IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M06/5/FURMA/SP2/ENG/TZ0/XX

22067102

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 16 May 2006 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. *[Total maximum mark: 26]*

Part A *[Maximum mark: 11]*

When Bill shoots an arrow at a target, he has a probability 0.6 of hitting the target. Each shot is independent of all other shots.

(a) Find the probability of

(i) hitting the target five times in eight shots;

(ii) hitting the target for the fifth time on the eighth shot. *[6 marks]*

(b) One morning, he decides to shoot arrows at the target and to stop as soon as he hits the target for the tenth time. Find the mean and standard deviation of the number of shots required. *[5 marks]*

(This question continues on the following page)

(Question 1 continued)

Part B *[Maximum mark: 15]*

Anne tosses a coin which has probability p of giving a head. Anne thinks that it is a fair coin for which $p = 0.5$. However, Anne's friend thinks that $p > 0.5$. In order to investigate the value of p, Anne decides to toss the coin 15 times.

(a) State appropriate null and alternative hypotheses. *[2 marks]*

Let X denote the number of heads obtained. Anne decides to reject the null hypothesis if $X \geq 11$.

(b) (i) What name is given to the region $X \geq 11$?

(ii) Explain what is meant by the significance level and find its value in this case. *[5 marks]*

It is known that $p = 0.6$.

(c) Find the probability of a Type II error. *[4 marks]*

(d) When Anne tosses the coin 15 times, she obtains 10 heads.

(i) What type of error does she commit?

(ii) Explain briefly the consequences of this error. *[4 marks]*

2. *[Total maximum mark: 31]*

Part A *[Maximum mark: 18]*

(a) Find $\begin{pmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -a & -b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ *[2 marks]*

(b) Let G be the set of matrices of the form

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{ for } a, b \in \mathbb{R}$$

Show that G is an Abelian group under matrix multiplication. *[10 marks]*

(c) Let F be the group of real ordered pairs under addition defined by

$(a, b) + (c, d) = (a + c, b + d)$.

Show that G is isomorphic to F. *[6 marks]*

Part B *[Maximum mark: 13]*

Consider the group $(H, \bullet)$ with identity element e.

(a) For $x, y \in H$, show that $(x \bullet y)^{-1} = y^{-1} \bullet x^{-1}$. *[4 marks]*

(b) Given $x, y \in H$, the relation R is defined as follows.

$x R y \Leftrightarrow$ there exists $z \in H$ such that $x = z \bullet y \bullet z^{-1}$.

Determine whether or not R is an equivalence relation. *[9 marks]*

3. *[Total maximum mark: 25]*

Part A *[Maximum mark: 11]*

(a) Use the Euclidean algorithm to find the greatest common divisor of 75 and 110. *[4 marks]*

(b) Consider the diophantine equation $110x + 75y = 45$.

(i) Explain briefly why your answer to part (a) shows that this equation has a solution.

(ii) Find a solution to this equation.

(iii) Hence find the general solution to this equation. *[7 marks]*

Part B *[Maximum mark: 14]*

The weights of the edges of a complete graph *G* are shown in the following table.

	A	B	C	D	E	F
A	–	5	4	7	6	2
B	5	–	6	3	5	4
C	4	6	–	8	1	6
D	7	3	8	–	7	3
E	6	5	1	7	–	3
F	2	4	6	3	3	–

(a) Determine whether or not *G* is planar. *[4 marks]*

(b) Starting at B, use Prim's algorithm to find and draw a minimum spanning tree for *G*. Your solution should indicate the order in which the vertices are added. State the total weight of your tree. *[10 marks]*

4. *[Total maximum mark: 12]*

Part A *[Maximum mark: 5]*

Estimate the range of values of x for which the Maclaurin approximation
$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$ is accurate to within 0.005. *[5 marks]*

Part B *[Maximum mark: 7]*

Determine whether the series $\sum_{n=0}^{\infty} (n+1)^{-n}$ is convergent or divergent. *[7 marks]*

