International Baccalaureate

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Diploma Programme subject in which this extended essay is registered: Mathematics

(For an extended essay in the area of languages, state the language and whether it is group 1 or group 2.)

Title of the extended essay: An Investigation of the Various Practical Uses of Differential Calculus in Geometry, Biology, Economics, and Physics

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The extended essay I am submitting is my own work (apart from guidance allowed by the International Baccalaureate).

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chose to write about applications of differential calculus because she found a great interest in it during her IB Math class. She wishes she had time to complete a deeper analysis of her topic; however, her busy schedule made it difficult so she is somewhat disappointed with the outcome of her essay. It was a pleasure meeting with when she was able to and her understanding of her topic was evident during our viva voce. I, too, wish she had more time to complete a more thorough investigation. Overall, however, I believe she did well and am satisfied with her essay.

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I spent [illegible] hours with the candidate discussing the progress of the extended essay.

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Assessment form (for examiner use only)

Criteria	Examiner 1	maximum	Examiner 2	maximum	Examiner 3
A research question	1	2		2	
B introduction	1	2		2	
C investigation	1	4		4	
D knowledge and understanding	1	4		4	
E reasoned argument	0	4		4	
F analysis and evaluation	0	4		4	
G use of subject language	0	4		4	
H conclusion	0	2		2	
I formal presentation	2	4		4	
J abstract	0	2		2	
K holistic judgment	0	4		4	
Total out of 36	6				

Mathematics Extended Essay

An Investigation of the Various Practical Uses of Differential Calculus in Geometry, Biology, Economics, and Physics

Candidate Number:

2031 Words

Abstract

Calculus is a field of math dedicated to analyzing and interpreting behavioral changes in terms of a dependent variable in respect to changes in an independent variable. The versatility of differential calculus and the derivative function is discussed and highlighted in regards to its applications to various other fields such as geometry, biology, economics, and physics. First, a background on derivatives is provided in regards to their origin and evolution, especially as apparent in the transformation of their notations so as to include various individuals and ways of denoting derivative properties. Then, a discussion of the various methods often employed to find a derivative was evident. Such methods included were the tedious and time-consuming, yet precise, method of first principles, the quick and accurate simple rules of differentiating functions with variables and constants, the chain rule which is especially useful for composite functions, the product and quotient rules which reduce time and effort in having to first calculate the product or quotient and then find the derivative, and the most convenient and fast – the graphing calculator. Then, these various methods' application to other fields of study is made evident in regards to optimization in geometry, time rate of change in biology, general rate of change in economics, and motion behaviors in physics. I chose this topic to study and investigate further because in my math class, we had just gone over the method of first principles which seemed tedious and time-consuming, so I ventured to find and apply other, perhaps faster yet still precise, methods of calculating derivatives.

258 Words

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Introduction

Often times, people disregard the potential of or fail to acknowledge the significance of mathematics in their everyday lives, perhaps because they do not think that they can apply an aspect of it to their lives, that it will not be applicable to their lives, or that they do not have the knowledge or motivation to obtain knowledge on how it can influence their lives. One field of mathematics which is especially useful, but often not considered to be relevant to practical life, is calculus – more specifically, differential calculus. This branch of math allows us to derive at behavioral changes over time, distance, intensity, etc. by graphing and interpreting quadratic, exponential, or logarithmic equations and analyzing their significance in regards to other points or lines. With the intent of highlighting that mathematics and its differential calculus division – no matter how seemingly complex or daunting – can and actually are very relevant, applicable, useful, and appreciated in various components of life.

The subject of this investigation was the application of differential calculus to other vital fields of study which are relevant to understanding mathematics and practical life. Such other fields include geometry, biology, economics, and physics, in which a component(s) of differential calculus can be applied and used to model and interpret changes in variables. In the essay, it is made evident that information on derivatives is provided and discussed in regards to what they are, their importance in mathematics, and their historical development with the contribution of various authors. Then, the various different methods established to calculate derivatives, including first principles, simple rules, and mere graphing technology, are discussed and highlighted in terms of how they are calculated and which is appropriate in specific situations as well as their comparable veracities. Most importantly, the final component prior to conclusive analysis is the actual designing, manipulating, and solving of problems relating to geometry, biology, economics, and physics. Hence, a correlation between mathematics, other subjects, and real-life situations is highlighted through investigation of derivatives and their respective utilization.

