INTERNATIONAL BACCALAUREATE
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MARKSCHEME

November 1999

MATHEMATICAL METHODS

Standard Level

Paper 2

20 pages

Paper 2 Markscheme

Instructions to Examiners

1 All marking must be done using a **red** pen.

2 **Abbreviations**

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

C Marks awarded for **Correct** statements

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 **Follow Through (ft) Marks**

Questions in this paper were constructed to enable a candidate to:

- show, step by step, what he or she knows and is able to do;
- use an answer obtained in one part of a question to obtain answers in the later parts of a question.

Thus errors made at any step of the solution can affect all working that follows. Furthermore, errors made early in the solution can affect more steps or parts of the solution than similar errors made later.

To limit the severity of the penalty for errors made at any step of a solution, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) penalise an error when it **first occurs**;

(ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent parts of the question;

(iii) award **M** marks for a correct method, and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The errors made by a candidate may be: arithmetical errors; errors in algebraic manipulation; errors in geometrical representation; use of an incorrect formula; errors in conceptual understanding.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	**M1**	Amount earned = \$ 600 × 1.02	✓	**M1**
= \$ 612	**A1**	= \$602	×	**A0**
\$ (306 × 1.02) + (306 × 1.04)	**M1**	Amount = 301 × 1.02 + 301 × 1.04	✓	**M1**
= \$ 630.36	**A1**	= \$ 620.06	✓	**A1(ft)**

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by '**(d)**' (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, the working out must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by '**(d)**' (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7 , 1,7 ; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan x.

5 Accuracy of Answers

(a) In the case when the accuracy of answers is **specified in the question** (for example: "all answers should be given to four significant figures") **A** marks are awarded **only if** the correct answers are given to the accuracy required.

(b) When the accuracy is **not** specified in the question, then the general rule applies:

> *Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.*

In this case, the candidate is **penalised once only IN EACH QUESTION** for giving a correct answer to the wrong degree of accuracy. Hence, on the **first** occasion in a question when a correct answer is given to the wrong degree of accuracy **A** marks are **not** awarded. But on **all subsequent occasions** in the same question when correct answers are given to the wrong degree of accuracy then **A** marks **are** awarded.

NOVEMBER 1999

Additional instructions for Assistant Examiners

1. SAMPLES

All examiners are reminded that samples should be sent to the Team Leader by the fastest means possible. IBCA will reimburse examiners for any costs incurred.

2. PAPER 2 EXAMINERS – PART MARKS

Assistant examiners are asked to indicate on candidates' scripts the part marks that they have allocated for each part. To help identify these marks, they have now been incorporated into the markschemes. Where the markscheme has part marks *e.g.* ***[3 marks]*** after a part solution, assistant examiners should note the candidates' marks for that part of the question alongside the solution. They should also write down the total for the question at the end of each question.

1. (a) $\begin{matrix} ① \, 6x+8y=39 \\ ② \, 8x-6y=27 \end{matrix}\Bigg\} \Rightarrow \begin{matrix} 36x+48y=234 \\ 64x-48y=216 \end{matrix}\Bigg\}$ (or equivalent) *(M1)*

$\Rightarrow 100x = 450 \Rightarrow x = 4.5$ *(A1)(AG)*

Using ① $27 + 8y = 39$ *(M1)*

$\Rightarrow 8y = 12$

$\Rightarrow y = 1.5$ *(A1)* ***[4 marks]***

(b) (RQ) is parallel to (SP) and therefore has equation *(M1)*

$6x + 8y = 6(3.5) + 8(8.5) = 21 + 68 = 89$

$\Rightarrow 6x + 8y = 89$ (accept $y = -\left(\frac{3}{4}\right)x + 11.125$, or similar) *(A1)*

At Q, $\begin{matrix} 6x+8y=89 \\ 8x-6y=27 \end{matrix}\Bigg\} \Rightarrow \begin{matrix} 48x+64y=712 \\ 48x-36y=162 \end{matrix}\Bigg\} \Rightarrow 100y = 550$

$6x + 8(5.5) = 89$ *(M1)*

$\Rightarrow 6x = 89 - 44 = 45;$

$x = 7.5$ *(AG)*

$y = 5.5$ *(A1)*

Note: If, instead of solving simultaneously the equations for (PQ) and (RQ) in order to show $x = 7.5$, the candidate substitutes $x = 7.5$ into either equation in order to get $y = 5.5$ make sure $(7.5, 5.5)$ is shown to satisfy the other equation.

