INTERNATIONAL BACCALAUREATE

MATHEMATICS

Higher Level

Monday 6 May 1996 (afternoon)

Paper 1 — 2 hours

This examination paper consists of 20 questions.

The maximum mark for each question is 4.

The maximum mark for this paper is 80.

This examination paper consists of 14 pages.

INSTRUCTIONS TO CANDIDATES

Write your candidate reference number in the box:

DO NOT open this examination paper until instructed to do so.

Answer ALL questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

EXAMINATION MATERIALS

Required/Essential:

IB Statistical Tables
Electronic calculator
Ruler and compasses

Allowed/Optional:

A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Trigonometrical identities:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$

$$\text{If } \tan\frac{\theta}{2} = t \text{ then } \sin\theta = \frac{2t}{1+t^2} \text{ and } \cos\theta = \frac{1-t^2}{1+t^2}$$

Integration by parts:

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Standard integrals:

$$\int \frac{\mathrm{d}x}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + c \quad (|x| < a)$$

Statistics: If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard deviation s are given by

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad s = \sqrt{\frac{\Sigma f_i (x_i - m)^2}{\Sigma f_i}}, \qquad i = 1, 2, \ldots, n$$

Binomial distribution:

$$p_x = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to the cover sheet together with this examination paper.

1. Express $\dfrac{12}{x^4 + x^3 - 2x^2}$ as the sum of **four** partial fractions.

Working:

Answer:

2. Find the area of the parallelogram determined by the vectors $\vec{a} = 3\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{b} = 2\vec{i} + \vec{j} - 4\vec{k}$.

Working:

Answer:

3. Describe clearly the locus in the complex plane defined by the equation

$$|z + 2i| = |2iz - 1|.$$

Working:

Answer:

4. Three suppliers A, B and C produce respectively 45%, 30% and 25% of the total number of a certain component that is required by a car manufacturer. The percentages of faulty components in each supplier's output are, again respectively, 4%, 5% and 6%.

What is the probability that a component selected at random is faulty?

Working:

Answer:

5. Find all the real numbers x for which $|4x + 1| > |2x - 3|$.

Working:

Answer:

6. Given that $f(x) = \dfrac{x}{x+2}$ and $g(x) = f\left(\dfrac{x-2}{3}\right)$, derive an expression for $g(x)$ and find its domain.

Working:

Answers:

7. Calculate the intervals between 0 and 2π in which the function

$$y(x) = x - 2 \sin x$$

is increasing.

Working:

Answer:

8. Determine the coefficients of $\frac{1}{x}$ and $\frac{1}{x^2}$ in the expansion of $\left(x + \frac{1}{x}\right)^9$.

Working:

Answers:

9. (a) Given that $\log_a c = \log_a b \times \log_b c$, express

$$\log_4 x^3 + \log_2 x^2 = \log_8 128^3$$

in terms of logarithms to the base 2.

(b) Hence, or otherwise, solve the equation in part (a).

Working:

Answers:

(a) _____

(b) _____

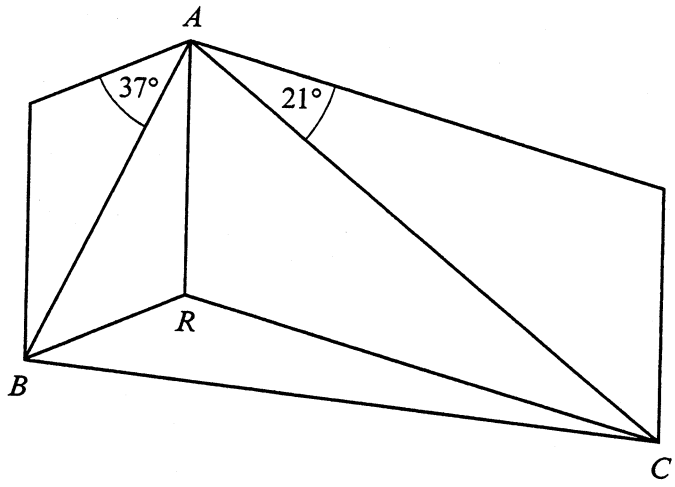
10. Find $\int \frac{dx}{\sqrt{8x - x^2 - 7}}$.

