M11/5/FURMA/SP2/ENG/TZ0/XX

22117102

International Baccalaureate®
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FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 6 May 2011 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

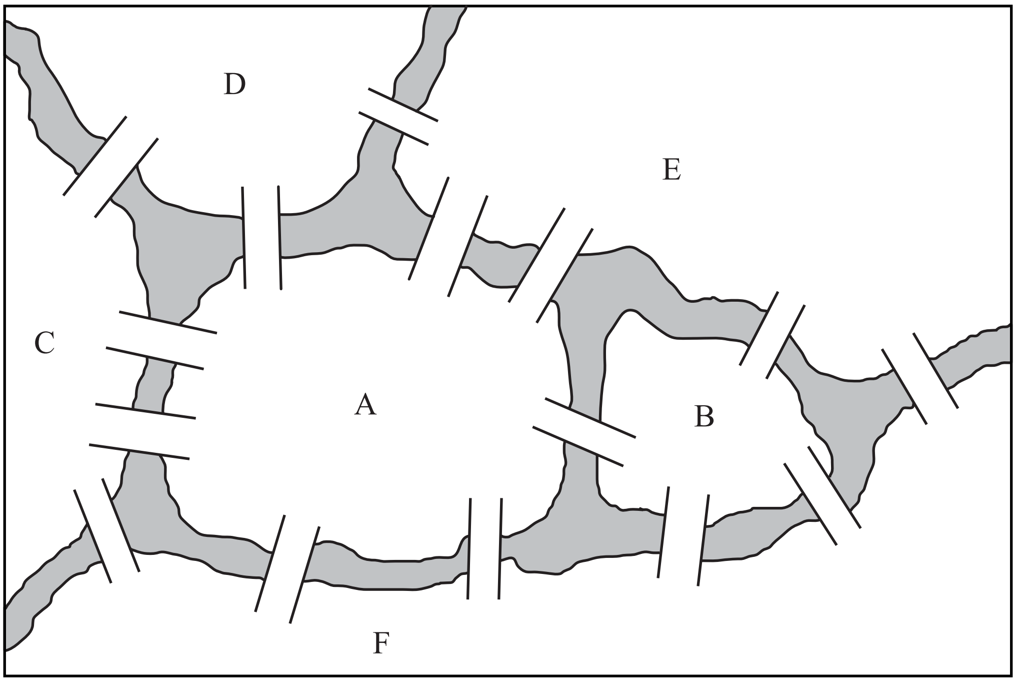
- Do not open this examination paper until instructed to do so.
- Answer all questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

2211-7102

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 12]*

A canal system divides a city into six land masses connected by fifteen bridges, as shown in the diagram below.



(a) Draw a planar graph to represent this map. *[2 marks]*

(b) Write down the adjacency matrix of the graph. *[2 marks]*

(c) List the degrees of each of the vertices. *[2 marks]*

(d) State with reasons whether or not this graph has

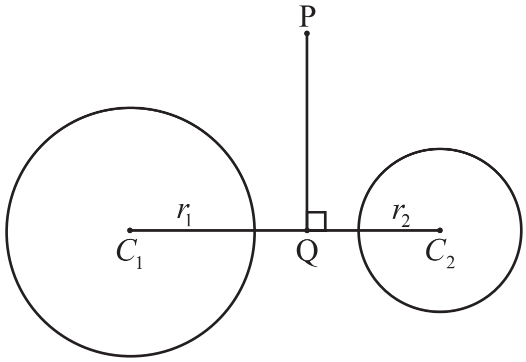
(i) an Eulerian circuit;

(ii) an Eulerian trail. *[4 marks]*

(e) Find the number of walks of length 4 from E to F. *[2 marks]*

2. *[Maximum mark: 18]*

(a) (i) Two non-intersecting circles, centres C_1, C_2 having radii r_1, r_2 respectively, where $r_1 > r_2$, are shown below.