[4 marks]

(c) $\vec{PQ} = \begin{pmatrix} 7.5-4.5 \\ 5.5-1.5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ *(A1)*

$\vec{QR} = \begin{pmatrix} 3.5-7.5 \\ 8.5-5.5 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ *(A1)*

$\vec{PQ} \cdot \vec{QR} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \end{pmatrix} = 3 \times (-4) + 4 \times 3$ *(M1)*

$= 0$ *(A1)* ***[4 marks]***

(d) Since $PQRS$ is a parallelogram, we need two equal sides including a 90° angle. *(M1)*

$\vec{PQ} \cdot \vec{QR} = 0 \Rightarrow P\hat{Q}R = 90^\circ$ *(A1)*

$PQ = \sqrt{(3^2 + 4^2)} = 5$ *(M1)*

$QR = \sqrt{((-4)^2 + 3^2)} = 5$ *(A1)*

$PQRS$ is a square *(AG)*

Note: Award *(A1)* only if only perpendicularity shown and no further explanation given.

[4 marks]

2. (a) Area $A = 0.1$ *(A1)* ***[1 mark]***

(b) **EITHER** Since $p(X \geq 12) = p(X \leq 8)$, *(M1)*

then 8 and 12 are symmetrically disposed around the mean. *(M1)(R1)*

Thus mean $= \dfrac{8+12}{2}$ *(M1)*

$= 10$ *(A1)*

> **Notes:** If a candidate says simply "by symmetry $\mu = 10$" with no further explanation award ***[3 marks] (M1, A1, R1)***. As a full explanation is requested award an additional *(A1)* for saying since $p(X < 8) = p(X > 12)$ and another *(A1)* for saying that the normal curve is symmetric.

OR $p(X \geq 12) = 0.1 \quad \Rightarrow p\left(Z \geq \dfrac{12-\mu}{\sigma}\right) = 0.1$ *(M1)*

$\Rightarrow p\left(Z \leq \dfrac{12-\mu}{\sigma}\right) = 0.9$

$p(X \leq 8) = 0.1 \quad \Rightarrow p\left(Z \leq \dfrac{8-\mu}{\sigma}\right) = 0.1$

$\Rightarrow p\left(Z \leq \dfrac{\mu-8}{\sigma}\right) = 0.9$ *(A1)*

So $\dfrac{12-\mu}{\sigma} = \dfrac{\mu-8}{\sigma}$ *(M1)*

$\Rightarrow 12 - \mu = \mu - 8$ *(M1)*

$\Rightarrow \mu = 10$ *(A1)*

[5 marks]

(c) $\Phi\left(\dfrac{12-10}{\sigma}\right) = 0.9$ *(A1)(M1)(A1)*

> **Note:** Award *(A1)* for $\left(\dfrac{12-10}{\sigma}\right)$, *(M1)* for standardising, and *(A1)* for 0.9

$\Rightarrow \dfrac{2}{\sigma} = 1.282$ (or 1.28) *(A1)*

$\sigma = \dfrac{2}{1.282}\left(\text{or } \dfrac{2}{1.28}\right)$ *(A1)*

$= 1.56$ (3 s.f.) *(AG)*

> **Note:** Working backwards from $\sigma = 1.56$ to show it leads the given data should receive a maximum of ***[3 marks]*** if done correctly.

[5 marks]

continued…

Question 2 continued

(d) $p(X \le 11) = p\left(Z \le \frac{11-10}{1.561}\right)$ (or 1.56) *(M1)(A1)*

Note: Award *(M1)* for standardising and *(A1)* for $\left(\frac{11-10}{1.561}\right)$.