In terms of the different subjects to which derivatives were applied, they each were focused on a feasible method of determining change. For example, in geometry, derivatives were applied to the optimization of a legitimate situation in which the manufacturing of a good was used as the variable for the calculation of dimensions of the good. Moreover, in biology, the time rate of change was calculated for polar bear populations over a given time period using differential calculus. For economics, the production costs of manufacturing and profiting from clothing is calculated utilizing derivatives. Changes in motion are also calculated using limits in differential calculus as applicable to physics velocity and acceleration situations. The versatility of differentiation and the core subject of derivatives are made evident in this investigation in the analysis of their application to various other fields of study and therefore practical life, in sometimes unexpected ways.

Chapter 1 – Background Information on Differential Calculus and the Derivative Function

This section will provide the information necessary to process and analyze the application of derivatives. Essentially, a historic overview of derivatives, involving the development of calculation methods for them, and their evolving notations is provided.

2.1: The Derivative Function

Differential calculus was developed by Sir Isaac Newton and Gottfried Wilhelm Leibniz in the 17^{th} century while attempting to find algebraic methods for finding solutions to real-life problems, such as sea navigation, dealing with gradients of tangents to curves at any point on the curve and with the rate of change in one variable with respect to another. Essentially, a derivative is a measure of how the behavior of a function changes as its input changes. For a real-valued function of a single real variable, the derivative at a specific point is equivalent to the slope of the tangent line to the graph of the function at that point. In most cases, the output y on a graph is dependent upon and changes with respect to changes in the input, the independent variable x. This dependence of y on changes in x indicates that y is a function of x. The derivative can be calculated using various methods, with the process of finding the derivative being referred to as differentiation.

2.2: Notations for Differentiation

Because differential calculus was developed at different times and places, and similar ideas were proposed by different individuals, different notations of differentiation have originated.

When the dependence of y on x is viewed as a functional relationship, meaning $y = f(x)$, one of the earliest notations of differentiation, is used. In this notation, introduced by Gottfried Leibniz, the first derivative is denoted by

$$\frac{dy}{dx}.$$

This notation allows variables to be specified for differentiation, where

$$\frac{dy}{dx} = \frac{dy}{du} \square \frac{du}{dx}.$$

One of the most common and modern notations is Lagrange's notation, in which the functional relationship of y in relation to x is also used, and a prime mark (') is used to denote the derivative of a function. Hence, the derivative of a function $f(x)$ is $f'(x)$. This notation is simpler, but Leibniz's notation is more indicative of the fact that the derivative is taken, especially in its use of d.

Although exclusively used for problems involving taking the derivative of a function where time is the independent variable, Newton's notation for differentiation (or the dot notation) places a filled dot over the function to represent a time derivative. In this case, as when $y = f(x)$, if $y = f(t)$, then "$\dot{y}$" denotes the first derivative of the function where behavioral changes over a period of time is measured.

Although other notations for differentiation such as Euler's exist, such notations are restricted to certain situations and are therefore not commonly used.

2.3: The n^{th} Derivative

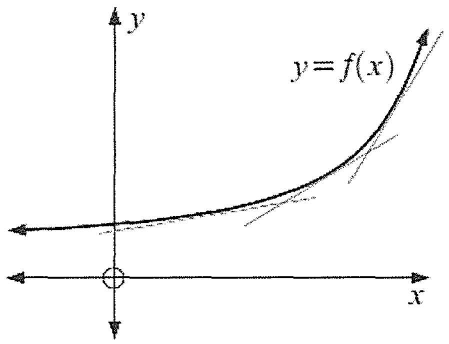
Often times, especially in the computation of velocity and acceleration in physics, the derivatives of derivatives are taken, with the derivative of the first derivative being the second derivative, and subsequent calculations being similar in manner.

