INTERNATIONAL BACCALAURÉAT BACHILLERATO
BACCALAUREATE INTERNATIONAL INTERNACIONAL

MATHEMATICAL METHODS

Standard Level

Tuesday 4 May 1999 (afternoon)

Paper 1 — 1 hour 30 minutes

A

Candidate name:	Candidate category & number:

This examination paper consists of 20 questions.
The maximum mark for each question is 4.
The maximum mark for this paper is 80.

INSTRUCTIONS TO CANDIDATES

Write your candidate name and number in the boxes above.

Do NOT open this examination paper until instructed to do so.

Answer ALL questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

B

QUESTIONS ANSWERED
ALL

C

EXAMINER	MODERATOR
TOTAL /80	TOTAL /80

D

IBCA
TOTAL /80

EXAMINATION MATERIALS

Required:
IB Statistical Tables
Calculator
Ruler and Compasses

Allowed:
A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Arithmetic series: $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

Geometric series: $S_n = \dfrac{a(r^n - 1)}{r - 1}$, $r \neq 1$

Arc length of a circle: $s = r\theta$

Area of a sector of a circle: $A = \frac{1}{2}r^2\theta$

Area of a triangle: $A = \frac{1}{2}ab\sin C$

Statistics: If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard deviation s are given by

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad s = \sqrt{\frac{\Sigma f_i (x_i - m)^2}{\Sigma f_i}}, \qquad i = 1, 2, \ldots, n$$

Newton-Raphson formula: (For finding a root of $f(x) = 0$)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Integration by parts: (Analytical Geometry and Further Calculus Option only)

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to this examination paper.

1. There were 1240 tickets sold for a concert. Of these, 780 were sold at \$24.50 each, while the remainder were sold at a reduction of 20%. Find the total money taken in ticket sales.

Working:

Answer: _____

2. (a) Factorise $x^2 - 3x - 10$.

(b) Solve the equation $x^2 - 3x - 10 = 0$.

Working:

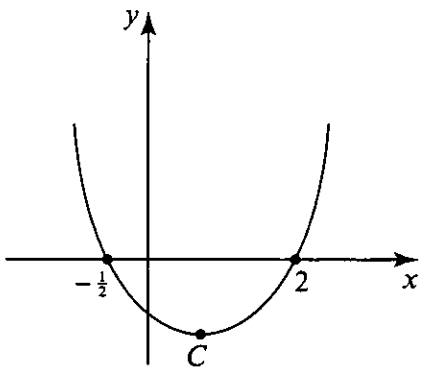
Answers:

(a) _____

(b) _____

3. The diagram represents the graph of the function

$$f : x \mapsto (x - p)(x - q) .$$



(a) Write down the values of p and q.

(b) The function has a minimum value at the point C. Find the x-coordinate of C.

Working:

Answers:

(a) _____

(b) _____

4. Find the equation of the normal to the curve with equation

$$y = x^3 + 1$$

at the point $(1, 2)$.

Working:

Answer:

5. $A(-4, -2)$, $B(2, 2)$, $C(-3, 3)$ are three points. M is the midpoint of $[AB]$.

(a) Find the coordinates of M.

(b) Find the gradient of (CM).

Working:

Answers:

(a)

(b)

6. Find the sum of the arithmetic series

$$17 + 27 + 37 + \ldots + 417 .$$

Working:

Answer:

7. A triangle has sides of length 4 , 5 , 7 units. Find, to the nearest tenth of a degree, the size of the largest angle.

Working:

Answer:

8. Given that $\int_1^{22} \frac{1}{3x-2}\,dx = \ln b$, find the value of b.

Working:

Answer: _____

9. Two unbiased six-sided dice are thrown; one is red and the other is green. Two numbers, between 1 and 6 inclusive, are shown. Find the probability that

(a) the number on the red die is even;

(b) the number on the green die is greater than the number on the red die.

Working:

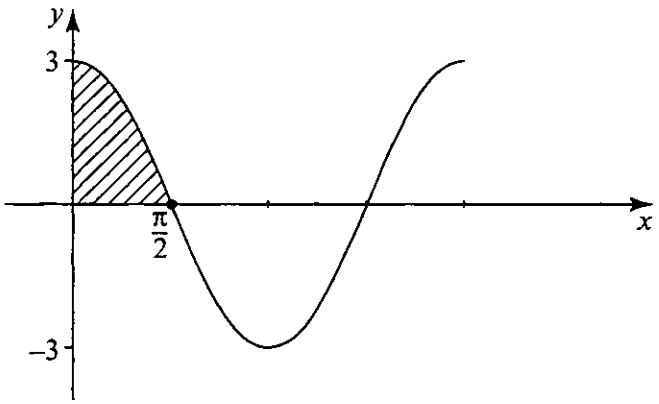
Answers:

(a) _____

(b) _____

10. The graph represents the function

$$f : x \mapsto p \cos x \, , \; p \in \mathbb{N} \, .$$



Find

(a) the value of p ;

(b) the area of the shaded region.