$= p(Z \le 0.6407)$ (or 0.641 or 0.64) *(A1)*

$= \Phi(0.6407)$ *(M1)*

$= 0.739$ (3 s.f.) *(A1)* ***[5 marks]***

3. (a) $T = \frac{h}{2}\{y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\}$ *(M1)*

$= \frac{0.5}{2}\left\{1 + 2\sqrt{2} + 2 \times 2 + 2\sqrt{8} + 4\right\}$ *(A2)*

Notes: Award *(A1)* if there is one error only. Award *(AO)* if there is more than one error.

$= \frac{1}{4}\left\{9 + 6\sqrt{2}\right\}$ *(A1)*

≈ 4.37 (3 s.f.) *(A1)*

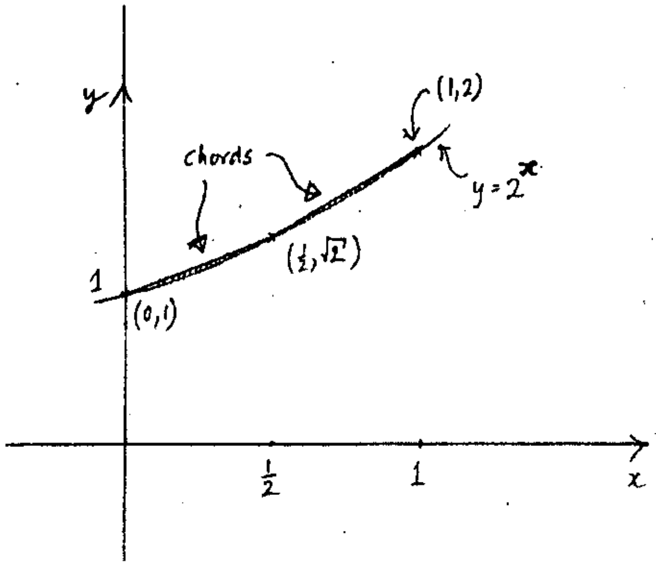
Notes: Candidates may calculate the required area by finding the areas of 4 separate trapezia of width 0.5. If this approach is taken, award *(A1)* for $h = 0.5$, award *(A1)* for 5 correct ordinates, *(M1)* for the correct formula for the area of a single trapezium, *(M1)* for adding the 4 areas together and *(A1)* for the correct answer. If some other rule or an incorrect form of the TR is used, do not award FT marks.

[5 marks]

continued...

Question 3 continued

(b)



On each interval the chords of the curve lie above the curve itself. ***(R2)***

Notes:	Accept valid explanations which do not use a diagram. Any valid explanation must make some mention of the fact that the straight lines of the trapezia have more area under them as they lie above the curve.

[2 marks]

(c)

① x	② 2^x	③ $1+\frac{1}{2}x+\frac{1}{2}x^2$
0	1	$1+0+0=1$
1	2	$1+\frac{1}{2}+\frac{1}{2}=2$
2	4	$1+\frac{1}{2}\times 2+\frac{1}{2}\times 4=1+1+2=4$

(M1)(A2)

Note:	Award ***(M1)*** for the table, ***(A1)*** each for column ② and column ③.

Column ② matches column ③, therefore the values agree. ***(R1)***

Note:	Award full marks for showing that $y=2^x$ and $y=1+\frac{1}{2}x+\frac{1}{2}x^2$ have the same value at each of the points $x=0, x=1$ and $x=2$.

[4 marks]

continued...

Question 3 continued

(d) $\int\left(1+\frac{1}{2}x+\frac{1}{2}x^2\right)\mathrm{d}x = x+\frac{1}{4}x^2+\frac{1}{6}x^3+c$ ***(A3)***

Note: Award ***(A1)*** for x, ***(A1)*** for $\frac{1}{4}x^2$, ***(A1)*** for $\frac{1}{6}x^3$. Do not penalise if the constant c is omitted.

