IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MARKSCHEME

MAY 2005

FURTHER MATHEMATICS

Standard Level

Paper 1

8 pages

Paper 1 Markscheme

Instructions to Examiners

1 Method of marking

(a) All marking must be done using a **red** pen.

(b) Marks should be noted on candidates' scripts as in the markscheme:

- show the breakdown of individual marks using the abbreviations (M1), (A2) etc.
- write down each part mark total, indicated on the markscheme (for example, [3marks]) – it is suggested that this be written at the end of each part, and underlined;
- write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are not awarded

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) penalise an error when it **first occurs**;

(ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;

(iii) award ***M*** marks for a correct method, and ***A(ft)*** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the follow through procedure:

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	***M1***	Amount earned = \$ 600 × 1.02	√	***M1***
= \$ 612	***A1***	= \$602	×	***A0***
\$ (306 × 1.02) + (306 × 1.04)	***M1***	Amount = 301 × 1.02 + 301 × 1.04	√	***M1***
= \$ 630.36	***A1***	= \$ 620.06	√	***A1*(ft)**

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, etc. Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

Mean = 7906/134 ***(M1)***
= 59 ***(A1)***

OR

Mean = 59 ***(G2)***

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect ie, once the correct answer is seen, ignore further working.

(c) As this is an international examination, **all alternative forms of notation** should be accepted. For example: 1.7, $1 \cdot 7$, 1,7; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan x.

5 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**. Award the marks as usual then write -1**(AP)** against the answer and also on the **front** cover.

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the AP.

OR

- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from May 2003).

Incorrect answers are wrong, and the accuracy penalty should not be applied to incorrect answers.

Examples

- A question leads to the answer 4.6789….
- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy : both should be penalised the first time this type of error occurs.
- 4.67 is incorrectly rounded - penalise on the first occurrence.

Note: All these "incorrect" answers may be assumed to come from 4.6789..., even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalised as being incorrect answers, not as examples of accuracy errors.

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

1. $f(x)=\sqrt[3]{1+x} \Rightarrow f'(x)=\frac{1}{3}(1+x)^{-\frac{2}{3}}$

$\Rightarrow f''(x)=-\frac{2}{9}(1+x)^{-\frac{5}{3}} \Rightarrow f^{(3)}(x)=\frac{10}{27}(1+x)^{-\frac{8}{3}}$ ***(M1)(A1)***

$f(x) \approx \sum_{i=0}^{3} \frac{f^{(i)}(0)}{i!}x^i = 1+\frac{1}{3}x-\frac{1}{9}x^2+\frac{5}{81}x^3$ ***(M1)(A1)***

$f(0.2) \approx 1.06272$ ***(A1)***

[5 marks]

2. If we have only one vertex we have no edges. ***(A1)***
Assume that a tree with n vertices has $n-1$ edges. ***(M1)***
If we add one vertex to the tree with n vertices we are going to connect it to just one of the vertices of the previous tree by exactly one edge, otherwise the graph will not be a tree. ***(R1)***
Since the previous tree had $n-1$ edges and the additional edge makes n edges for $n+1$ vertices. ***(A1)***
Thus, true for $n \Rightarrow$ true for $n+1$, hence proved by induction. ***(R1)***

[5 marks]

3. (a) $ab^2a=b \Rightarrow a^2b^2a=ab$
$eb^2a=ab \Rightarrow ab=b^2a$ ***(M1)(A1)(AG)***

(b) $(ab^2)^2=ab^2ab^2=(ab^2a)b^2$ ***(M1)(A1)***
$=bb^2=b^3=e$ ***(A1)(AG)***

[5 marks]

4. H_0: "The handedness and the gender of a person are independent."
H_1: "The handedness and the gender of a person are not independent."

