M14/5/FURMA/HP1/ENG/TZ0/XX/M

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MARKSCHEME

May 2014

FURTHER MATHEMATICS

Higher Level

Paper 1

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) are often dependent on the preceding ***M*** mark.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.* do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates may not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

The method of dealing with accuracy errors on a whole paper basis by means of the Accuracy Penalty ***(AP)*** *no longer applies.*

Instructions to examiners about such numerical issues will be provided on a question by question basis within the framework of mathematical correctness, numerical understanding and contextual appropriateness.

The rubric on the front page of each question paper is given for the guidance of candidates. The markscheme (MS) may contain instructions to examiners in the form of "Accept answers which round to n significant figures (sf)". Where candidates state answers, required by the question, to fewer than n sf, award ***A0****. Some intermediate numerical answers may be required by the MS but not by the question. In these cases only award the mark(s) if the candidate states the answer exactly or to at least 2sf.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 1, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation

The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. converting to base 10

$(551662)_7 = 2 + 6\times 7 + 6\times 7^2 + 1\times 7^3 + 5\times 7^4 + 5\times 7^5$ ***(M1)***

$= 96721$ ***(A1)***

$\sqrt{96721} = 311$ ***A1***

converting back to base 7

7)<u>311</u> ***(M1)***

)<u>44</u>(3

)<u>6</u>(2 ***(A1)***

it follows that $\sqrt{(551662)_7} = (623)_7$ ***A1***

Note: Accept 623.

Total [6 marks]

2. use of $y \to y + h\dfrac{\mathrm{d}y}{\mathrm{d}x}$ ***(M1)***

x	y	dy/dx	hdy/dx	
0	1	1	0.1	***(A1)***
0.1	1.1	1.33	0.133	***A1***
0.2	1.233	1.866516337	0.1866516337	***A1***
0.3	1.419651634	2.834181181	0.283418118	***A1***
0.4	1.703069752			***(A1)***

Note: After the first line, award ***A1*** for each subsequent y value, provided it is correct to 3sf.

approximate value of $y(0.4) = 1.70$ ***A1***

Note: Accept 1.7 or any answers that round to 1.70.

Total [7 marks]

3. (a) $G(t) = \frac{1}{4}t + \frac{1}{2}t^2 + \frac{1}{4}t^3$ ***M1A1***

$= \frac{t(1+t)^2}{4}$ ***AG***

[2 marks]

(b) (i) PGF of $Y = (G(t))^4 \left(= \left(\frac{t(1+t)^2}{4} \right)^4 \right)$ ***A1***

(ii) $\mathrm{P}(Y = 8) =$ coefficient of t^8 ***(M1)***

$= \frac{{}^8\mathrm{C}_4}{256}$ ***(A1)***

$= \frac{35}{128}$ (0.273) ***A1***

Note: Accept 0.27 or answers that round to 0.273.

[4 marks]

Total [6 marks]

4. (a) the eigenvalues satisfy $\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$ ***(M1)***

$\lambda^2 - (a+d)\lambda + ad - bc = 0$ ***A1***

using the sum and product properties of the roots of a quadratic equation ***R1***

$\lambda_1 + \lambda_2 = a + d$, $\lambda_1\lambda_2 = ad - bc = \det(\boldsymbol{M})$ ***AG***

[3 marks]

(b) let $f(\lambda) = \lambda^2 - (a+d)\lambda + ad - bc$

putting $b = 1 - a$ and $d = 1 - c$, consider ***M1***

$f(1) = 1 - a - 1 + c + a - ac - c + ac = 0$ ***A1***

therefore $\lambda = 1$ is an eigenvalue ***AG***

[2 marks]

Note:	Allow substitution for *b*, *c* into the quadratic equation for λ followed by solution of this equation.

(c) using any valid method ***(M1)***

the eigenvalues are 1 and -1 ***A1***

an eigenvector corresponding to $\lambda = 1$ satisfies

$\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ **or** $\begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***M1A1***

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ **or** any multiple ***A1***

an eigenvector corresponding to $\lambda = -1$ satisfies

$\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix}$ **or** $\begin{pmatrix} 3 & -1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***M1***

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ **or** any multiple ***A1***

[7 marks]

Note:	Award ***M1A1A1*** for calculating the first eigenvector and ***M1A1*** for the second irrespective of the order in which they are calculated.