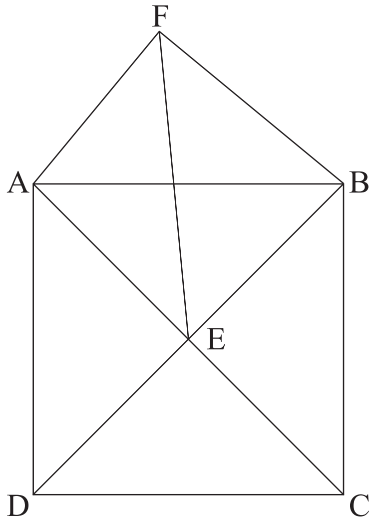
5. *[Maximum mark: 10]*

Consider the differential equation $\frac{dy}{dx} - 2y = \sin x$ with boundary condition $y = 1$ when $x = 0$.

Use four steps of Euler's method starting at $x = 0$, with interval $h = 0.1$, to find an approximate value for y when $x = 0.4$. *[10 marks]*

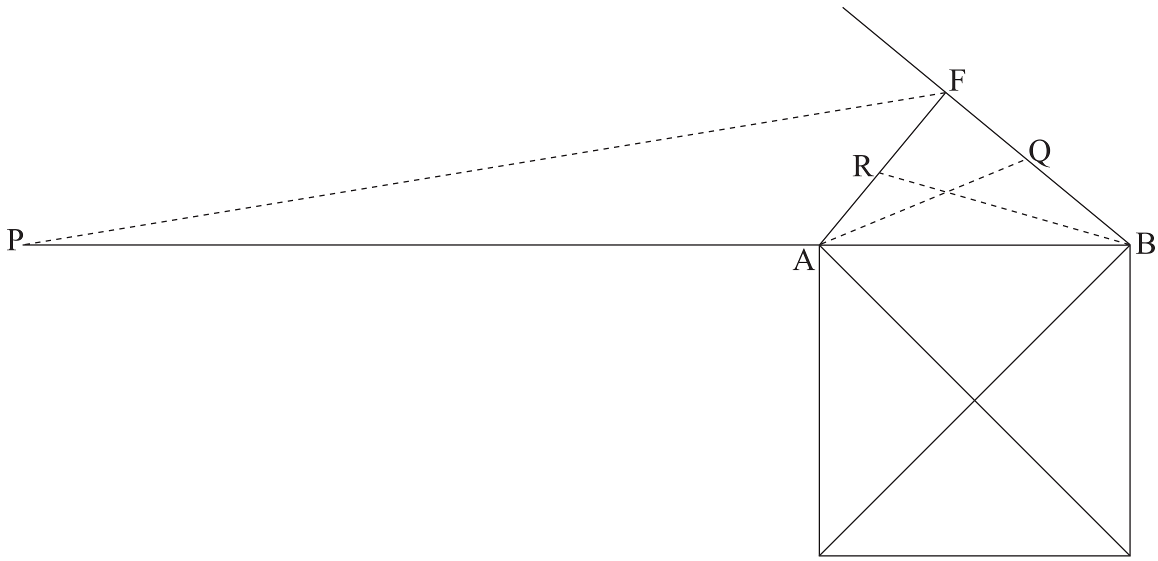
6. *[Maximum mark: 16]*

The following diagram shows a square ABCD with diagonals intersecting at E. The point F is outside the square such that $\hat{AFB}$ is a right angle and $AF < BF$.



(a) The lengths BF and AF are denoted respectively by *a* and *b*. Find an expression for EF in terms of *a* and *b*. *[8 marks]*

(b) In the triangle AFB, the external bisector of angle *F* and the internal bisectors of angles *A* and *B* meet the opposite sides at P, Q and R respectively, as shown in the diagram below.



Show that P, Q and R are collinear. *[8 marks]*