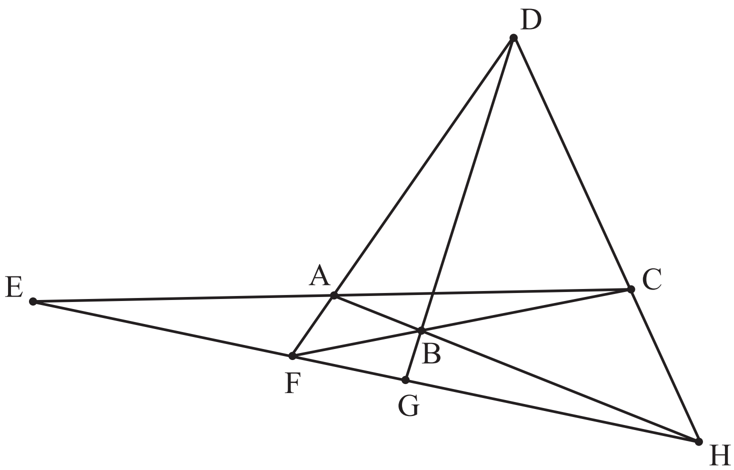


The point P has the same power with respect to each of the two circles. The perpendicular from P to $[C_1C_2]$ intersects $[C_1C_2]$ at Q.

Show that $C_1\text{Q} - C_2\text{Q} = \dfrac{r_1^{\ 2} - r_2^{\ 2}}{C_1C_2}$.

(ii) Given that circle with centre C_1 has equation $x^2 + y^2 + 2x - 10y + 17 = 0$ and that circle with centre C_2 has equation $x^2 + y^2 - 10x + 6y + 30 = 0$, show that $\dfrac{C_1\text{Q}}{C_2\text{Q}} = \dfrac{21}{19}$. *[12 marks]*

(b) ABCD is a quadrilateral. (AD) and (BC) intersect at F and (AB) and (CD) intersect at H. (DB) and (CA) intersect (FH) at G and E respectively. This is shown in the diagram below.



Prove that $\dfrac{\text{HG}}{\text{GF}} = -\dfrac{\text{HE}}{\text{EF}}$. *[6 marks]*

3. *[Maximum mark: 24]*

(a) In an opinion poll of 1400 people, 852 said they preferred Swiss chocolate to any other kind. Calculate a 95 % confidence interval for the proportion of people who prefer Swiss chocolate. *[5 marks]*

(b) An α % confidence interval based on a sample of size 600 for the proportion of people preferring Swiss cheese to other kinds was calculated to be $[0.2511,\ 0.3155]$. Calculate

(i) the number of people in the sample who preferred Swiss cheese;

(ii) the value of α. *[9 marks]*

(c) Stating null and alternative hypotheses carry out a χ^2 test at the 5 % level to decide if the following data can be modelled by the binomial distribution $\mathrm{B}(5,\ 0.35)$.

x	0	1	2	3	4	5
f	15	49	65	36	10	5

[10 marks]

4. *[Maximum mark: 14]*

(a) Given the linear congruence $ax \equiv b \pmod{p}$, where $a,\ b \in \mathbb{Z}$, p is a prime and $\gcd(a,\ p) = 1$, show that $x \equiv a^{p-2} b \pmod{p}$. *[4 marks]*

(b) (i) Solve $17x \equiv 14 \pmod{21}$.

(ii) Use the solution found in part (i) to find the general solution to the Diophantine equation $17x + 21y = 14$. *[10 marks]*

5. *[Maximum mark: 28]*

(a) Find the value of $\lim_{x \to 0} \left(\frac{1}{x} - \cot x \right)$. *[6 marks]*

(b) Find the interval of convergence of the infinite series

$$\frac{(x+2)}{3\quad 1} + \frac{(x+2)^2}{3^2\quad 2} + \frac{(x+2)^3}{3^3\quad 3} + \quad .$$ *[10 marks]*

(c) (i) Find the Maclaurin series for $\ln(1+\sin x)$ up to and including the term in x^3.

(ii) **Hence** find a series for $\ln(1-\sin x)$ up to and including the term in x^3.

(iii) Deduce, by considering the difference of the two series, that

$$\ln 3 \approx \frac{\pi}{3}\left(1 + \frac{\pi^2}{216}\right).$$ *[12 marks]*

6. *[Maximum mark: 24]*

(a) (i) Draw the Cayley table for the set $S = \{0, 1, 2, 3, 4, 5\}$ under addition modulo six $(+_6)$ and hence show that $\{S, +_6\}$ is a group.

(ii) Show that the group is cyclic and write down its generators.

(iii) Find the subgroup of $\{S, +_6\}$ that contains exactly three elements. *[11 marks]*

(b) Prove that a cyclic group with exactly one generator cannot have more than two elements. *[4 marks]*

(c) H is a group and the function $\Phi : H \to H$ is defined by $\Phi(a) = a^{-1}$, where a^{-1} is the inverse of a under the group operation. Show that Φ is an isomorphism **if and only if** H is Abelian. *[9 marks]*