Total [12 marks]

5. (a) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$

$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \dots$ ***A1***

$\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$ ***(M1)A1***

Note: Accept any valid (otherwise) method.

[3 marks]

(b) $\mathrm{P}\left(X \equiv 1(\mathrm{mod}\ 2)\right) = \mathrm{P}\left(X = 1, 3, 5, \dots\right)$ ***(M1)***

$= e^{-\mu}\left(\mu + \frac{\mu^3}{3!} + \frac{\mu^5}{5!} + \dots\right)$ ***A1***

$= \frac{e^{-\mu}(e^{\mu} - e^{-\mu})}{2}$ ***A1***

$= \frac{1}{2} - \frac{1}{2}e^{-2\mu}$ ***A1***

$\left(a = \frac{1}{2}, b = -\frac{1}{2}, c = -2\right)$

[4 marks]

Total [7 marks]

6. (a) gradient of OU $= \frac{2au}{au^2} = \frac{2}{u}$ ***A1***

gradient of OV $= \frac{2av}{av^2} = \frac{2}{v}$ ***A1***

since the lines are perpendicular,

$\frac{2}{u} \times \frac{2}{v} = -1$ ***M1***

so $v = -\frac{4}{u}$ ***AG***

[3 marks]

(b) coordinates of W are $\left(\frac{a(u^2+v^2)}{2}, \frac{2a(u+v)}{2}\right)$ ***M1***

$= \left(\frac{a}{2}\left(u^2 + \frac{16}{u^2}\right), a\left(u - \frac{4}{u}\right)\right)$ ***A1***

[2 marks]

(c) putting

$x = \frac{a}{2}\left(u^2 + \frac{16}{u^2}\right); \; y = a\left(u - \frac{4}{u}\right)$ ***M1***

it follows that

$y^2 = a^2\left(u^2 + \frac{16}{u^2} - 8\right)$ ***A1***

$= 2ax - 8a^2$ ***AG***

[2 marks]

Note: Accept verification.

(d) since $y^2 = 2a(x - 4a)$ ***(M1)***

the vertex is at $(4a, 0)$ ***A1***

[2 marks]

Total [9 marks]

7. (a) (i) $\bar{x} = 213.2$ ***A1***

$s = 3.0728\ldots$ ***(A1)***

$s^2 = 9.442$ ***A1***

(ii) [211.0, 215.4] ***A1A1***

Note: Accept 211 in place of 211.0.

Note: Apart from the above note, accept any answers which round to the correct 4 significant figure answers.

[5 marks]

(b) use of the fact that the width of the interval is $2t \times \frac{s}{\sqrt{n}}$ ***(A1)***

so that $3.4 = 2t \times \frac{3.0728\ldots}{\sqrt{10}}$ ***M1***

$t = 1.749$ ***A1***

degrees of freedom $= 9$ ***(A1)***

$\mathrm{P}(T > 1.749) = 0.0571$ ***(M1)***

confidence level $= 1 - 2 \times 0.0571 = 0.886\ (88.6\%)$ ***A1***

Note: Award the $\mathrm{DF} = 9$ ***(A1)*** mark if the following line has 0.00337 on the RHS.

[6 marks]

Note: Accept any answer which rounds to 88.6%.

Total [11 marks]

8. (a) reflexive

$x^{-1}x = e \in H$ ***A1***

therefore *xRx* and *R* is reflexive ***R1***

symmetric

Note: Accept the word commutative.

let *xRy* so that $x^{-1}y \in H$ ***M1***

the inverse of $x^{-1}y$ is $y^{-1}x \in H$ ***A1***

therefore *yRx* and *R* is symmetric ***R1***

transitive

let *xRy* and *yRz* so $x^{-1}y \in H$ and $y^{-1}z \in H$ ***M1***

therefore $x^{-1}y\, y^{-1}z = x^{-1}z \in H$ ***A1***

therefore *xRz* and *R* is transitive ***R1***

hence *R* is an equivalence relation ***AG***

[8 marks]