Working:

Answer:

_____

11. A reconnaissance airplane A, flying at a height of three thousand metres above the point R on the surface of the sea, spots a freighter B at an angle of depression 37° and a tanker C at an angle of depression 21°, shown in the figure. The angle BAC is 110°.



Find, to the nearest metre,

(a) the distance CA between the plane and the tanker;

(b) the distance BC between the two ships.

Working:

Answers:

(a) _____

(b) _____

12. How many times must a pair of dice be thrown so that there is a better than even chance of obtaining a double, that is, the same number on both dice?

Working:

Answer:

13. Solve the differential equation

$$\frac{dy}{dx} + 5y = e^{8x}$$

given that $y = \frac{3}{2}$ when $x = 0$.

Working:

Answer:

14. Solve the equation

$$\det\begin{pmatrix} 1 & 4 & 1 \\ x & 2 & 2 \\ x^2 & 2 & 1 \end{pmatrix} = \det\begin{pmatrix} 1 & 3 \\ x & 3 \end{pmatrix}.$$

Working:

Answers:

15. Find the equation of the tangent to the curve

$$3x^2 - 2xy + y^2 = 9$$

at the point $(2, 3)$.

Working:

Answer:

16. Using all the letters of CALCUTTA ,

(a) how many different arrangements of letters can be found?

(b) how many of these arrangements begin and end with the letter C?

Working:

Answers:

(a) _____

(b) _____

17. Let X be a continuous random variable with the probability density function

$$f(x) = \frac{x}{8} + c, \quad 0 \leqslant x \leqslant 4$$

$$= 0, \qquad \text{elsewhere .}$$

Find the value of c and the expected value of X .

Working:

Answers:

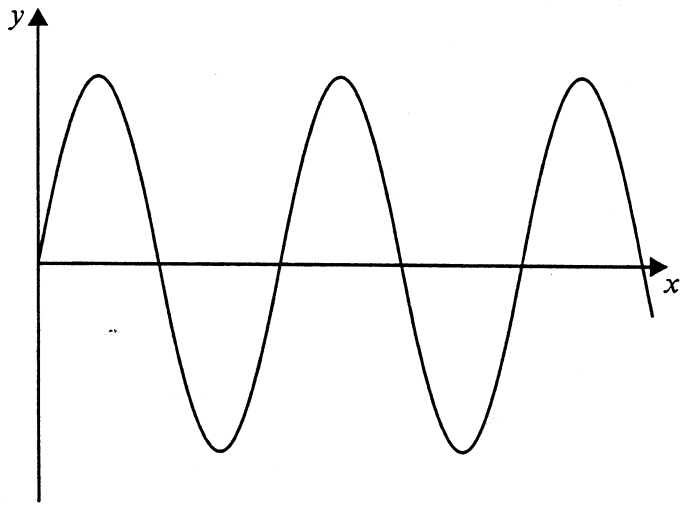
_____

18. The sum of the perimeter of a square of side a cm and the circumference of a circle of radius r cm is 240 cm . What is the value of r if the sum of the areas is a minimum?

Working:

Answer:

19. The diagram below shows part of the graph of the function $y = a \sin bx$ for $x > 0$. Sketch on the same axes the graph of $y = \frac{a}{2} \sin \frac{bx}{2}$.



Working:

20. Given that α and β are the roots of the quadratic equation $x^2 - 5x - 3 = 0$ write down the values of $\alpha\beta$ and $\alpha + \beta$.

Hence, or otherwise, find the quadratic equation whose roots are $\frac{1}{\alpha + 1}$ and $\frac{1}{\beta + 1}$.

Working:

Answers: