INTERNATIONAL BACCALAUREATE
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MARKSCHEME

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MATHEMATICAL METHODS

Standard Level

Paper 1

13 pages

1. $\$24.50 \times 780 = \$19110$ *(A1)*

$(1240 - 780) \times 24.50 \times 0.8 = \$9016$ *(M1)(A1)*

Total $\$28126$ *(A1)*

Answer: $\$28126$ *(C4)*

2. (a) $x^2 - 3x - 10 = (x-5)(x+2)$ *(M1)(A1)*

(b) $x^2 - 3x - 10 = 0 \Rightarrow (x-5)(x+2) = 0$ *(M1)*

$\Rightarrow x = 5$ or $x = -2$ *(A1)*

Answers: (a) $(x-5)(x+2)$ *(C2)*

(b) $x = 5$ or $x = -2$ *(C2)*

3. (a) $p = -\frac{1}{2}, q = 2$ *(A1)(A1)*

or vice versa

(b) By symmetry C is midway between p, q *(M1)*

Note: This *(M1)* may be gained by implication

$\Rightarrow x$-coordinate is $\frac{-\frac{1}{2}+2}{2} = \frac{3}{4}$ *(A1)*

Answer: (a) $p = -\frac{1}{2}, q = 2$ *(C2)*

(b) $x = \frac{3}{4}$ *(C2)*

4. $y = x^3 + 1$

$\frac{dy}{dx} = 3x^2$

= Slope of tangent at any point

Therefore at point where $x = 1$, slope $= 3$ *(M1)*

$\Rightarrow$ Slope of normal $= -\frac{1}{3}$ *(M1)(A1)*

$\Rightarrow$ Equation of normal: $y - 2 = -\frac{1}{3}(x - 1)$

$3y - 6 = -x + 1$

$x + 3y - 7 = 0$ *(A1)*

Answer: $x + 3y - 7 = 0$ *(C4)*

Note: Accept equivalent forms e.g. $y = -\frac{1}{3}x + 2\frac{1}{3}$

5. (a) $M\left(\frac{-4+2}{2}, \frac{-2+2}{2}\right) = (-1, 0)$ *(M1)(A1)*

(b) Gradient $CM = \frac{0-3}{-1+3}$ *(M1)*

$= -\frac{3}{2}$ *(A1)*

Answers: (a) $(-1, 0)$ *(C2)*

(b) $-\frac{3}{2}$ *(C2)*

6. $17+27+37+\ldots+417$

$17+(n-1)10=417$ *(M1)*

$10(n-1)=400$

$n=41$ *(A1)*

$S_{41}=\frac{41}{2}(2(17)+40(10))$ *(M1)*

$=41(17+200)$

$=8897$ *(A1)*

OR

$S_{41}=\frac{41}{2}(17+417)$ *(M1)*

$=\frac{41}{2}(434)$

$=8897$ *(A1)*

Answer: 8897 *(C4)*

7. **Note:** Award *(M1)* for identifying the largest angle.

$\cos\alpha=\frac{4^2+5^2-7^2}{2\times4\times5}$ *(M1)*

$=-\frac{1}{5}$ *(A1)*

$\Rightarrow\alpha=101.5^\circ$ *(A1)*

OR Find other angles first

$\beta=44.4^\circ$ $\quad\gamma=34.0^\circ$ *(M1)*

$\Rightarrow\alpha=101.6^\circ$ *(A1)(A1)*

Answer: $\alpha=101.5^\circ$ or 101.6° *(C4)*

Note: Award *(C3)* if not given to the correct accuracy

8. $\int_1^{22} \frac{dx}{3x-2} = \frac{1}{3}\ln[3x-2]_1^{22}$ *(A1)(A1)*

$= \frac{1}{3}[\ln 64 - \ln 1]$

$= \frac{1}{3}\ln 64$

$= \ln 4$ *(A1)*

$\Rightarrow b = 4$ *(A1)*

Answer: $b = 4$ *(C4)*

9. (a) $p(\text{red even}) = \frac{1}{2}$ *(A1)*

$p(\text{green number} > \text{red number}) = \frac{15}{36}$ *(M2)(A1)*

(b)

Note:	Award up to *(M2)* if they do not get the right answer, *e.g.* counting 61, 62, 63, 64, 65, 51....or diagram.