(b) the identity is 0 so the inverse of 3 is -3 ***(R1)***

the equivalence class of 3 contains x where $-3 + x \in H$ ***(M1)***

$-3 + x = 4n \, (n \in \mathbb{Z})$ ***(M1)***

$x = 3 + 4n \, (n \in \mathbb{Z})$ ***A1***

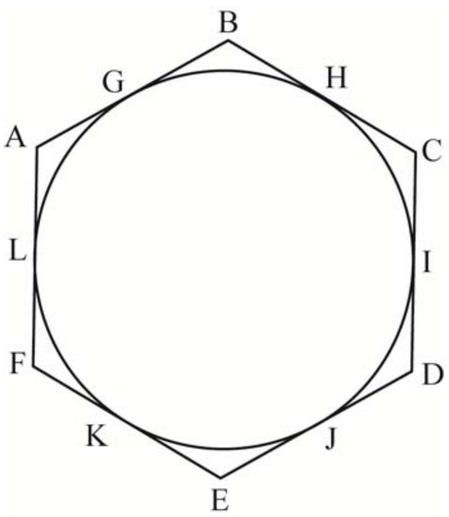
Note: Accept $\{\ldots -5,\ -1,\ 3,\ 7,\ \ldots\}$ or $x \equiv 3 \pmod 4$.

Note: If no other relevant working seen award ***A3*** for $\{3 + 4n\}$ or $\{\ldots -5,\ -1,\ 3,\ 7,\ \ldots\}$ seen anywhere.

[4 marks]

Total [12 marks]

9.



A1

the lengths of the two tangents from a point to a circle are equal *(R1)*
so that
$AG = LA$
$GB = BH$
$CI = HC$
$ID = DJ$
$EK = JE$
$KF = FL$ *A1*
adding,
$(AG + GB) + (CI + ID) + (EK + KF) = (BH + HC) + (DJ + JE) + (FL + LA)$ *M1A1*

$AB + CD + EF = BC + DE + FA$ *AG*

Total [5 marks]

10. (a) successive powers of $\boldsymbol{A}$ are given by

$$\boldsymbol{A}^2 = \begin{pmatrix} 5 & 7 & 6 \\ 6 & 9 & 5 \\ 7 & 10 & 9 \end{pmatrix}$$ ***(M1)A1***

$$\boldsymbol{A}^3 = \begin{pmatrix} 24 & 35 & 25 \\ 25 & 36 & 29 \\ 35 & 51 & 36 \end{pmatrix}$$ ***A1***

it follows, considering elements in the first rows, that

$5a + b + c = 24$

$7a + 2b = 35$

$6a + b = 25$ ***M1A1***

solving, ***(M1)***

$(a, b, c) = (3, 7, 2)$ ***A1***

[7 marks]

Note: Accept any other three correct equations.

Note: Accept the use of the Cayley–Hamilton Theorem.

(b) (i) it has been shown that

$\boldsymbol{A}^3 = 3\boldsymbol{A}^2 + 7\boldsymbol{A} + 2\boldsymbol{I}$

multiplying by $\boldsymbol{A}^{-1}$, ***M1***

$\boldsymbol{A}^2 = 3\boldsymbol{A} + 7\boldsymbol{I} + 2\boldsymbol{A}^{-1}$ ***A1***

whence

$\boldsymbol{A}^{-1} = 0.5\boldsymbol{A}^2 - 1.5\boldsymbol{A} - 3.5\boldsymbol{I}$ ***A1***

(ii) substituting powers of $\boldsymbol{A}$,

$$\boldsymbol{A}^{-1} = 0.5\begin{pmatrix} 5 & 7 & 6 \\ 6 & 9 & 5 \\ 7 & 10 & 9 \end{pmatrix} - 1.5\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix} - 3.5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ ***M1***

$$= \begin{pmatrix} -2.5 & 0.5 & 1.5 \\ 1.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 \end{pmatrix}$$ ***A1***

[5 marks]

Note: Follow through their equation in (b)(i).

Note: Line (ii) of (ii) must be seen.