Working:

Answers:

(a) _____

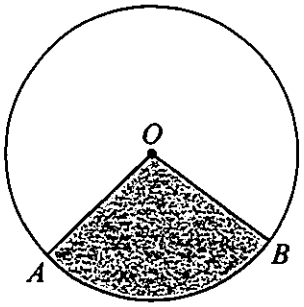
(b) _____

11. If $A = \begin{pmatrix} 2p & 3 \\ -4p & p \end{pmatrix}$ and $\det A = 14$, find the possible values of p.

Working:

Answers:

12. O is the centre of the circle which has a radius of 5.4 cm.

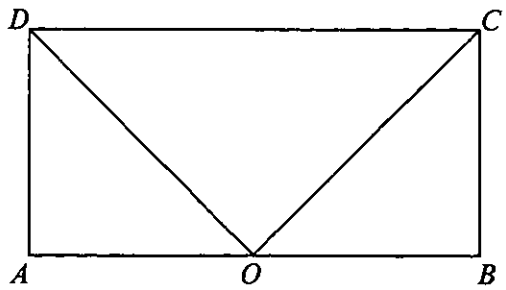


The area of the shaded sector OAB is 21.6 cm^2. Find the length of the minor arc AB.

Working:

Answer:

13. $ABCD$ is a rectangle and O is the midpoint of $[AB]$.



Express each of the following vectors in terms of $\overrightarrow{OC}$ and $\overrightarrow{OD}$:

(a) $\overrightarrow{CD}$

(b) $\overrightarrow{OA}$

(c) $\overrightarrow{AD}$

Working:

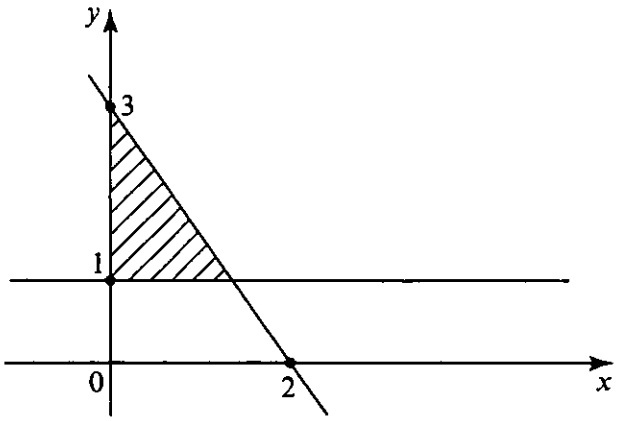
Answers:

(a) _____

(b) _____

(c) _____

14. The shaded region shown is defined by three inequalities. The boundary lines are included in the region.



Find these inequalities.

Working:

Answers:

15. Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$.

Working:

Answer:

16. Find the coefficient of x^5 in the expansion of $(3x-2)^8$.

Working:

Answer:

17. Differentiate with respect to x :

(a) $\sqrt{3-4x}$

(b) $e^{\sin x}$

Working:

Answers:

(a) _____

(b) _____

18. The vectors $\vec{i}$, $\vec{j}$ are unit vectors along the x-axis and y-axis respectively. The vectors $\vec{u} = -\vec{i} + 2\vec{j}$ and $\vec{v} = 3\vec{i} + 5\vec{j}$ are given.

(a) Find $\vec{u} + 2\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$.

A vector $\vec{w}$ has the same direction as $\vec{u} + 2\vec{v}$, and has a magnitude of 26.

(b) Find $\vec{w}$ in terms of $\vec{i}$ and $\vec{j}$.

Working:

Answers:

(a) _____

(b) _____

19. Two functions f and g are defined as follows:

$$f(x) = \cos x, \qquad 0 \leqslant x \leqslant 2\pi;$$
$$g(x) = 2x + 1, \qquad x \in \mathbb{R}.$$

Solve the equation $(g \circ f)(x) = 0$.

Working:

Answer:

20. The equation $x - 2\sin x = 0$ is to be solved using the Newton-Raphson method, with starting value $x_0 = 2$.

Evaluate x_1, correct to three significant figures.

Working:

Answer: