IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 16 May 2007 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. *[Maximum mark: 8]*

The point $P(x, y)$ moves in such a way that its distance from the point $(1, 0)$ is three times its distance from the point $(-1, 0)$.

(a) Find the equation of the locus of P. *[4 marks]*

(b) Show that this equation represents a circle and state its radius and the coordinates of its centre. *[4 marks]*

2. *[Maximum mark: 8]*

Calculate the following limits

(a) $\lim\limits_{x \to 0} \dfrac{2^x - 1}{x}$; *[3 marks]*

(b) $\lim\limits_{x \to 0} \dfrac{(1+x^2)^{\frac{3}{2}} - 1}{\ln(1+x) - x}$. *[5 marks]*

3. *[Maximum mark: 12]*

(a) Show that the set S of numbers of the form $2^m \times 3^n$, where $m, n \in \mathbb{Z}$, forms a group $\{S, \times\}$ under multiplication. *[6 marks]*

(b) Show that $\{S, \times\}$ is isomorphic to the group of complex numbers $m + n\mathrm{i}$ under addition, where $m, n \in \mathbb{Z}$. *[6 marks]*

4. *[Maximum mark: 12]*

(a) Use the Euclidean Algorithm to show that 275 and 378 are relatively prime. *[5 marks]*

(b) Find the general solution to the diophantine equation $275x + 378y = 1$. *[7 marks]*

5. *[Maximum mark: 9]*

Solve the differential equation $x\dfrac{dy}{dx}+2y=\sqrt{1+x^2}$

given that $y=1$ when $x=\sqrt{3}$. *[9 marks]*

6. *[Maximum mark: 11]*

The weights, X kg, of male birds of a certain species are normally distributed with mean 4.5 kg and standard deviation 0.2 kg. The weights, Y kg, of female birds of this species are normally distributed with mean 2.5 kg and standard deviation 0.15 kg.

(a) (i) Find the mean and variance of $2Y-X$.

   (ii) Find the probability that the weight of a randomly chosen male bird is more than twice the weight of a randomly chosen female bird. *[6 marks]*

(b) Two randomly chosen male birds and three randomly chosen female birds are placed together on a weighing machine for which the recommended maximum weight is 16 kg. Find the probability that this maximum weight is exceeded. *[5 marks]*