Total [12 marks]

11. (a) $r = -0.163$ ***A2***

[2 marks]

(b) (i) $H_0 : \rho = 0 : H_1 : \rho \neq 0$ ***A1***

(ii) $t = r\sqrt{\dfrac{n-2}{1-r^2}} = -0.468\ldots$ ***(A1)***

$DF = 8$ ***(A1)***

p-value $= 2 \times 0.326\ldots = 0.652$ ***A1***

since $0.652 > 0.05$, we accept H_0 ***R1***

[5 marks]

Note: Award ***(A1)(A1)A0*** if the p-value is given as 0.326 without prior working.

Note: Follow through their p-value for the ***R1***.

(c) (i) $y = -0.257x + 5.22$ ***A1***

Note: Accept answers which round to –0.26 and 5.2.

(ii) no, because X and Y have been shown to be independent (or equivalent) ***A1***

[2 marks]

Total [9 marks]

12. (a) let T_n denote the nth term

consider

$$\frac{T_{n+1}}{T_n} = \frac{x^{(n+1)}}{2^{2(n+1)}\left(2[n+1]^2-1\right)} \times \frac{2^{2n}\left(2n^2-1\right)}{x^n}$$ ***M1***

$$= \frac{x}{2^2} \times \frac{\left(2n^2-1\right)}{\left(2[n+1]^2-1\right)}$$ ***A1***

$$\to \frac{x}{4} \text{ as } n \to \infty$$ ***A1***

so the radius of convergence is 4 ***A1***

[4 marks]

(b) we need to consider $x = \pm 4$ ***R1***

$$S(4) = \sum_{n=1}^{\infty} \frac{1}{\left(2n^2-1\right)}$$ ***A1***

$$S(4) < \sum_{n=1}^{\infty} \frac{1}{n^2}$$ ***M1***

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent; therefore by the comparison test $S(4)$ is convergent ***R1***

$$S(-4) = \sum_{n=1}^{\infty} \frac{(-1)^n}{\left(2n^2-1\right)}$$ ***A1***

EITHER

this series is convergent because it is absolutely convergent ***R1***

OR

this series is alternating and is convergent ***R1***

THEN

the interval of convergence is therefore [– 4, 4] ***A1***

[7 marks]

Note: The final ***A1*** is independent of any of the previous marks.

Total [11 marks]

13. we need to show that f is injective and surjective ***(R1)***

Note: Award ***R1*** if seen anywhere in the solution.

<u>injective</u>

let (a, b) and $(c, d) \in \mathbb{R}^+ \times \mathbb{R}^+$, and let $f(a, b) = f(c, d)$ ***M1***

it follows that

$ab = cd$ and $\frac{a}{b} = \frac{c}{d}$ ***A1***

multiplying these equations,

$a^2 = c^2 \Rightarrow a = c$ and therefore $b = d$ ***A1***

since $f(a, b) = f(c, d) \Rightarrow (a, b) = (c, d)$, f is injective ***R1***

Note: Award ***R1*** if stated anywhere as needing to be shown.

<u>surjective</u>

let $(p, q) \in \mathbb{R}^+ \times \mathbb{R}^+$

consider $f(x, y) = (p, q)$ so $xy = p$ and $\frac{x}{y} = q$ ***M1A1***

multiplying these equations,

$x^2 = pq$ so $x = \sqrt{pq}$ and therefore $y = \sqrt{\frac{p}{q}}$ ***A1***

so given $(p, q) \in \mathbb{R}^+ \times \mathbb{R}^+$, $\exists (x, y) \in \mathbb{R}^+ \times \mathbb{R}^+$ such that $f(x, y) = (p, q)$ which shows that f is surjective ***R1***

Note: Award ***R1*** if stated anywhere as needing to be shown.

f is therefore a bijection ***AG***

Total [9 marks]

14. (a) (i) completing the square,

$$\left(x+\frac{d}{2}\right)^2+\left(y+\frac{e}{2}\right)^2-\frac{d^2}{4}-\frac{e^2}{4}+f=0$$ ***M1A1***

whence the centre C is the point $\left(-\frac{d}{2},-\frac{e}{2}\right)$ and the radius is $\sqrt{\frac{d^2}{4}+\frac{e^2}{4}-f}$ ***AG***

(ii) $\text{CP}^2=\left(a+\frac{d}{2}\right)^2+\left(b+\frac{e}{2}\right)^2$ ***(A1)***

let Q denote the point of contact of one of the tangents from P to the circle.