$\int_0^2\left(1+\frac{1}{2}x+\frac{1}{2}x^2\right)\mathrm{d}x = \left[x+\frac{1}{4}x^2+\frac{1}{6}x^3\right]_0^2$ ***(M1)***

$= 2+\frac{1}{4}\times 2^2+\frac{1}{6}\times 2^3$

$= 2+1+\frac{4}{3}$

$= 4\frac{1}{3}$ ***(A1)*** ***[5 marks]***

(e) $\int 2^x\,\mathrm{d}x = \int \mathrm{e}^{x\ln 2}\mathrm{d}x$ ***(M1)***

$= \frac{\mathrm{e}^{x(\ln 2)}}{\ln 2}+c$ ***(A1)***

$= \frac{2^x}{\ln 2}+c$ ***(AG)***

$\int_0^2 2^x\,\mathrm{d}x = \left[\frac{2^x}{\ln 2}\right]_0^2$ ***(M1)***

$= \frac{1}{\ln 2}(2^2-2^0)$ ***(A1)***

$= \frac{3}{\ln 2}$ ***(A1)*** ***[5 marks]***

(f) Relative error $= \dfrac{4\frac{1}{3}-\frac{3}{\ln 2}}{\frac{3}{\ln 2}}$ ***(M1)(A1)***

$= 0.00121$

$= 0.121\,\%$ ***(A1)***

Notes: Accept $-0.121\,\%$ if the order of the terms in the numerator was reversed. The answer need not be converted to a percent to receive full marks.

[3 marks]

4. (a) $f(1)=3 \quad f(5)=3$ *(A1)(A1)* ***[2 marks]***

(b) **EITHER** distance between successive maxima = period *(M1)*

$=5-1$ *(A1)*

$=4$ *(AG)*

OR Period of $\sin kx = \dfrac{2\pi}{k}$; *(M1)*

so period $=\dfrac{2\pi}{\frac{\pi}{2}}$ *(A1)*

$=4$ *(AG)* ***[2 marks]***

(c) **EITHER** $A\sin\left(\dfrac{\pi}{2}\right)+B=3$ and $A\sin\left(\dfrac{3\pi}{2}\right)+B=-1$ *(M1)(M1)*

$\Leftrightarrow A+B=3, -A+B=-1$ *(A1)(A1)*

$\Leftrightarrow A=2, B=1$ *(AG)(A1)*

OR Amplitude= A *(M1)*

$A=\dfrac{3-(-1)}{2}=\dfrac{4}{2}$ *(M1)*

$A=2$ *(AG)*

Midpoint value = B *(M1)*

$B=\dfrac{3+(-1)}{2}=\dfrac{2}{2}$ *(M1)*

$B=1$ *(A1)*

Note:	As the values of $A=2$ and $B=1$ are likely to be quite obvious to a bright student, do not insist on too detailed a proof.

[5 marks]

(d) $f(x)=2\sin\left(\dfrac{\pi}{2}x\right)+1$

$f'(x)=\left(\dfrac{\pi}{2}\right)2\cos\left(\dfrac{\pi}{2}x\right)+0$ *(M1)(A2)*

Note:	Award *(M1)* for the chain rule, *(A1)* for $\left(\dfrac{\pi}{2}\right)$, *(A1)* for $2\cos\left(\dfrac{\pi}{2}x\right)$.

$=\pi\cos\left(\dfrac{\pi}{2}x\right)$ *(A1)*

Notes:	Since the result is given, make sure that reasoning is valid. In particular, the final *(A1)* is for simplifying the result of the chain rule calculation. If the preceding steps are not valid, this final mark should not be given. Beware of "fudged" results.

[4 marks]

continued...

