M09/5/FURMA/SP1/ENG/TZ0/XX

22097101

International Baccalaureate®
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FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 14 May 2009 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. *[Maximum mark: 11]*

The relation R is defined on the set $\mathbb{Z}$ by aRb if and only if $4a+b=5n$, where a, b, $n \in \mathbb{Z}$.

(a) Show that R is an equivalence relation. *[8 marks]*

(b) State the equivalence classes of R. *[3 marks]*

2. *[Maximum mark: 6]*

The random variable X has a Poisson distribution. Given that $\mathrm{P}(X>2)=0.6$, find

(a) the mean of the distribution; *[4 marks]*

(b) the mode of the distribution. *[2 marks]*

3. *[Maximum mark: 8]*

Triangle ABC has points D, E and F on sides [BC], [CA] and [AB] respectively; [AD], [BE] and [CF] intersect at the point P. If $3\mathrm{BD}=2\mathrm{DC}$ and $\mathrm{CE}=4\mathrm{EA}$, calculate the ratios

(a) $\mathrm{AF}:\mathrm{FB}$; *[4 marks]*

(b) $\mathrm{AP}:\mathrm{PD}$. *[4 marks]*

4. *[Maximum mark: 6]*

Prove that $3k+2$ and $5k+3$, $k \in \mathbb{Z}$ are relatively prime.

5. *[Maximum mark: 8]*

A circle has radius R cm where R is uniformly distributed on the interval $[1, 4]$.

(a) Find an expression for $F(r)$, $r \in [1, 4]$, where F is the cumulative distribution function of R. *[3 marks]*

(b) The area of the circle is A cm^2. Show that, for $a \in [\pi, 16\pi]$,

$$\mathrm{P}(A \le a) = \frac{1}{3}\left(\sqrt{\frac{a}{\pi}} - 1\right).$$ *[3 marks]*

(c) Deduce the probability density function of A in the interval $[\pi, 16\pi]$. *[2 marks]*

6. *[Maximum mark: 21]*

(a) Find the value of $\sum_{n=1}^{\infty} \frac{3}{9n^2 + 3n - 2}$. *[6 marks]*

(b) (i) Sum the series $\sum_{r=0}^{\infty} x^r$.

(ii) **Hence**, using sigma notation, deduce a series for

(a) $\frac{1}{1+x^2}$;

(b) $\arctan x$;

(c) $\frac{\pi}{6}$. *[11 marks]*

(c) Show that $\sum_{n=1}^{100} n! \equiv 3 \pmod{15}$. *[4 marks]*