Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

Thursday 12 May 2022 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

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7 pages

Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 27]

This question uses statistical tests to investigate whether advertising leads to increased profits for a grocery store.

Aimmika is the manager of a grocery store in Nong Khai. She is carrying out a statistical analysis on the number of bags of rice that are sold in the store each day. She collects the following sample data by recording how many bags of rice the store sells each day over a period of 90 days.

Number of bags of rice sold	0	1	2	3	4	5	6	7	8	9	10
Number of days	1	8	12	11	19	14	13	8	2	0	2

She believes that her data follows a Poisson distribution.

(a) (i) Find the mean and variance for the sample data given in the table. [2]

(ii) Hence state why Aimmika believes her data follows a Poisson distribution. [1]

(b) State one assumption that Aimmika needs to make about the sales of bags of rice to support her belief that it follows a Poisson distribution. [1]

Aimmika knows from her historic sales records that the store sells an average of 4.2 bags of rice each day. The following table shows the expected frequency of bags of rice sold each day during the 90 day period, assuming a Poisson distribution with mean 4.2.

Number of bags of rice sold	≤ 1	2	3	4	5	6	7	≥ 8
Expected frequency	a	11.903	16.665	b	14.698	10.289	6.173	c

(c) Find the value of a, of b, and of c. Give your answers to 3 decimal places. [5]

(This question continues on the following page)

(Question 1 continued)

Aimmika decides to carry out a χ^2 goodness of fit test at the 5% significance level to see whether the data follows a Poisson distribution with mean 4.2.

(d) (i) Write down the number of degrees of freedom for her test. [1]

(ii) Perform the χ^2 goodness of fit test and state, with reason, a conclusion. [7]

Aimmika claims that advertising in a local newspaper for 300 Thai Baht (THB) per day will increase the number of bags of rice sold. However, Nichakarn, the owner of the store, claims that the advertising will **not** increase the store's overall profit.

Nichakarn agrees to advertise in the newspaper for the next 60 days. During that time, Aimmika records that the store sells 282 bags of rice with a profit of 495 THB on each bag sold.

(e) Aimmika wants to carry out an appropriate hypothesis test to determine whether the number of bags of rice sold during the 60 days increased when compared with the historic sales records.

(i) By finding a critical value, perform this test at a 5% significance level. [6]

(ii) Hence state the probability of a Type I error for this test. [1]

(f) By considering the claims of both Aimmika and Nichakarn, explain whether the advertising was beneficial to the store. [3]

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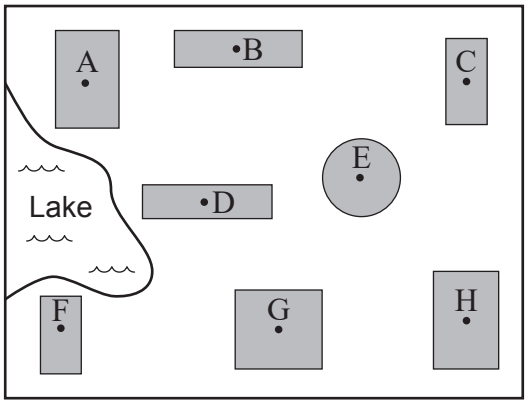
2. [Maximum mark: 28]

This question compares possible designs for a new computer network between multiple school buildings, and whether they meet specific requirements.

A school's administration team decides to install new fibre-optic internet cables underground. The school has eight buildings that need to be connected by these cables. A map of the school is shown below, with the internet access point of each building labelled A–H.



Jonas is planning where to install the underground cables. He begins by determining the distances, in metres, between the underground access points in each of the buildings.

He finds $AD = 89.2\,\text{m}$, $DF = 104.9\,\text{m}$ and $A\hat{D}F = 83^\circ$.

(a) Find AF. [3]

The cost for installing the cable directly between A and F is $\$21\,310$.

(b) Find the cost per metre of installing this cable. [2]

Jonas estimates that it will cost $\$110$ per metre to install the cables between all the other buildings.

(c) State why the cost for installing the cable between A and F would be higher than between the other buildings. [1]