Question 4 continued

(e) (i) $y = k - \pi x$ is a tangent $\Rightarrow -\pi = \pi\cos\left(\frac{\pi}{2}x\right)$ ***(M1)***

$$\Rightarrow -1 = \cos\left(\frac{\pi}{2}x\right)$$ ***(A1)***

$$\Rightarrow \frac{\pi}{2}x = \pi \text{ or } 3\pi \text{ or } \ldots$$

$$\Rightarrow x = 2 \text{ or } 6 \ \ldots$$ ***(A1)***

Since $0 \le x \le 5$, we take $x = 2$, so the point is $(2, 1)$ ***(A1)***

(ii) Tangent line is: $y = -\pi(x-2)+1$ ***(M1)***

$$y = (2\pi + 1) - \pi x$$

$$k = 2\pi + 1$$ ***(A1)*** ***[6 marks]***

(f) $f(x) = 2 \Rightarrow 2\sin\left(\frac{\pi}{2}x\right) + 1 = 2$ ***(A1)***

$$\Rightarrow \sin\left(\frac{\pi}{2}x\right) = \frac{1}{2}$$ ***(A1)***

$$\Rightarrow \frac{\pi}{2}x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \text{ or } \frac{13\pi}{6}$$

$$x = \frac{1}{3} \text{ or } \frac{5}{3} \text{ or } \frac{13}{3}$$ ***(A1)(A1)(A1)*** ***[5 marks]***

5. (i) (a) Mid-point of $[PR] = \left(\frac{-8+14}{2}, \frac{2+6}{2}\right)$

$= (3, 4)$ *(A1)(A1)* **[2 marks]**

(b) **EITHER** $(x-3)^2 + (y-4)^2 = \left(\frac{PR}{2}\right)^2$ *(M1)*

$PR^2 = (14-(-8))^2 + (6-2)^2 = 22^2 + 4^2 = 500$ *(M1)*

$x^2 - 6x + 9 + y^2 - 8y + 16 = \frac{500}{4}$ *(A1)*

$x^2 + y^2 - 6x - 8y - 100 = 0$

$A = -8 \quad B = -100$ *(A1)(A1)*

OR Substituting $(-8, 2)$ and $(14, 6)$ into the equation of the circle gives: *(M1)*

$2A + B + 116 = 0, \quad 6A + B + 148 = 0$ (or equivalent) *(A1)(A1)*

$\Rightarrow \quad A = -8, \quad B = -100$ *(A1)(A1)*

[5 marks]

(c) (i) $8^2 + 14^2 - 48 - 112 - 100 = 64 + 196 - 260 = 0$ *(M1)*

(ii) **EITHER** $\vec{PQ} \cdot \vec{QR} = \begin{pmatrix} 8--8 \\ 14-2 \end{pmatrix} \cdot \begin{pmatrix} 14-8 \\ 6-14 \end{pmatrix} = \begin{pmatrix} 16 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -8 \end{pmatrix} = 96 - 96 = 0$ *(M1)*

$\Rightarrow \theta = 90^\circ$ *(R1)*

OR $PQ^2 + QR^2 = 16^2 + 12^2 + 6^2 + (-8)^2 = 500 = PR^2$ *(M1)*

$\Rightarrow$ Pythagoras satisfied *(R1)*

OR Q satisfies equation $\Rightarrow Q$ on circle $\Rightarrow P\hat{Q}R = 90^\circ$ *(M1)*
(angle subtended by diameter) *(R1)*

[3 marks]

continued...

Question 5 continued.

(ii) (a) $y = \ln x \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x}$ *(A1)*

when $x = e$, $\dfrac{dy}{dx} = \dfrac{1}{e}$

tangent line: $y = \left(\dfrac{1}{e}\right)(x - e) + 1$ *(M1)*

$y = \dfrac{1}{e}(x) - 1 + 1 = \dfrac{x}{e}$ *(A1)*

$x = 0 \Rightarrow y = \dfrac{0}{e} = 0$ *(M1)*

$(0, 0)$ is on line *(AG)* ***[4 marks]***

(b) $\dfrac{d}{dx}(x\ln x - x) = (1) \times \ln x + x \times \left(\dfrac{1}{x}\right) - 1 = \ln x$ *(M1)(A1)(AG)*

Note: Award ***(M1)*** for applying the product rule, and ***(A1)*** for $(1) \times \ln x + x \times \left(\dfrac{1}{x}\right)$

[2 marks]

(c) Area = area of triangle – area under curve *(M1)*

$= \left(\dfrac{1}{2} \times e \times 1\right) - \int_1^e \ln x \, dx$ *(A1)*

$= \dfrac{e}{2} - [x\ln x - x]_1^e$ *(A1)*

$= \dfrac{e}{2} - \{(e\ln e - 1\ln 1) - (e - 1)\}$ *(A1)*

$= \dfrac{e}{2} - \{e - 0 - e + 1\}$

$= \dfrac{1}{2}e - 1.$ *(AG)* ***[4 marks]***

continued...

