



**MATHEMATICS**

**Higher Level**

Tuesday 3 November 1998 (afternoon)

Paper 1

2 hours

**A**

Candidate name:	Candidate category & number:								
	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> </tr> </table>								
<p>This examination paper consists of 20 questions. The maximum mark for each question is 4. The maximum mark for this paper is 80.</p> <p style="text-align: center;"><b>INSTRUCTIONS TO CANDIDATES</b></p> <p>Write your candidate name and number in the boxes above.</p> <p>Do NOT open this examination paper until instructed to do so.</p> <p>Answer ALL questions in the spaces provided.</p> <p>Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.</p>									

**B**

QUESTIONS ANSWERED
ALL

**C**

EXAMINER	MODERATOR
TOTAL /80	TOTAL /80

**D**

IBCA
TOTAL /80

**EXAMINATION MATERIALS**

Required:

IB Statistical Tables

Calculator

Ruler and compasses

Allowed:

A simple translating dictionary for candidates not working in their own language

Millimetre square graph paper

### FORMULAE

**Trigonometrical identities:**

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \cos^2 \theta - \sin^2 \theta$$

$$\text{If } \tan \frac{\theta}{2} = t \text{ then } \sin \theta = \frac{2t}{1+t^2} \text{ and } \cos \theta = \frac{1-t^2}{1+t^2}$$

**Integration by parts:**

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

**Standard integrals:**

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c \quad (|x| < a)$$

**Statistics:** If  $(x_1, x_2, \dots, x_n)$  occur with frequencies  $(f_1, f_2, \dots, f_n)$  then the mean  $m$  and standard deviation  $s$  are given by

$$m = \frac{\sum f_i x_i}{\sum f_i}, \quad s = \sqrt{\frac{\sum f_i (x_i - m)^2}{\sum f_i}}, \quad i = 1, 2, \dots, n$$

**Binomial distribution:**

$$p_x = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to this examination paper.

1. Let  $f(x) = \frac{1}{x+1}$  and  $g(x) = \sqrt{x^2 - 1}$ . Find

(a)  $f^{-1}(x)$ ;

(b)  $(f \circ g)(x)$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

2. Solve the system of simultaneous equations:

$$\begin{aligned}x + 2y &= 5 \\ 4^x &= 8^y\end{aligned}$$

*Working:*

*Answer:*

3. Let  $P(x) = x^3 - 3x^2 + 4x + c$ .

(a) Calculate  $P(-2)$ .

(b) If the remainder when  $P(x)$  is divided by  $(x + 2)$  is  $-23$ , find the value of  $c$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

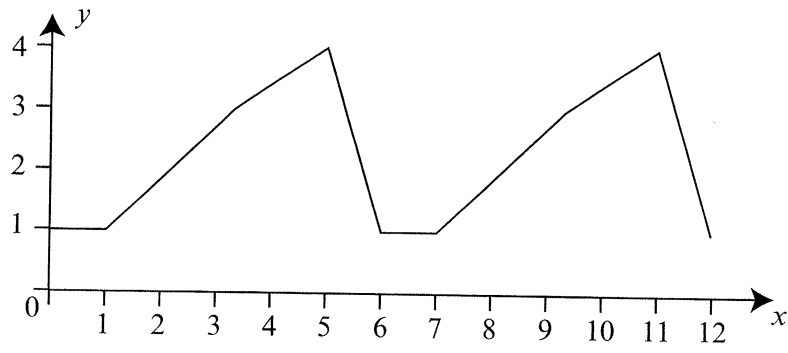
(b) \_\_\_\_\_

4. Find the non-zero value of  $k$  if the coefficients of  $x^3$  and  $x^4$  in the expansion of  $(1 + kx)^5$  are equal.

*Working:*

*Answer:*

5. A part of the graph of the periodic function  $f(x)$  is given below.



(a) Find its period.

(b) Find  $f(41)$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

6. The discrete random variable  $X$  has the probability distribution given in the table below.

$x$	$p(X = x)$
0	$k$
1	$\frac{1}{4}$
2	$\frac{1}{4}$
3	$3k$

(a) Find the value of  $k$ .