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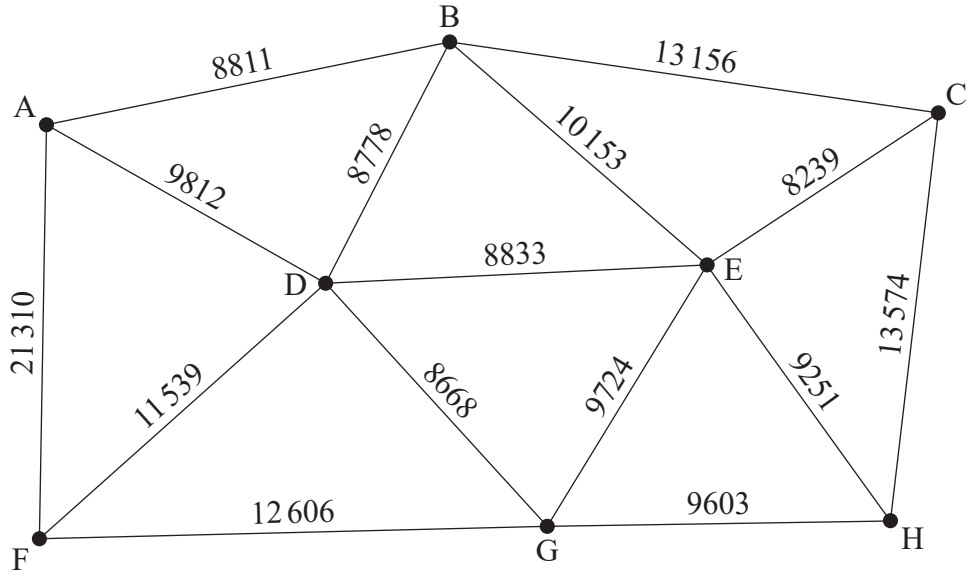
(Question 2 continued)

Jonas creates the following graph, S, using the cost of installing the cables between two buildings as the weight of each edge.



The computer network could be designed such that each building is directly connected to at least one other building and hence all buildings are indirectly connected.

(d) (i) By using Kruskal's algorithm, find the minimum spanning tree for S, showing clearly the order in which edges are added. [3]

(ii) Hence find the minimum installation cost for the cables that would allow all the buildings to be part of the computer network. [2]

The computer network fails if any part of it becomes unreachable from any other part. To help protect the network from failing, every building could be connected to at least two other buildings. In this way if one connection breaks, the building is still part of the computer network. Jonas can achieve this by finding a Hamiltonian cycle within the graph.

(e) State why a path that forms a Hamiltonian cycle does not always form an Eulerian circuit. [1]

(f) Starting at D, use the nearest neighbour algorithm to find the upper bound for the installation cost of a computer network in the form of a Hamiltonian cycle.

Note: Although the graph is not complete, in this instance it is not necessary to form a table of least distances. [5]

(g) By deleting D, use the deleted vertex algorithm to find the lower bound for the installation cost of the cycle. [6]

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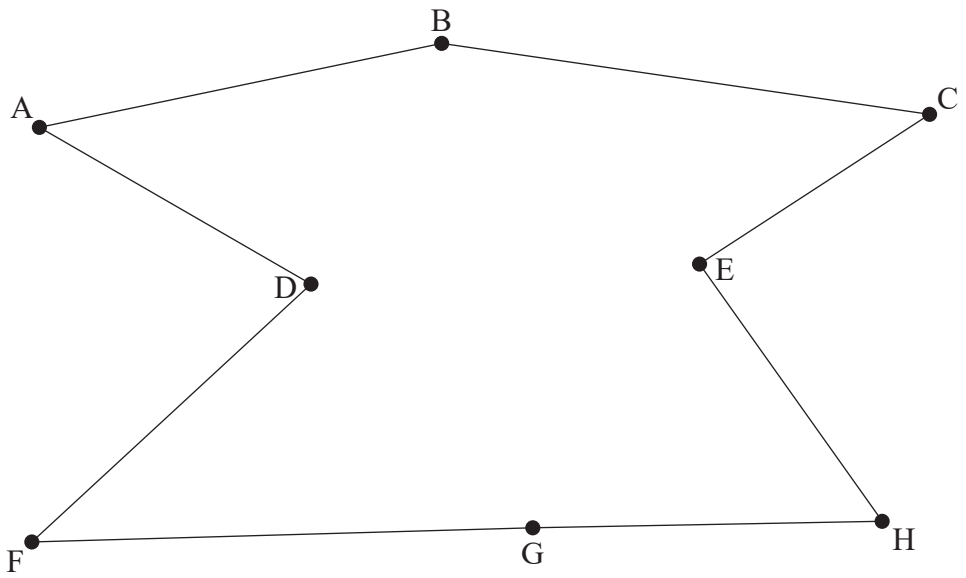
(Question 2 continued)

After more research, Jonas decides to install the cables as shown in the diagram below.



Each individual cable is installed such that each end of the cable is connected to a building's access point. The connection between each end of a cable and an access point has a 1.4% probability of failing after a power surge.

For the network to be successful, each building in the network must be able to communicate with every other building in the network. In other words, there must be a path that connects any two buildings in the network. Jonas would like the network to have less than a 2% probability of failing to operate after a power surge.

(h) Show that Jonas's network satisfies the requirement of there being less than a 2% probability of the network failing after a power surge. [5]

References: