



**MATHEMATICS**

**Higher Level**

Tuesday 5 May 1998 (afternoon)

Paper 1

2 hours

**A**

Candidate name:	Candidate category & number:								
	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%; height: 20px;"></td> <td style="width: 10%; height: 20px;"></td> <td style="width: 10%; height: 20px;"></td> <td style="width: 10%; height: 20px;"></td> <td style="width: 10%; height: 20px;"></td> <td style="width: 10%; height: 20px;"></td> <td style="width: 10%; height: 20px;"></td> <td style="width: 10%; height: 20px;"></td> </tr> </table>								
<p>This examination paper consists of 20 questions. The maximum mark for each question is 4. The maximum mark for this paper is 80.</p> <p style="text-align: center;"><b>INSTRUCTIONS TO CANDIDATES</b></p> <p>Write your candidate name and number in the boxes above.</p> <p>Do NOT open this examination paper until instructed to do so.</p> <p>Answer ALL questions in the spaces provided.</p> <p>Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.</p>									

**B**

QUESTIONS ANSWERED
ALL

**C**

EXAMINER	MODERATOR
TOTAL /80	TOTAL /80

**D**

IBCA
TOTAL /80

**EXAMINATION MATERIALS**

**Required**

- IB Statistical Tables
- Calculator
- Ruler and compasses

**Allowed**

- A simple translating dictionary for candidates not working in their own language
- Millimetre square graph paper

**FORMULAE**

**Trigonometrical identities:**

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \cos^2 \theta - \sin^2 \theta$$

$$\text{If } \tan \frac{\theta}{2} = t \text{ then } \sin \theta = \frac{2t}{1+t^2} \text{ and } \cos \theta = \frac{1-t^2}{1+t^2}$$

**Integration by parts:**

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

**Standard integrals:**

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c \quad (|x| < a)$$

**Statistics:** If  $(x_1, x_2, \dots, x_n)$  occur with frequencies  $(f_1, f_2, \dots, f_n)$  then the mean  $m$  and standard deviation  $s$  are given by

$$m = \frac{\sum f_i x_i}{\sum f_i} \quad s = \sqrt{\frac{\sum f_i (x_i - m)^2}{\sum f_i}}, \quad i = 1, 2, \dots, n$$

**Binomial distribution:**

$$p_x = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to this examination paper.

1. Given that  $\cos \theta = \frac{2}{5}$  and  $\sin \theta < 0$ , find the exact values of  $\sin \theta$ ,  $\tan \theta$  and  $\sec \theta$ .

*Working:*

*Answers:*

2. A discrete random variable  $X$  has the following probability distribution.

$x$	$p(X = x)$
0	$\frac{1}{8}$
1	$3k$
2	$\frac{1}{6}k$
3	$\frac{1}{4}$
4	$\frac{1}{6}k$

- (a) Find the exact value of  $k$ .
- (b) Calculate  $p(0 < X < 4)$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

3. Find all real values of  $x$  so that  $3^{x^2-1} = (\sqrt{3})^{126}$ .

*Working:*

*Answers:*

4. Express the complex number  $z = -2 + i2\sqrt{3}$  in the form  $re^{i\theta}$  where  $\theta \in \mathbb{R}$ , and  $r > 0$ .

*Working:*

*Answer:*

5. For two independent events  $A$  and  $B$ ,  $p(A|B) = \frac{1}{4}$  and  $p(A \cap B) = \frac{1}{32}$ . Find  $p(A)$  and  $p(B)$ .

*Working:*

*Answers:*

6. The mean test score for a mathematics class was 60 with a standard deviation of 10. Assuming that the test scores are normally distributed, find the proportion of students scoring more than 80 in the given test.

*Working:*

*Answer:*

7. The first, second and the  $n$ th terms of an arithmetic sequence are 2, 6, and 58, respectively.
- (a) Find the value of  $n$ .
  - (b) For that value of  $n$ , find the exact value of the sum of  $n$  terms of a geometric sequence whose first term is 2 and common ratio is  $\frac{1}{2}$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

8. When a biased die is rolled the numbers from 1 to 6 appear according to the following probability distribution.

Number on the uppermost face of the die	1	2	3	4	5	6
Probability	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

If  $X$  is the random variable which takes on values 1 through 6 of the above table find the exact values of  $E(X)$  and  $\text{Var}(X)$ .

