



**FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1**

Monday 19 May 2008 (afternoon)

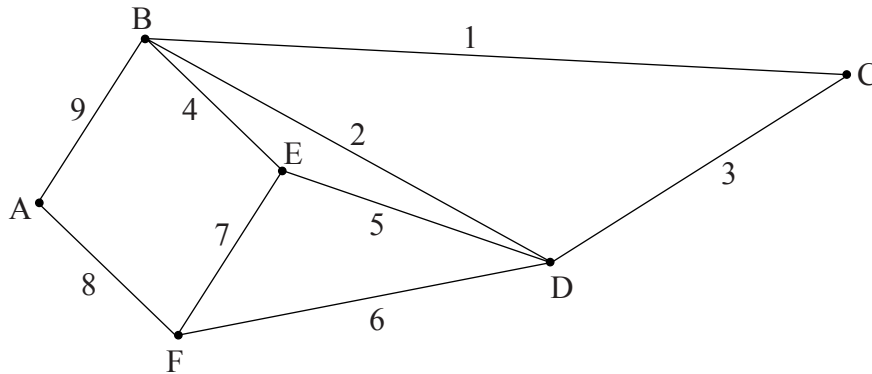
1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. [Maximum mark: 11]



The above diagram shows the weighted graph G .

- (a) Determine whether or not G is bipartite. [2 marks]
- (b) (i) Write down the adjacency matrix for G .
(ii) Find the number of distinct walks of length 4 beginning and ending at A. [4 marks]
- (c) Starting at A, use Prim's algorithm to find and draw the minimum spanning tree for G . Your solution should indicate clearly the way in which the tree is constructed. [5 marks]

2. [Maximum mark: 12]

The group $\{G, *\}$ is defined on the set $G = \{1, 2, 3, 4, 5, 6\}$ where $*$ denotes multiplication modulo 7.

- (a) Draw the Cayley table for $\{G, *\}$. [3 marks]
- (b) (i) Determine the order of each element of $\{G, *\}$.
(ii) Find all the proper subgroups of $\{G, *\}$. [6 marks]
- (c) Solve the equation

$$x * 6 * x = 3 \text{ where } x \in G. \quad [3 \text{ marks}]$$

3. [Maximum mark: 15]

The heights, x metres, of the 241 new entrants to a men’s college were measured and the following statistics calculated.

$$\sum x = 412.11, \quad \sum x^2 = 705.5721$$

- (a) Calculate unbiased estimates of the population mean and the population variance. [3 marks]
- (b) The Head of Mathematics decided to use a χ^2 test to determine whether or not these heights could be modelled by a normal distribution. He therefore divided the data into classes as follows.

| | | | | | | |
|------------------|------------|----------------------|----------------------|----------------------|----------------------|---------------|
| Interval | $x < 1.60$ | $1.60 \leq x < 1.65$ | $1.65 \leq x < 1.70$ | $1.70 \leq x < 1.75$ | $1.75 \leq x < 1.80$ | $x \geq 1.80$ |
| Frequency | 5 | 34 | 70 | 72 | 48 | 12 |

- (i) State suitable hypotheses.
- (ii) Calculate the value of the χ^2 statistic and state your conclusion using a 10% level of significance. [12 marks]

4. [Maximum mark: 11]

The triangle ABC is isosceles and $AB = BC = 5$. D is the midpoint of AC and $BD = 4$.

- (a) Find the lengths of the tangents from A, B and D to the circle inscribed in the triangle ABD. [6 marks]
- (b) Find the power of the point A with respect to this circle. [2 marks]
- (c) This inscribed circle touches [AD] at the point E. Show that the points A, E, D, C form a harmonic ratio. [3 marks]

5. [Maximum mark: 11]

Solve the following differential equation

$$(x+1)(x+2) \frac{dy}{dx} + y = x+1$$

giving your answer in the form $y = f(x)$.