Question 5 continued

(iii) (a) $y = x(x-4)^2$

(i) $y = 0 \Leftrightarrow x = 0$ or $x = 4$ *(A1)*

(ii) $\dfrac{dy}{dx} = 1(x-4)^2 + x \times 2(x-4) = (x-4)(x-4+2x)$

$= (x-4)(3x-4)$ *(A1)*

$\dfrac{dy}{dx} = 0 \Leftrightarrow x = 4$ or $x = \dfrac{4}{3}$ *(A1)*

$$\left.\begin{aligned} x = 1 \Rightarrow \frac{dy}{dx} = (-3)(-1) = 3 > 0 \\ x = 2 \Rightarrow \frac{dy}{dx} = (-2)(2) = -4 < 0 \end{aligned}\right\} \Rightarrow \frac{4}{3} \text{ is a } \textbf{maximum}$$ *(R1)*

Note: A second derivative test may be used

$x = \dfrac{4}{3} \Rightarrow y = \dfrac{4}{3} \times \left(\dfrac{4}{3} - 4\right)^2 = \dfrac{4}{3} \times \left(\dfrac{-8}{3}\right)^2 = \dfrac{4}{3} \times \dfrac{64}{9} = \dfrac{256}{27}$

$\left(\dfrac{4}{3}, \dfrac{256}{27}\right)$ *(A1)*

Note: Proving that $\left(\dfrac{4}{3}, \dfrac{256}{27}\right)$ is a maximum is not necessary to receive full credit of ***[4 marks]*** for this part.

(iii) $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}((x-4)(3x-4)) = \dfrac{d}{dx}(3x^2 - 16x + 16) = 6x - 16$ *(A1)*

$\dfrac{d^2y}{dx^2} = 0 \Leftrightarrow 6x - 16 = 0$ *(M1)*

$\Leftrightarrow x = \dfrac{8}{3}$ *(A1)*

$x = \dfrac{8}{3} \Rightarrow y = \dfrac{8}{3}\left(\dfrac{8}{3} - 4\right)^2 = \dfrac{8}{3}\left(\dfrac{-4}{3}\right)^2 = \dfrac{8}{3} \times \dfrac{16}{9} = \dfrac{128}{27}$

$\left(\dfrac{8}{3}, \dfrac{128}{27}\right)$ *(A1)*

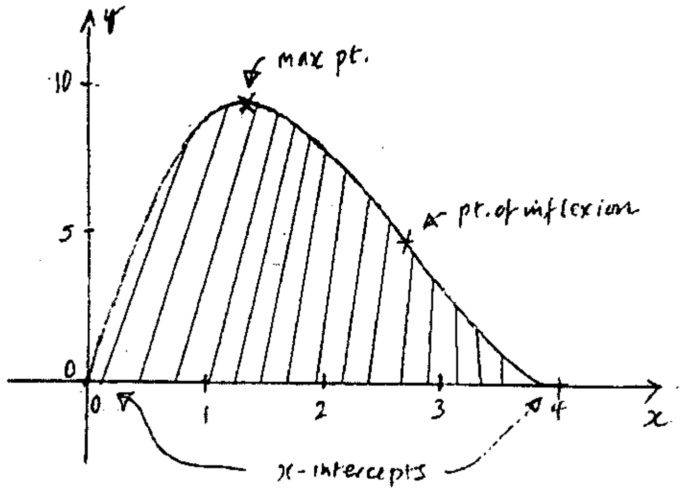
Note: GDC use is likely to give the answer $(1.33, 9.48)$. If this answer is given with no explanation, award *(A2)*. If the answer is given with the explanation "used GDC" or equivalent, award full credit.