*Working:*

*Answers:*

9. Find all values of  $x$  in the interval  $0 \leq x \leq 2\pi$ , so that

$$\sin x \tan x = \sin x .$$

*Working:*

*Answers:*



10. Find the acute angle between the planes

$$2x + 3y - z = 7 \quad \text{and} \quad 7x - y + 3z = -5$$

to the nearest tenth of a degree.

*Working:*

*Answer:*

11. Given  $f(x) = \frac{\arcsin x}{\ln x}$ , find  $f'(x)$ .

*Working:*

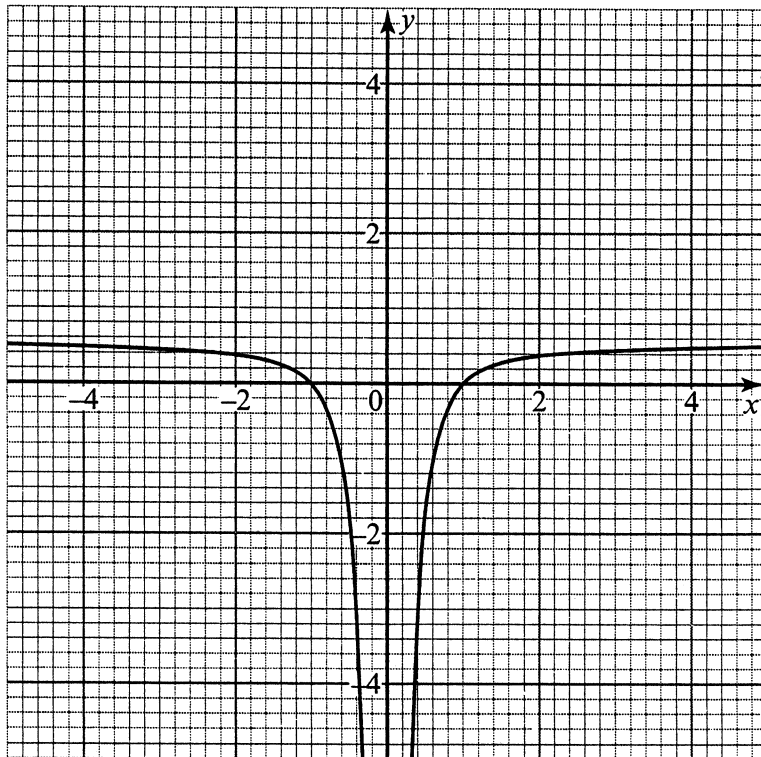
*Answer:*

12. Find the area completely enclosed by the curve  $y^2 = x^2 - x^4$ ,  $-1 \leq x \leq 1$ .

*Working:*

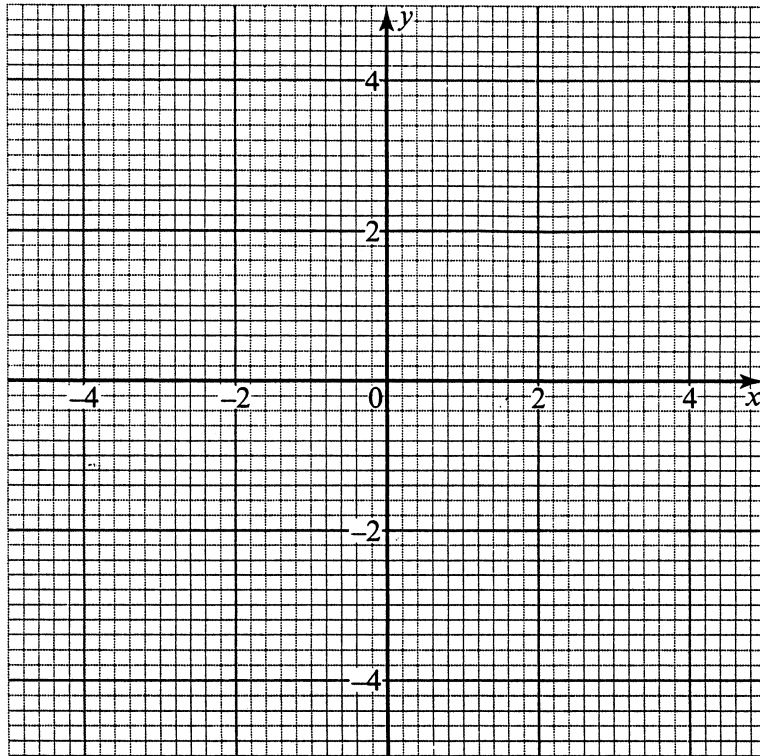
*Answer:*

13. Given below is the graph of a function  $y = f(x)$  with asymptotes  $x = 0$  and  $y = \frac{1}{2}$ .

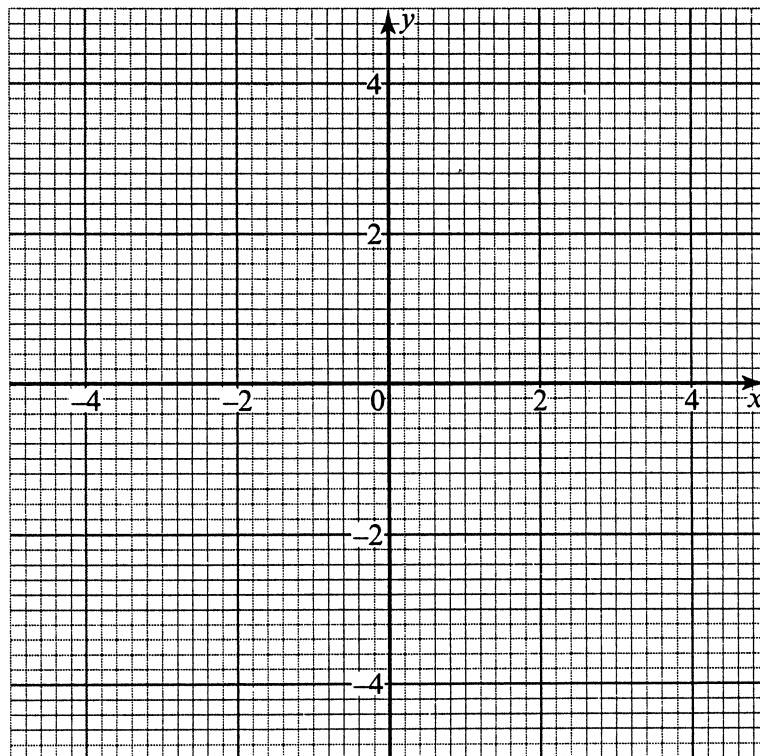


On the axes provided, sketch the graphs of the following functions including their asymptotes.

(a)  $y = |f(x)|$



(b)  $y = \frac{1}{f(x)}$



14. Let  $0 < x < \infty$ , and  $y(x) > 0$ . Solve the differential equation  $\frac{dy}{dx} = y \cos x$ ,  $y = 1$  when  $x = \frac{\pi}{2}$ .

*Working:*

*Answer:*

---

15. The probability density function of a random variable  $X$  is given by

