INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 13 November 2001 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

You are advised to start each new question on a new page. A correct answer with **no** *indication of the method used will usually receive* **no** *marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

1. *[Maximum mark: 14]*

(i) In a candy factory sweets are packed in bags whose masses are distributed normally with a mean of 100 g and standard deviation of 1 g. Find the probability that the mass of 10 bags selected at random will be within 5 g of the expected mass? *[4 marks]*

(ii) A hospital in a town has recorded the number of newborn babies per day during a period of 100 days, with the following results:

Number of babies (x_i)	0	1	2	3	4	5
Number of days	8	28	31	18	9	6

(a) Show that the mean number of newborn babies per day is 2.1 . *[1 mark]*

(b) It is believed that this distribution may be modelled by a Poisson distribution. Some of the expected frequencies are given in the table below.

x_i	f_o	f_e
0	8	a
1	28	25.7
2	31	b
3	18	18.9
4	9	9.9
5	6	9.9
6 or more	0	c

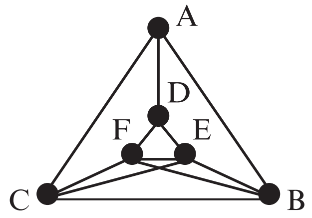
(i) Calculate values of a, b and c.

(ii) Test, at the 5% level of significance, whether or not the given distribution can reasonably be modelled by a Poisson distribution. *[9 marks]*

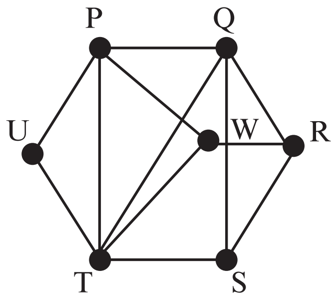
2. *[Maximum mark: 19]*

(i) (a) Which of the following graphs, if any, are planar? Justify your answer.

(i)



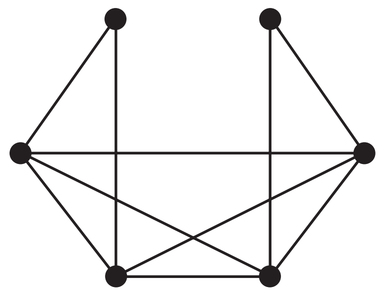
(ii)



[6 marks]

(b) **Ore's theorem:** In a simple graph G with n vertices, where $n \geq 3$, if $\deg A + \deg B \geq n$ for each pair of two non-adjacent vertices A, B in G then G is Hamiltonian.

Use the theorem to determine whether the following graph is Hamiltonian and find, if possible, a Hamiltonian cycle.



[4 marks]

(ii) Find all positive integers n smaller than 500 such that $n \equiv 4 \pmod{19}$ and $n \equiv 1 \pmod{11}$. *[9 marks]*

3. *[Maximum mark: 23]*

(i) $\boldsymbol{M}$ is the set of all $n \times n$ matrices. A relation R is defined on $\boldsymbol{M}$ as follows:

$\boldsymbol{A}\ R\ \boldsymbol{B}$ if and only if there exists an invertible matrix $\boldsymbol{X}$ such that $\boldsymbol{B} = \boldsymbol{X}^{-1}\boldsymbol{A}\boldsymbol{X}$. Prove that R is an equivalence relation. *[8 marks]*

(ii) Show that the intersection of two subgroups of a group is a subgroup of that group. *[4 marks]*

(iii) Let $\mathbb{Z}_n$ be the group of integers under addition modulo n.

(a) Find all subgroups of $\mathbb{Z}_3 \times \mathbb{Z}_3$. *[6 marks]*

(b) Hence determine the number of subgroups of $\mathbb{Z}_p \times \mathbb{Z}_p$, when p is prime. *[5 marks]*

4. *[Maximum mark: 22]*

(i) Let the functions $f(x)$ and $g(x)$ be defined by $f(x) = 3 - 2x$ and $g(x) = e^{1-x}$.

(a) Consider the equation $f(x) = g(x)$.

(i) Find the **exact** solution to this equation.

(ii) Use the Newton-Raphson method with a starting value $x_0 = 0$ to find an approximate solution to this equation. Give your answer correct to three decimal places.

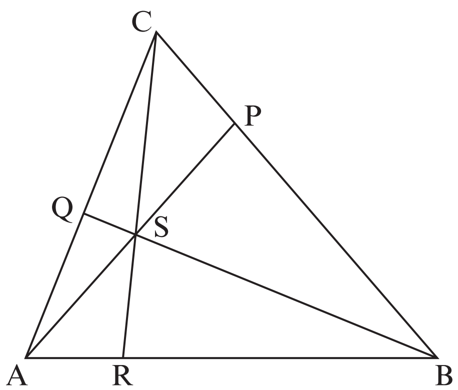
(iii) Use Rolle's theorem to prove that these solutions are the only two solutions to this equation. *[10 marks]*

(b) Let the area between the curves of $y = f(x)$ and $y = g(x)$ be denoted by A. Given that $h(x) = f(x) - g(x)$, and that $h^{(4)}(x) = e^{1-x}$, use Simpson's rule with 8 intervals to show that the maximum error in evaluating A does not exceed 0.00002. *[5 marks]*

(ii) Use the Maclaurin series expansion to approximate $\sin 3°$, giving your answer correct to five decimal places. *[7 marks]*

5. *[Maximum mark: 22]*

(i) In triangle ABC , the points P , Q and R are on the sides [BC] , [CA] and [AB] respectively. The lines (AP) , (BQ) and (CR) contain a common point S .



(a) Show that the ratio of AR to BR is equal to the ratio of the areas of the triangles ARS and RBS . *[2 marks]*

(b) Hence prove Ceva's theorem. *[5 marks]*

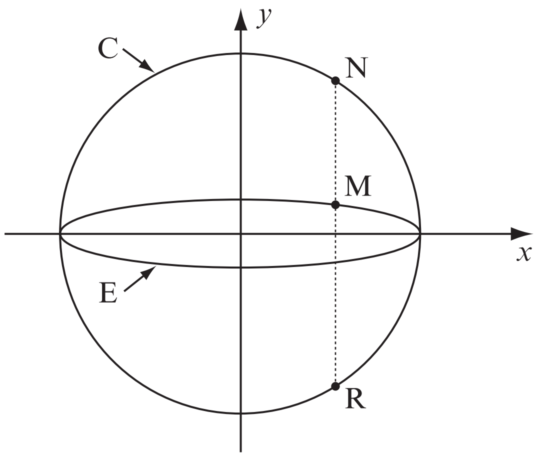
(This question continues on the following page)

(Question 5 continued)

(ii) An ellipse E and a circle C are defined by the following parametric equations.

$$\text{E}: x = 4\cos t,\ \ y = \sin t, \qquad \text{C}: x = 4\cos t,\ \ y = 4\sin t\,.$$

The points M on E and N on C have the same value s for the parameter t, where $s \in \left]0, \frac{\pi}{2}\right[$, and the point R on C has the value $-s$ for the parameter t.



(a) The normal to E through M cuts the diameter of C through N at the point P. Show that the point P, as s varies, lies on a circle, and find its radius. *[10 marks]*

(b) The normal to E through M cuts the diameter of C through R at the point Q. Describe the locus of Q. *[5 marks]*