(b) Calculate  $E(X)$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

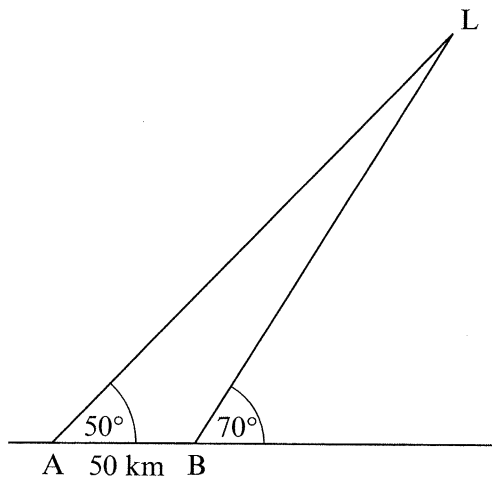
7. Given the vectors  $\vec{a} = -2\vec{i} + p\vec{j} - \vec{k}$  and  $\vec{b} = \vec{i} + 3(p + 4)\vec{j} + (2p - 5)\vec{k}$ , find all values of  $p$ , for which  $\vec{a}$  is perpendicular to  $\vec{b}$ .

*Working:*

*Answers:*



8. A ship travels from A to B and observes a lighthouse at L.



Using the distance and angles given in the diagram, find the distance of the ship at point B from the lighthouse at L.

*Working:*

*Answer:*

9. If  $f(x) = \ln(6x^2 - 5x - 6)$ , find

- (a) the exact domain of  $f(x)$ ;
- (b) the range of  $f(x)$ .

<i>Working:</i>	<i>Answers:</i> (a) _____ (b) _____
-----------------	---

10. If  $z = a + bi$ , with  $a, b \in \mathbb{R}$ , find all ordered pairs  $(a, b)$  such that  $z^2 = -5 + 12i$ .

<i>Working:</i>	<i>Answers:</i> _____
-----------------	--------------------------

11. Find all values of  $m$  such that the equation  $mx^2 - 2(m + 2)x + m + 2 = 0$  has

- (a) two real roots;
- (b) two real roots, one positive and one negative.

<i>Working:</i>	<i>Answers:</i> (a) _____ (b) _____
-----------------	---

12. Events  $A$  and  $B$  are given such that  $p(A \cap B) = \frac{1}{5}$ ,  $p(B | A) = \frac{1}{2}$ , and  $p(A | B) = \frac{3}{10}$ . Find

- (a)  $p(B)$ ;
- (b)  $p(A)$ ;
- (c)  $p(A \cup B)$ .

<i>Working:</i>	<i>Answers:</i> (a) _____ (b) _____ (c) _____
-----------------	--

13. Given the equation  $3 \cos x - 4 \sin x = -5$ , solve for  $\tan x$  expressing your answer **exactly** in the form  $\tan x = \frac{p}{q}$ , where  $p, q \in \mathbb{Z}$ .

*Working:*

*Answer:*

14. Given the following two matrices,

$$M = \begin{pmatrix} 1 & -1 & -2 \\ 1 & 1 & -2 \\ 1 & 2 & a \end{pmatrix} \text{ and } M^{-1} = \frac{1}{2} \begin{pmatrix} b & -5 & 4 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{pmatrix},$$

find the values of  $a$  and  $b$ .