$$f(x) = \begin{cases} 6x(1+x), & 0 < x < k; \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the value of  $k$ .
- (b) Find the exact value of the mean of  $X$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

16. The equation of a curve is given by  $x^3 + y^3 = 6xy$ .

Find the equation of the tangent to the given curve at the point (3, 3).

*Working:*

*Answer:*

17. Find  $\int \arctan x \, dx$ .

*Working:*

*Answer:*

18. Find all values of  $k$  for which the system of equations

$$\begin{aligned} 2x - 2y + kz &= 0 \\ x + 4z &= 0 \\ kx + y + z &= 0 \end{aligned}$$

has a nonzero solution [that is,  $(x, y, z) \neq (0, 0, 0)$ ].

*Working:*

*Answers:*

19. Given the function  $f(x) = e^{3x^2} + \sqrt{x^2 - 4}$ , find the domain and range of  $f(x)$ .

*Working:*

*Answers:*

20. If  $z = x + iy$  and  $z^* = x - iy$ , where  $x, y \in \mathbb{R}$  and  $i = \sqrt{-1}$ , find

(a) the equation of the circle in the  $x$ - $y$  plane which is given by

$$|z - 2 - i\sqrt{3}| = (\sqrt{2}) |z^* - 1 + i\sqrt{3}|;$$

(b) the centre and radius of this circle.

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_