[9 marks]

continued…

Question 5 (iii) continued

(b)



(A3)

Note: Award *(A1)* for intercepts, *(A1)* for maximum and *(A1)* for point of inflexion.

[3 marks]

(c) (i) See diagram above *(A1)*

(ii) $0 < y < 10$ for $0 \le x \le 4$ *(R1)*

So $\int_0^4 0\mathrm{d}x < \int_0^4 y\mathrm{d}x < \int_0^4 10\mathrm{d}x \Rightarrow 0 < \int_0^4 y\mathrm{d}x < 40$ *(R1)* *[3 marks]*

(d) $\int_0^4 x(x-4)^2\,\mathrm{d}x$ let $u = x - 4$ *(AG)*

$\Rightarrow u + 4 = x \Rightarrow 1 = \dfrac{\mathrm{d}x}{\mathrm{d}u}$ (or $\mathrm{d}u = \mathrm{d}x$) *(A1)*

x	u
0	–4
4	0

(A1)

$\int_0^4 x(x-4)^2\,\mathrm{d}x = \int_{-4}^0 (u+4)u^2(1)\,\mathrm{d}u = \int_{-4}^0 (u^3 + 4u^2)\,\mathrm{d}u$ *(A1)*

$= \left[\dfrac{u^4}{4} + \dfrac{4u^3}{3}\right]_{-4}^0$ *(A1)*

$= -64 + \dfrac{4(64)}{3}$

$= 21\dfrac{1}{3}$ *(A1)* *[5 marks]*

6. (i) (a) Require area under graph = 1 *(M1)*

Area $= (1 \times k) + \left(\frac{1}{2} \times 3 \times k\right) = \frac{5}{2}k$; (or by integration) *(M1)*

$\frac{5}{2}k = 1 \Rightarrow k = 0.4$ *(M1)(AG)*

Note: Showing area $= 1$ given that $k = 0.4$ should receive full credit.

[3 marks]

(b) $\left.\begin{array}{ll}(1, 0.4): & ① \; 0.4 = a - b \\ (4, 0): & ② \; 0 = a - 4b\end{array}\right\}$ *(M1)*

$\Rightarrow 3b = 0.4 \Rightarrow b = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$ *(M1)(AG)*

Hence in ②: $a = 4b \Rightarrow a = \frac{8}{15}$ *(A1)* *[3 marks]*

(c) (i) $p(X \geq 2) = \frac{1}{2}\left(\frac{8}{15} - \frac{2}{15} \times 2\right)(4 - 2)$ (or by integration) *(M1)*

$= \frac{4}{15}$ *(A1)*

(ii) $p(0.5 \leq X \leq 2) = 0.5 \times 0.4 + \left(0.6 - \frac{4}{15}\right)$ *(M2)*

Note: Award *(M1)* for 0.5×0.4 and *(M1)* for $0.6 - \frac{4}{15}$

$= 0.2 + \frac{5}{15} = \frac{1}{5} + \frac{1}{3}$

$= \frac{8}{15}$ *(A1)* *[5 marks]*

Note: If a candidate calculates this probability by adding or subtracting two appropriate areas, award *(M1)* for calculating each clearly identified area, and *(A1)* for adding/subtracting.

continued...

Question 6 continued

(d) $E(X) = \int_0^4 x f(x)\,\mathrm{d}x$ *(M1)*

$= \int_0^1 0.4x\,\mathrm{d}x + \int_1^4 \left(\frac{8}{15} - \frac{2}{15}x\right)x\,\mathrm{d}x$ *(M1)*

$= \frac{2}{5}\left[\frac{1}{2}x^2\right]_0^1 + \frac{2}{15}\left[2x^2 - \frac{x^3}{3}\right]_1^4$ *(A1)*

$= \frac{1}{5} + \frac{2}{15}\left\{2(15) - \frac{1}{3}(63)\right\}$ *(A1)*

$= \frac{1}{5} + \frac{2}{15}\{9\} = \frac{1}{5} + \frac{6}{5} = \frac{7}{5}$

$= 1.4$ *(A1)* ***[5 marks]***

(e) **EITHER** let $x_m = 4 - b$; then $\frac{1}{2} = b\left(\frac{2}{15}b\right)\frac{1}{2}$ *(M2)*

