M09/5/FURMA/SP2/ENG/TZ0/XX

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FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 15 May 2009 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. *[Total mark: 23]*

Part A *[Maximum mark: 9]*

(a) Assuming the series for e^x, find the first five terms of the Maclaurin series for

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

[3 marks]

(b) (i) Use your answer to (a) to find an approximate expression for the cumulative distribution function of $N(0, 1)$.

(ii) **Hence** find an approximate value for $P(-0.5 \le Z \le 0.5)$, where $Z \sim N(0, 1)$. *[6 marks]*

Part B *[Maximum mark: 14]*

A machine fills containers with grass seed. Each container is supposed to weigh 28 kg. However the weights vary with a standard deviation of 0.54 kg. A random sample of 24 bags is taken to check that the mean weight is 28 kg.

(a) State and justify an appropriate test procedure giving the null and alternate hypotheses. *[5 marks]*

(b) What is the critical region for the sample mean if the probability of a Type I error is to be 3.5 %? *[7 marks]*

(c) If the mean weight of the bags is actually 28.1 kg, what would be the probability of a Type II error? *[2 marks]*

2. *[Total mark: 24]*

Part A *[Maximum mark: 16]*

(a) (i) Show that $\mathbb{Z}_4$ (the set of integers modulo 4) together with the operation $+_4$ (addition modulo 4) form a group G. You may assume associativity.

(ii) Show that G is cyclic. *[9 marks]*

(b) Using Cayley tables or otherwise, show that G and $H = (\{1, 2, 3, 4\}, \times_5)$ are isomorphic where $\times_5$ is multiplication modulo 5. State clearly all the possible bijections. *[7 marks]*

Part B *[Maximum mark: 8]*

A group has exactly three elements, the identity element e, h and k. Given the operation is denoted by $\otimes$, show that

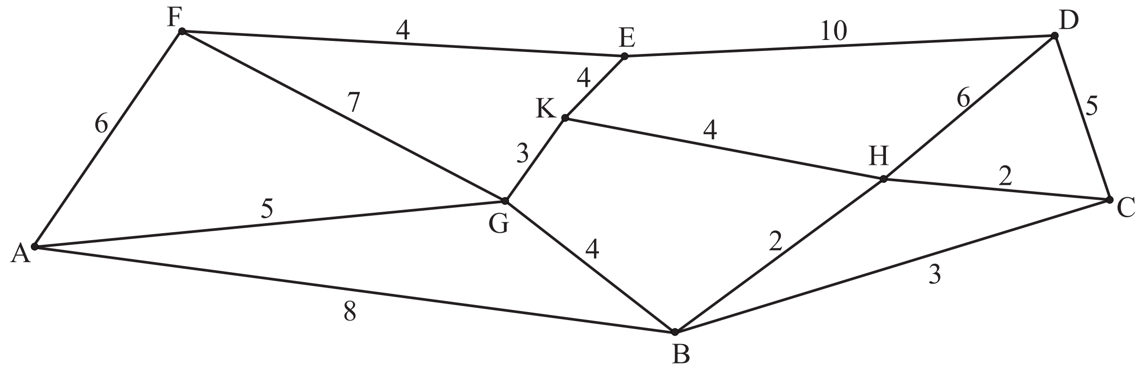
(a) $h \otimes k = e$; *[3 marks]*

(b) the group is cyclic. *[5 marks]*

3. *[Total mark: 25]*

Part A *[Maximum mark: 13]*

(a) Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph shown below. State the weight of the tree. *[5 marks]*



(b) For the travelling salesman problem defined by this graph, find

(i) an upper bound;

(ii) a lower bound. *[8 marks]*

Part B *[Maximum mark: 12]*

(a) Given that the integers m and n are such that $3 \mid (m^2 + n^2)$, prove that $3 \mid m$ and $3 \mid n$. *[7 marks]*

(b) **Hence** show that $\sqrt{2}$ is irrational. *[5 marks]*

4. *[Total mark: 22]*

Part A *[Maximum mark: 14]*

The function $f(x)$ is defined by the series $f(x) = 1 + \frac{(x+2)}{3\times 1} + \frac{(x+2)^2}{3^2 \times 2} + \frac{(x+2)^3}{3^3 \times 3} + \ldots$

(a) Write down the general term. *[1 mark]*

(b) Find the interval of convergence. *[13 marks]*

Part B *[Maximum mark: 8]*

Solve the differential equation $(u + 3v^3)\frac{\mathrm{d}v}{\mathrm{d}u} = 2v$, giving your answer in the form $u = f(v)$.

5. *[Total mark: 26]*

Part A *[Maximum mark: 8]*

Prove that the interior bisectors of two of the angles of a non-isosceles triangle and the exterior bisector of the third angle, meet the sides of the triangle in three collinear points.

Part B *[Maximum mark: 18]*

(a) An equilateral triangle QRT is inscribed in a circle. If S is any point on the arc QR of the circle,

(i) prove that $\mathrm{ST} = \mathrm{SQ} + \mathrm{SR}$;

(ii) show that triangle RST is similar to triangle PSQ where P is the intersection of [TS] and [QR];

(iii) using your results from parts (i) and (ii) deduce that $\frac{1}{\mathrm{SP}} = \frac{1}{\mathrm{SQ}} + \frac{1}{\mathrm{SR}}$. *[10 marks]*

(b) Perpendiculars are drawn from a point P on the circumcircle of triangle LMN to the three sides. The perpendiculars meet the sides [LM], [MN] and [LN] at the points E, F and G respectively.

Prove that $\mathrm{PL} \times \mathrm{PF} = \mathrm{PM} \times \mathrm{PG}$. *[8 marks]*