x^3}{y} = 3$$

$$xy - y = x^2 + \frac{9}{4}$$

*Working:*

*Answers:*

17. A rectangle is drawn so that its lower vertices are on the  $x$ -axis and its upper vertices are on the curve  $y = \sin x$ , where  $0 \leq x \leq \pi$ .

(a) Write down an expression for the area of the rectangle.

(b) Find the maximum area of the rectangle.

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

18. Find the values of  $a > 0$ , such that  $\int_a^{a^2} \frac{1}{1+x^2} dx = 0.22$ .

*Working:*

*Answers:*

19. Let  $f: x \mapsto e^{\sin x}$ .

(a) Find  $f'(x)$ .

There is a point of inflexion on the graph of  $f$ , for  $0 < x < 1$ .

(b) Write down, but do not solve, an equation in terms of  $x$ , that would allow you to find the value of  $x$  at this point of inflexion.

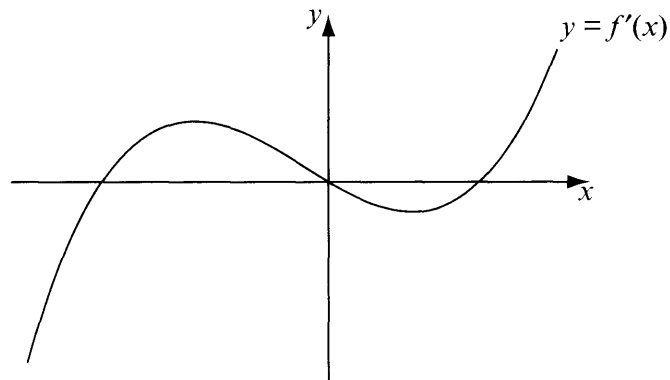
*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

20. The diagram shows the graph of  $y = f'(x)$ .



Indicate, and label clearly, **on the graph**

- (a) the points where  $y = f(x)$  has minimum points;
- (b) the points where  $y = f(x)$  has maximum points;
- (c) the points where  $y = f(x)$  has points of inflexion.

*Working:*