For example, in physics, when velocity and acceleration are calculated, the second derivative is utilized in order to see where the curve meets the derivative again.

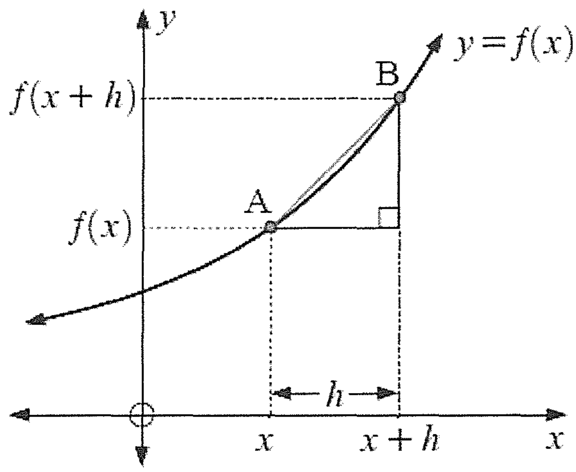
Chapter 3 – Different Methods of Calculating Derivatives

3.1: First Principles

For functions with curves with equation $y = f(x)$, the gradients of the tangents or the slope points are at different points. Often times, the purpose is to determine a gradient function so that x is replaced with a specific point a to find $y = f(x)$ at $x = $ a.



Considering a function $y = f(x)$, where point A is $(x, f(x))$ and B is $(x+h, f(x+h))$.



The chord AB has the gradient

$$= \frac{f(x+h) - f(x)}{x + h - x}$$

$$= \frac{f(x+h) - f(x)}{h}.$$

As B approaches A, then the slope of AB approaches the slope of A. Hence, the gradient of the tangent at the variable point $(x, f(x))$ is the limiting value of

$$\frac{f(x+h)-f(x)}{h}$$

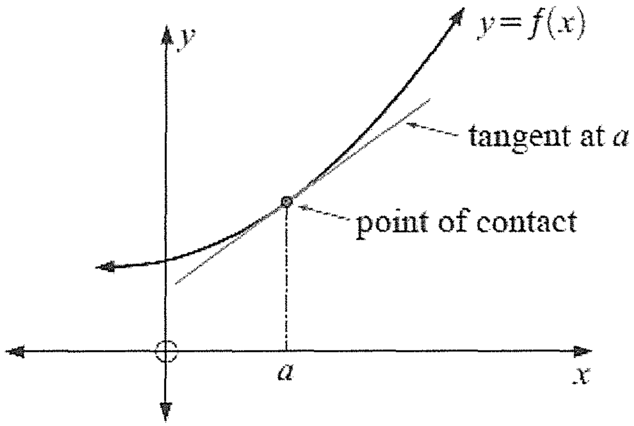
as h approaches 0, leading to the following first principles rule for determining derivatives.

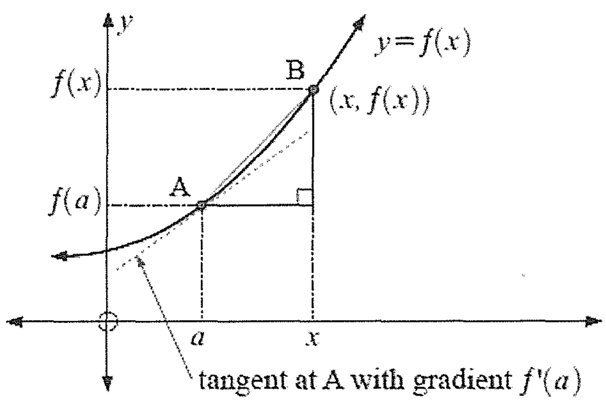
$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

However, since the gradient, or slope, is dependent on x, the limit formula is a function.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

When finding the derivative at a defined point where $x = a$, x in the limit formula for first principles is replaced with a.