Expected	Left	Right
Female	47.6	352.4
Male	71.4	528.6

(M1)
(A1)

$\chi^2_{calc}=\sum \frac{\left(|f_e-f_o|-0.5\right)^2}{f_e}=0.668$ ***(M1)(A1)***

We do not have enough evidence to reject H_0 since
EITHER $0.668 < 6.64$
OR p-value $= 0.414 > 0.01$ ***(R1)***

[5 marks]

5. The characteristic equation is $r^2 - 5r + 6 = 0 \Rightarrow r_1 = 2,\ r_2 = 3$ ***(M1)(A1)***

The general solution is $a_n = A \times 2^n + B \times 3^n$. ***(A1)***

If the solution is a power of 3, then $A = 0$ ***(A1)***

so then the initial conditions are $a_1 = 3B,\ a_2 = 9B,\ B = 3^k,\ k \in \mathbb{N}$ ***(A1)***

[5 marks]

6. $a_n = \dfrac{3n-2}{2^{\frac{n}{3}}},\quad n = 1,\ 2,\ 3...$ ***(A1)***

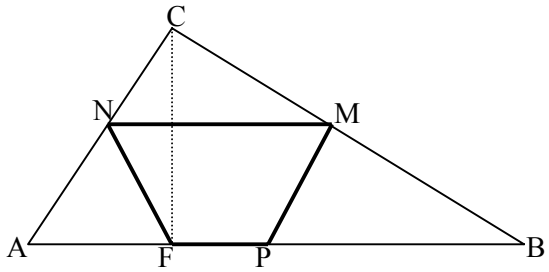
$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\dfrac{3n+1}{2^{\frac{n+1}{3}}}}{\dfrac{3n-2}{2^{\frac{n}{3}}}} = \lim_{n\to\infty}\frac{3n+1}{3n-2}\times 2^{-\frac{1}{3}}$$ ***(M1)(A1)***

$= 2^{-\frac{1}{3}} < 1$ ***(A1)***

The series converges. ***(R1)***

[5 marks]

7. (a)



Observing the right-angled triangle AFC we can deduce that N is the centre of the circumscribed circle, so $AN = NF$. ***(M1)(A1)***

Since $MP = \dfrac{1}{2}AC = AN = FN$. ***(A1)***

(b) *Hence:* The trapezium PFNM is isosceles, so the angles along the parallel sides are equal. $M\hat{N}F = N\hat{M}P,\ N\hat{F}P = F\hat{P}M$ ***(A1)***

Since the sum of the opposite angles is equal and equal to 180°, the trapezium is cyclic. ***(R1)***

Otherwise:

Using the nine point circle theorem we can conclude that all of the points ***(M1)***

are on the circle and therefore the trapezium is cyclic. ***(R1)***

[5 marks]

8. (a) $X \sim \mathrm{N}\left(\mu = 5.2, \sigma = \frac{0.7}{\sqrt{50}}\right) \Rightarrow \mathrm{P}(X < 5) = 0.0217$ ***(M1)(A1)***

(b) $X \sim \mathrm{N}\left(\mu = 5.2, \sigma = \frac{0.7}{\sqrt{n}}\right) \Rightarrow \Phi\left(\frac{5.3 - 5.2}{\frac{0.7}{\sqrt{n}}}\right) > 0.8$ ***(M1)***

$\frac{0.1\sqrt{n}}{0.7} > 0.842$ ***(A1)***

$n > 34.7 = 35$ ***(A1)***

[5 marks]

9. The parametric equation of an ellipse is $\begin{cases} x = a\cos t + h \\ y = b\sin t + k \end{cases}$, where a and b are major and/or minor half axes, while (h, k) is the centre of the ellipse. ***(M1)***

So the centre is at $(3, -1)$. ***(A1)***

Since $a = 2, b = 3, c = \sqrt{b^2 - a^2} \Rightarrow c = \sqrt{5}$. ***(A1)***

The major axis is parallel to the y-axis. ***(A1)***

So the Foci are at $\mathrm{F}_{1,2} = \left(3, -1 \pm \sqrt{5}\right)$. ***(A1)***

[5 marks]

10.

Reflexive:	[AB] and [BA] have the same midpoint.	***(A1)***
Symmetrical:	If [AD] and [BC] have the same midpoint then [CB] and [DA] have the same midpoint.	***(A1)***
Transitive:	If [AD] and [BC] have a common midpoint then $\mathrm{AB} = \mathrm{CD}$ and (AB) is parallel to (CD).	
	If [CF] and [DE] have a common midpoint then $\mathrm{CD} = \mathrm{EF}$ and (CD) is parallel to (EF).	***(R1)***
	Then $\mathrm{AB} = \mathrm{EF}$ and (AB) is parallel to (EF).	***(A1)***
	If they are collinear or they form a parallelogram then [AF] and [BE] have the common midpoint.	***(A1)***

[5 marks]