M07/5/FURMA/SP2/ENG/TZ0/XX

22077102

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 17 May 2007 (morning)

2 hours

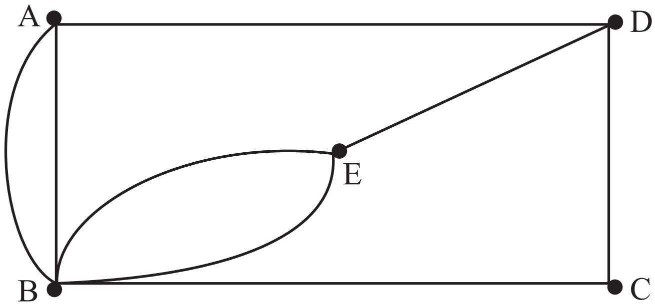
INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. *[Maximum mark: 24]*

(a)



The diagram above shows the graph G.

(i) Explain briefly why G has no Eulerian circuit.

(ii) Determine whether or not G is bipartite.

(iii) Write down the adjacency matrix of G. Hence find the number of walks of length 4 beginning at A and ending at C. *[12 marks]*

(b) The cost adjacency matrix of a graph with vertices P, Q, R, S, T, U is given by

	P	Q	R	S	T	U
P	–	8	–	–	–	4
Q	8	–	7	–	2	3
R	–	7	–	6	3	–
S	–	–	6	–	9	–
T	–	2	3	9	–	7
U	4	3	–	–	7	–

Use Dijkstra's Algorithm to find the length of the shortest path between the vertices P and S. Show all the steps used by the algorithm and list the order of the vertices in the path. *[12 marks]*

2. *[Maximum mark: 27]*

(a) The random variable X is thought to follow a binomial distribution $\mathrm{B}(4, p)$. In order to investigate this belief, a random sample of 100 observations on X was taken with the following results.

Value of x	Frequency
0	17
1	32
2	19
3	18
4	14

(i) State suitable hypotheses for testing this belief.

(ii) Calculate the mean of these data and hence estimate the value of p.

(iii) Calculate an appropriate value of χ^2 and state your conclusion, using a 1 % significance level. *[19 marks]*

(b) An automatic machine is used to fill bottles of water. The amount delivered, Y ml, may be assumed to be normally distributed with mean μ ml and standard deviation 8 ml. Initially, the machine is adjusted so that the value of μ is 500. In order to check that the value of μ remains equal to 500, a random sample of 10 bottles is selected at regular intervals, and the mean amount of water, $\bar{y}$, in these bottles is calculated. The following hypotheses are set up.

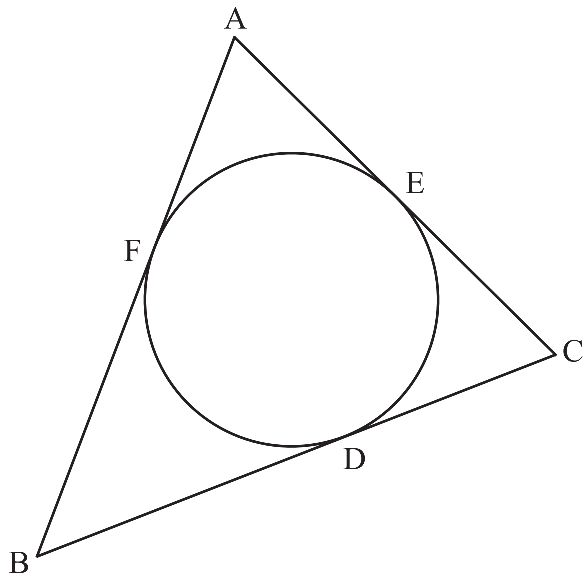
$$\mathrm{H}_0 : \mu = 500 \ ; \ \ \mathrm{H}_1 : \mu \neq 500$$

The critical region is defined to be $(\bar{y} < 495) \cup (\bar{y} > 505)$.

(i) Find the significance level of this procedure.

(ii) Some time later, the actual value of μ is 503. Find the probability of a Type II error. *[8 marks]*

3. *[Maximum mark: 21]*



(a) The diagram shows triangle ABC together with its inscribed circle. Show that [AD], [BE] and [CF] are concurrent. *[8 marks]*

(b) PQRS is a parallelogram and T is a point inside the parallelogram such that the sum of $\mathrm{P\hat{T}Q}$ and $\mathrm{R\hat{T}S}$ is 180°. Show that $\mathrm{PT \times TR + ST \times TQ = PQ \times QR}$. *[13 marks]*

4. *[Maximum mark: 22]*

(a) The function f is defined by $f(x) = \dfrac{e^x + e^{-x}}{2}$.

(i) Obtain an expression for $f^{(n)}(x)$, the nth derivative of $f(x)$ with respect to x.

(ii) Hence derive the Maclaurin series for $f(x)$ up to and including the term in x^4.

(iii) Use your result to find a rational approximation to $f\left(\dfrac{1}{2}\right)$.

(iv) Use the Lagrange error term to determine an upper bound to the error in this approximation. *[13 marks]*

(b) Use the integral test to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ is convergent or divergent. *[9 marks]*

5. *[Maximum mark: 26]*

(a) The relation R is defined for $x, y \in \mathbb{Z}^+$ such that xRy if and only if $3^x \equiv 3^y \pmod{10}$.

(i) Show that R is an equivalence relation.

(ii) Identify all the equivalence classes. *[11 marks]*

(b) Let S denote the set $\left\{x \,\middle|\, x = a + b\sqrt{3},\ a, b \in \mathbb{Q},\ a^2 + b^2 \neq 0\right\}$.

(i) Prove that S is a group under multiplication.

(ii) Give a reason why S would not be a group if the conditions on a, b were changed to $a, b \in \mathbb{R}$, $a^2 + b^2 \neq 0$. *[15 marks]*