*Working:*

*Answers:*

15. Consider the two planes  $P$  and  $Q$ , where

$$P: 6x - 2y + 3z + 8 = 0;$$

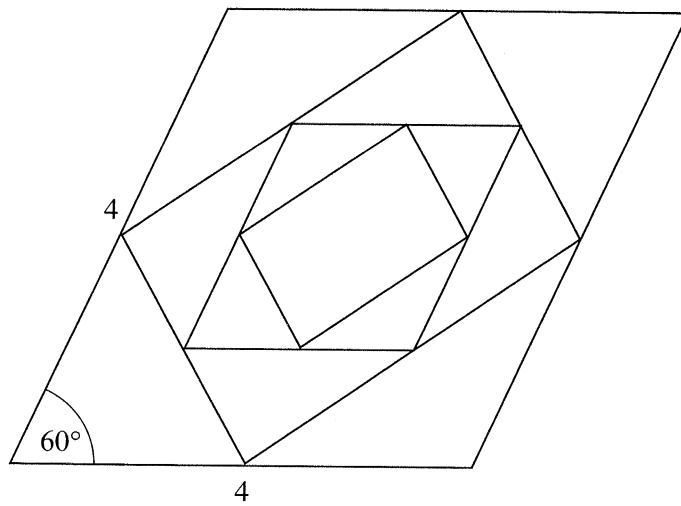
$$Q: x - 3y + 5 = 0.$$

Find the equation of a third plane  $R$  which is perpendicular to both  $P$  and  $Q$  and contains the point  $(2, 2, 3)$ .

*Working:*

*Answer:*

16. In the diagram below, each successive parallelogram is formed by joining the midpoints of the sides of the previous parallelogram. The largest parallelogram has sides of length 4.



**Diagram not drawn to scale**

If the pattern shown is infinite, find the sum of the areas of the parallelograms.

*Working:*

*Answer:*

17. The probability density function of a random variable is given by

$$f(x) = \begin{cases} \frac{x}{\sqrt{3}} \sqrt{x^2 - 1}, & 1 \leq x \leq k; \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of  $k$ .

*Working:*

*Answer:*

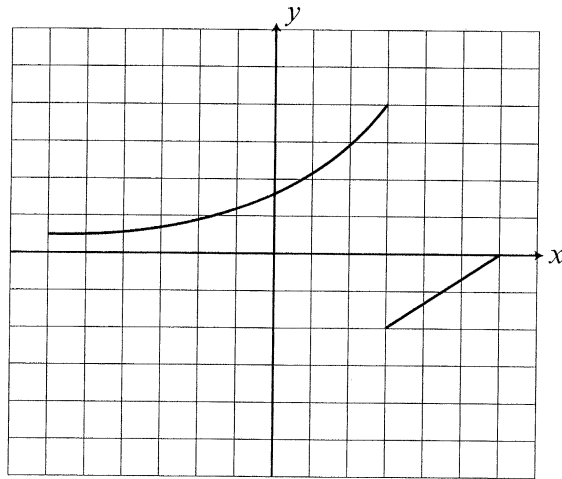
18. The vectors  $\vec{v} = \vec{i} + 2\vec{j} - 3\vec{k}$ ,  $\vec{u} = 2\vec{i} + 2\vec{j} + 3\vec{k}$ , and  $\vec{w} = \vec{i} + (2 - t)\vec{j} + (t + 1)\vec{k}$  are given.

Find the parameter  $t$  such that the three vectors  $\vec{u}$ ,  $\vec{v}$  and  $\vec{w}$  are coplanar.

*Working:*

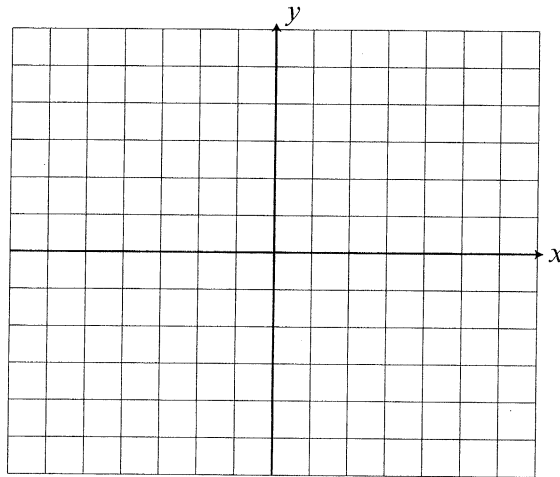
*Answer:*

19. The graph of a function  $f(x)$  for  $-6 \leq x \leq 6$  is given below.



Sketch the graphs of the following functions.

- (a)  $f^{-1}(x)$





20. Let

$$F(x) = \int_0^x \frac{dt}{1 + e^{t^2}}.$$

- (a) Find  $f(x) = F'(x)$ .
- (b) For what values of  $k$  does the equation  $f(x) = k$  have a real solution?

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_