INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

May 2000

MATHEMATICAL METHODS

Standard Level

Paper 1

1. $a = 5$

$a + 3d = 40$ (may be implied) *(M1)*

$d = \dfrac{35}{3}$ *(A1)*

$T_2 = 5 + \dfrac{35}{3}$ *(A1)*

$= 16\dfrac{2}{3}$ or $\dfrac{50}{3}$ or 16.7 (3 s.f.) *(A1)* *(C4)*

[4 marks]

2. (a) $f^{-1}(2) \Rightarrow 3x + 5 = 2$ *(M1)*

$x = -1$ *(A1)* *(C2)*

(b) $g(f(-4)) = g(-12+5)$

$= g(-7)$ *(A1)*

$= 2(1+7)$

$= 16$ *(A1)* *(C2)*

[4 marks]

3. (a)

	Boy	Girl	Total
TV	**13**	**25**	**38**
Sport	33	29	**62**
Total	46	**54**	100

$\mathrm{P(TV)} = \dfrac{38}{100}$ *(A1)* *(C2)*

(b) $\mathrm{P(TV \mid Boy)} = \dfrac{13}{46}$ $(= 0.283$ to 3 s.f.$)$ *(A2)* *(C2)*

Notes: Award ***(A1)*** for numerator and ***(A1)*** for denominator. Accept equivalent answers.

[4 marks]

4. $\boldsymbol{u} + \boldsymbol{v} = 4\boldsymbol{i} + 3\boldsymbol{j}$ *(A1)*

Then $a(4\boldsymbol{i} + 3\boldsymbol{j}) = 8\boldsymbol{i} + (b-2)\boldsymbol{j}$

$4a = 8$

$3a = b - 2$ *(A1)*

Whence $a = 2$ *(A1)* *(C2)*

$b = 8$ *(A1)* *(C2)*

[4 marks]

5. (a) $\log_2 5 = \dfrac{\log_a 5}{\log_a 2}$ *(M1)*

$= \dfrac{y}{x}$ *(A1)* *(C2)*

(b) $\log_a 20 = \log_a 4 + \log_a 5$ or $\log_a 2 + \log_a 10$ *(M1)*

$= 2\log_a 2 + \log_a 5$

$= 2x + y$ *(A1)* *(C2)*

[4 marks]

6. $3\cos x = 5\sin x$

$\Rightarrow \dfrac{\sin x}{\cos x} = \dfrac{3}{5}$ *(M1)*

$\Rightarrow \tan x = 0.6$ *(A1)*

$x = 31^\circ$ or $x = 211^\circ$ (to the nearest degree) *(A1)(A1)* *(C2)(C2)*

Note: Deduct ***[1 mark]*** if there are more than two answers.

[4 marks]

7. Required vector will be parallel to $\begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ *(M1)*

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ *(A1)*

Hence required equation is $\boldsymbol{r} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 4 \\ -5 \end{pmatrix}$ *(A1)(A1)* *(C4)*

Note: Accept alternative answers, *e.g.* $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ -5 \end{pmatrix}$.

[4 marks]

8. $y = x^2 - x$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x - 1 =$ gradient at any point. *(M1)*

Line parallel to $y = 5x$

$\Rightarrow 2x - 1 = 5$ *(M1)*

$x = 3$ *(A1)*

$y = 6$ *(A1)*

Point $(3, 6)$ *(C2)(C2)*

[4 marks]

9. $f'(x) = \cos x \Rightarrow f(x) = \sin x + C$ ***(M1)***

$f\left(\frac{\pi}{2}\right) = -2 \Rightarrow \quad -2 = \sin\left(\frac{\pi}{2}\right) + C$ ***(M1)***

$C = -3$ ***(A1)***

$f(x) = \sin x - 3$ ***(A1)*** ***(C4)***

[4 marks]

10. $S = \frac{u_1}{1-r} = \frac{\frac{2}{3}}{1-\left(-\frac{2}{3}\right)}$ ***(M1)(A1)***

$= \frac{2}{3} \times \frac{3}{5}$ ***(A1)***

$= \frac{2}{5}$ ***(A1)*** ***(C4)***

[4 marks]

11. $(a+b)^{12}$

Coefficient of a^5b^7 is $\binom{12}{5} = \binom{12}{7}$ ***(M1)(A1)***

$= 792$ ***(A2)*** ***(C4)***

[4 marks]

12. $\sin A = \frac{5}{13} \Rightarrow \cos A = \pm\frac{12}{13}$ ***(A1)***

But A is obtuse $\Rightarrow \cos A = -\frac{12}{13}$ ***(A1)***

$\sin 2A = 2\sin A \cos A$ ***(M1)***

$= 2 \times \frac{5}{13} \times \left(-\frac{12}{13}\right)$

$= -\frac{120}{169}$ ***(A1)*** ***(C4)***

[4 marks]

13. $4x^2 + 4kx + 9 = 0$

Only one solution $\Rightarrow b^2 - 4ac = 0$ *(M1)*

$16k^2 - 4(4)(9) = 0$ *(A1)*

$k^2 = 9$

$k = \pm 3$ *(A1)*

But given $k > 0$, $k = 3$ *(A1)* *(C4)*

OR

One solution $\Rightarrow (4x^2 + 4kx + 9)$ is a perfect square *(M1)*

$4x^2 + 4kx + 9 = (2x \pm 3)^2$ by inspection *(A2)*

given $k > 0$, $k = 3$ *(A1)* *(C4)*

[4 marks]

14. (a) C has equation $x = 2^y$ *(A1)*

i.e. $y = \log_2 x$ *(A1)* *(C2)*

OR Equation of B is $x = \log_2 y$ *(A1)*

Therefore equation of C is $y = \log_2 x$ *(A1)* *(C2)*

(b) Cuts x-axis $\Rightarrow \log_2 x = 0$

$x = 2^{\circ}$ *(A1)*

$x = 1$

Point is $(1, 0)$ *(A1)* *(C2)*

[4 marks]

15. (a) $\dfrac{f(5+h) - f(5)}{h} = \dfrac{(5.1)^3 - 5^3}{0.1}$

$= 76.51$ (or 76.5 to 3 s.f.) *(A1)* *(C1)*

(b) $\lim\limits_{h \to 0} \dfrac{f(5+h) - f(5)}{h} = f'(5)$ *(M1)*

$= 3(5)^2$ *(A1)*

$= 75$ *(A1)* *(C3)*

[4 marks]