Answers: (a) $p(\text{red even}) = \frac{1}{2}$ *(C1)*

(b) $p(\text{green} > \text{red}) = \frac{5}{12}$ *(C3)*

10. (a) $p = 3$ *(A1)*

(b) Area $= \int_0^{\frac{\pi}{2}} 3\cos x\,dx$ *(M1)*

$= [3\sin x]_0^{\frac{\pi}{2}}$ *(A1)*

$= 3$ square units *(A1)*

Answers: (a) $p = 3$ *(C1)*

(b) 3 square units *(C3)*

11. $2p^2 + 12p = 14$ *(M1)(A1)*

$p^2 + 6p - 7 = 0$

$(p+7)(p-1) = 0$ *(A1)*

$p = -7$ or $p = 1$ *(A1)*

Note: Both answers are required for the final *(A1)*

Answer: $p = -7$ or $p = 1$ *(C4)*

12. $AB = r\theta$

$= \frac{1}{2}r^2\theta \times \frac{2}{r}$ *(M1)(A1)*

$= 21.6 \times \frac{2}{5.4}$ *(A1)*

$= 8$ cm *(A1)*

OR $\frac{1}{2} \times (5.4)^2\theta = 21.6$

$\Rightarrow \theta = \frac{4}{2.7}$ $(= 1.481 \text{ radians})$ *(M1)*

$AB = r\theta$ *(A1)*

$= 5.4 \times \frac{4}{2.7}$ *(M1)*

$= 8$ cm *(A1)*

Answer: 8 cm *(C4)*

13. (a) $\vec{CD} = \vec{OD} - \vec{OC}$ *(A1)*

(b) $\vec{OA} = \frac{1}{2}\vec{CD}$

$= \frac{1}{2}(\vec{OD} - \vec{OC})$ *(A1)*

(c) $\vec{AD} = \vec{OD} - \vec{OA}$

$= \vec{OD} - \frac{1}{2}(\vec{OD} - \vec{OC})$ *(A1)*

$= \frac{1}{2}\vec{OD} + \frac{1}{2}\vec{OC}$ *(A1)*

Answers: (a) $\vec{CD} = \vec{OD} - \vec{OC}$ *(C1)*

(b) $\vec{OA} = \frac{1}{2}(\vec{OD} - \vec{OC})$ *(C1)*

(c) $\vec{AD} = \frac{1}{2}\vec{OD} + \frac{1}{2}\vec{OC}$ *(C2)*

Note: Deduct ***[1 mark]*** (once only) if appropriate vector notation is omitted.

14. Shaded region is above $y = 1$ i.e. $y \geq 1$ *(A1)*
And to the right of y-axis i.e. $x \geq 0$ *(A1)*
Also it is bounded by line passing through $(0, 3)$ and $(2, 0)$
This line is $3x + 2y = 6$ by inspection or otherwise *(A1)*
Then test some point e.g. $(0, 0)$ to get the inequality $3x + 2y \leq 6$ *(A1)*

Answer: $x \geq 0, y \geq 1, 3x + 2y \leq 6$ *(C1)(C1)(C2)*

Notes: Deduct ***[1 mark]*** if one or more strict inequalities are given.
Do not award ***(C)*** marks if the final answer is given as an equation.

15. $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

$3^{2x-2} = 3^{-2x}$ *(M1)(A1)*

$2x - 2 = -2x$ *(A1)*

$x = \frac{1}{2}$ *(A1)*

Answer: $x = \frac{1}{2}$ *(C4)*

16. Required term is $\binom{8}{5}(3x)^5(-2)^3$ *(A1)(A1)(A1)*

Therefore the coefficient of x^5 is $56\times243\times-8$
$=-108864$ *(A1)*

Answer: -108864 *(C4)*

17. (a) $y=\sqrt{3-4x}=(3-4x)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}(3-4x)^{-\frac{1}{2}}(-4)$ *(A1)(A1)*

Note: Award *(A1)* for each element, to a maximum of *[2 marks]*

$=-\frac{2}{\sqrt{3-4x}}$ *(A0)*

(b) $y=e^{\sin x}$

$\frac{dy}{dx}=(\cos x)(e^{\sin x})$ *(A1)(A1)*

Note: Award *(A1)* for each element

Answers: (a) $\frac{dy}{dx}=-\frac{2}{\sqrt{3-4x}}$ *(C2)*

(b) $\frac{dy}{dx}=(\cos x)(e^{\sin x})$ *(C2)*

18. (a) $\vec{u}=-\vec{i}+2\vec{j}\qquad\vec{v}=3\vec{i}+5\vec{j}$
$\vec{u}+2\vec{v}=5\vec{i}+12\vec{j}$ *(A1)*

(b) $|\vec{u}+2\vec{v}|=\sqrt{5^2+12^2}$
$=13$ *(A1)*

Vector $\vec{w}=\frac{26}{13}(5\vec{i}+12\vec{j})$ *(A1)*

$=10\vec{i}+24\vec{j}$ *(A1)*

Answers: (a) $\vec{u}+2\vec{v}=5\vec{i}+12\vec{j}$ *(C1)*

(b) $\vec{w}=10\vec{i}+24\vec{j}$ *(C3)*

19. $(g \circ f)(x) = 0 \quad \Rightarrow \quad 2\cos x + 1 = 0$ *(M1)*

$\Rightarrow \quad \cos x = -\frac{1}{2}$ *(A1)*

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ *(A1)(A1)*

Answer: $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ *(C4)*

Note: Accept 120°, 240°

20. $f(x) = x - 2\sin x$

$f'(x) = 1 - 2\cos x$ *(A1)*

$x_1 = 2 - \frac{x_0 - 2\sin x_0}{1 - 2\cos x_0}$

$= 2 - \frac{2 - 2\sin 2}{1 - 2\cos 2}$ *(A1)*

$= 2 - \frac{0.181406}{1.832294}$ *(A1)*

$= 2 - 0.0990$

$= 1.901$

$= 1.90$ to 3 s.f. *(A1)*

Answer: $x_1 = 1.90$ to 3 s.f. *(C4)*