Note:	Award ***(M1)*** for $\frac{1}{2}$ on left, and ***(M1)*** for $\frac{2}{15}b$

$\Rightarrow b^2 = 7.5$ *(A1)*

$\Rightarrow b \approx 2.74$

$x_m = 1.26$ (3 s.f.) *(A1)*

OR $0.4 + \int_1^{x_m} \frac{(8-2x)}{15}\,\mathrm{d}x = 0.5$ *(M1)*

$\Rightarrow x_m^{\ 2} - 8x_m + 8.5 = 0$ *(A1)*

$\Rightarrow x_m = 1.26$ or 6.74 *(A1)*

$\Rightarrow x_m = 1.26$ *(A1)*

[4 marks]

continued...

Question 6 continued

(ii) (a) $p(4 \text{ heads}) = \binom{8}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{8-4}$ ***(M1)***

$= \frac{8\times7\times6\times5}{1\times2\times3\times4}\times\left(\frac{1}{2}\right)^8$

$= \frac{70}{256} \cong 0.273$ (3 s.f.) ***(A1)*** ***[2 marks]***

(b) $p(3 \text{ heads}) = \binom{8}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{8-3} = \frac{8\times7\times6}{1\times2\times3}\times\left(\frac{1}{2}\right)^8$

$= \frac{56}{256} \approx 0.219$ (3 s.f.) ***(A1)*** ***[1 mark]***

(c) $p(5 \text{ heads}) = p(3 \text{ heads})$ (by symmetry) ***(M1)***

$p(3 \text{ or } 4 \text{ or } 5 \text{ heads}) = p(4) + 2p(3)$ ***(M1)***

$= \frac{70+2\times56}{256} = \frac{182}{256}$

≈ 0.711 (3 s.f.) ***(A1)*** ***[3 marks]***

(d) $n = 64, a = \frac{1}{2}$

$\mu = na = 64\times\frac{1}{2}$ ***(M1)***

$= 32$ ***(A1)***

$\sigma^2 = nab = 64\times\frac{1}{2}\times\frac{1}{2} = 16;$ ***(M1)***

$\sigma = 4$ ***(A1)***

Note:	The request "calculate" does not require "show". Therefore, award full marks for correct answers. GDCs can give these results.

[4 marks]

continued...

Question 6 (ii) continued

Note:	In parts (e) and (f), appropriate use of a graphic display calculator will give the correct answers. Award marks as follows:	
	(e) $p(23.5 \le R^* \le 40.5)$	*(M1)*
	$= 0.966$	*(A3)*
	(f) $p(R^* \ge 39.5)$	*(M1)*
	$\approx 3\,\%$	*(A3)*

(e) Applying the continuity correction to 24 and 40 gives 23.5, 40.5 respectively *(M1)*

Require $p(23.5 \le R^* \le 40.5) = 2 \times \Phi\left(\dfrac{40.5 - 32}{4}\right) - 1$ *(M1)(M1)*

$= 2 \times \Phi(2.125) - 1$ *(A1)*

$= 0.966$ (3 s.f.) *(A1)* *[5 marks]*

(f) H_0: coin fair, $p(\text{heads}) = 0.5$;
H_1: coin biased to heads, $p(\text{heads}) > 0.5$

$p(40 \text{ or more heads}) = p(R^* \ge 39.5)$ *(M1)*

$= p\left(Z \ge \dfrac{39.5 - 32}{4}\right)$

$= p(Z \ge 1.875)$ *(A1)*

$= 1 - \Phi(1.875)$ *(A1)*

$= 1 - 0.9696$

$\approx 3\,\%$ *(A1)*

This is less than 5 %, so suspicion is **confirmed**. *(R1)*

Note:	6(f) was disregarded and candidates who attempted Question 6 were compensated appropriately.

[5 marks]