$\text{CQ}^2=\frac{d^2}{4}+\frac{e^2}{4}-f$ ***(A1)***

using Pythagoras' Theorem in triangle CPQ, ***M1***

$L^2=\left(a+\frac{d}{2}\right)^2+\left(b+\frac{e}{2}\right)^2-\left(\frac{d^2}{4}+\frac{e^2}{4}-f\right)$

$=a^2+b^2+da+eb+f=g(a,b)$ ***A1***

therefore $L=\sqrt{g(a,b)}$ ***AG***

[6 marks]

(b) (i) the x-coordinates of R, S satisfy

$x^2+(mx)^2-6x-2mx+6=0$ ***M1***

$(1+m^2)x^2-(6+2m)x+6=0$ ***A1***

(ii) $L^2=g(0,0)=6$ ***A1***

let x_1, x_2 denote the two roots. Then $x_1x_2=\frac{6}{1+m^2}$ ***A1***

$\text{OR}=\sqrt{x_1^{\,2}+(mx_1)^2}=x_1\sqrt{1+m^2}$ and $\text{OS}=x_2\sqrt{1+m^2}$ ***M1***

therefore

$\text{OR}\times\text{OS}=x_1x_2(1+m^2)=6$ ***A1***

so that $\text{OR}\times\text{OS}=L^2$ ***AG***

[6 marks]

Total [12 marks]

15. (a) using Fermat's little theorem, ***(M1)***

$a^{p-1} \equiv 1 (\text{mod } p)$ ***A1***

multiplying both sides of the congruence by a^{p-2}, ***(M1)***

$a^{p-1}x \equiv a^{p-2}b (\text{mod } p)$ ***A1***

$x \equiv a^{p-2}b (\text{mod } p)$ ***AG***

[4 marks]

(b) (i) the solution is

$x \equiv 7^{17} \times 13 (\text{mod } 19)$ ***A1***

consider

$7^3 = 343 \equiv 1 (\text{mod } 19)$ ***(A1)***

Note: Other powers are possible.

therefore

$x \equiv \left(7^3\right)^5 \times 7^2 \times 13 (\text{mod } 19)$ ***(A1)***

$\equiv 7^2 \times 13 (\text{mod } 19)$ ***(A1)***

$\equiv 10 (\text{mod } 19)$ ***A1***

(ii) using any method, including trial and error, the solution to the second congruence is given by $x \equiv 32 (\text{mod } 7)$ (or equivalent) ***(A1)***

a simultaneous solution is $x = 67$ (or equivalent, *eg* -66) ***A1***

the full solution is $x = 67 + 133N$ (where $N \in \mathbb{Z}$) (or equivalent) ***A1***

Note: Do not ***FT*** an incorrect answer from (i).

[8 marks]

Total [12 marks]

16. (a) the right coset containing a has the form $\{ha \mid h \in H\}$ ***A1***

[1 mark]

Note: From here on condone the use of left cosets.

(b) let b, c be distinct elements of H. Then, given $a \in G$, by the Latin square property of the Cayley table, ba and ca are distinct ***A1***

therefore each element of H corresponds to a unique element in the coset which must therefore contain n elements ***R1***

[2 marks]

(c) let d be any element of G. Then since H contains the identity e, $ed = d$ will be in a coset ***R1***

therefore every element of G will be contained in a coset which proves that the union of all the cosets is G ***R1***

[2 marks]

(d) let the cosets of b and c $(b, c \in G)$ contain a common element so that $pb = qc$ where $p, q \in H$. Let r denote any other element $\in H$ ***M1***

then

$rb = rp^{-1}qc$ ***A1***

since $rp^{-1}q \in H$, this shows that all the other elements are common and the cosets are equal ***R1***

since not all cosets can be equal, there must be other cosets which are disjoint ***R1***

[4 marks]

(e) the above results show that G is partitioned into a number of disjoint subsets containing n elements so that N must be a multiple of n ***R1***

[1 mark]

Total [10 marks]