3.2: Simple Rules

Because first principles can be a tedious method for determining derivatives of function, simple differentiation rules can be resorted to in order to decrease time and increase speed and efficiency. In a function, differentiating different aspects of a function are different. Differentiating the constant c in a function results in the derivative equaling 0 while differentiating x^n results in nx^{n-1}.

For example, $f(x) = x^3$ results in $f'(x) = 3x^2$ and $f(x) = 2x^2+1$ leads to the derivative equaling $4x$.

3.3: The Chain Rule

The chain rule is applicable to complicated functions when they can be written as composites of two or simpler functions.

For example, $y = (x^2 + 3x)^4$ could be written as $y = u^4$, where $u = x^2 + 3x$. Hence, if $y = u^2$, then

$$\frac{dy}{dx} = 2u \times \frac{du}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Now consider $y = (2x+1)^3$ which has the form $y = u^3$ where $u = 2x + 1$.

Expanding we have

$$\begin{aligned} y &= (2x+1)^3 \\ &= (2x)^3 + 3(2x)^2 1 + 3(2x)1^2 + 1^3 \quad \{\text{binomial expansion}\} \\ &= 8x^3 + 12x^2 + 6x + 1 \\ \therefore \quad \frac{dy}{dx} &= 24x^2 + 24x + 6 \\ &= 6(4x^2 + 4x + 1) \\ &= 6(2x+1)^2 \\ &= 3(2x+1)^2 \times 2 \\ &= 3u^2 \times \frac{du}{dx} \text{ which is again } \frac{dy}{du}\frac{du}{dx}. \end{aligned}$$

3.4: The Product Rule

If $f(x) = u(x) + v(x)$ then $f'(x) = u'(x) + v'(x)$.

In this case, the derivative of a sum of two functions is the sum of the derivatives. However, this cannot be applied to finding the derivative of the product of two functions. If the two functions can be stated as u and v, then the derivative of the functions' product is the sum of the derivative of one function times the other function and the derivative of the other function times the first function. Therefore,

$$f'(x) = u'v + uv'.$$

3.5: The Quotient Rule

The quotient rule is applicable to situations in which one function is being divided by another function. In this case, the derivative of the problem is the quotient of the difference of the derivative of one function times the other function and the derivative of the other function and the first function. For a function $Q(x)$, the derivative is denoted by:

$$Q'(x) = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{[v(x)]^2}$$

3.6: Graphing Calculator

All the previously discussed methods are tedious, time-consuming, and do not ensure accuracy because human involvement and interpretation may be inaccurate or be hindered by human fault. However, when time is not available, a graphing calculator may be used in order to quickly determine the derivative of a function and also see the graph of the function as well as plot the different points at which the gradient of the function can be calculated. In order to utilize a graphing calculator, in this case, a TI-84 Plus, the *Y=* button is followed by the entry of the function, such as $(4x^2+1)^3$. Then, the 2^{ND} and TRACE buttons are pressed, with the resulting screen allowing options of calculation, and the sixth choice is selected in order to calculate the derivative of the function, and tracing across the resulting graph allows for the gradient to the tangent to be computed at different points.

Chapter 4 – Conclusion

Many different methods of calculating derivatives have been discussed in the previous sections. Of the different methods, there are different applications of them in different fields of study, including geometry, biology, economics, and physics.

In geometry, optimizing using derivatives is especially valuable. For example, if a geometric figure calls for its external features to be manufactured, manufacturers would prefer if the amount of material they have to use to build the product is minimized because that would in turn minimize cost. Hence, they use derivatives to calculate the maximum dimensions for the production of that material. For example, if a manufacturing company makes cans out of metal, they can use derivatives to ensure that the maximum dimensions are used with the minimal amount of material. Moreover, in biology, population sizes differ over time, so derivatives could be applied in terms of time rates of change where population sizes are dependent on the time and the progression of changes. Moreover, in economics, maximum profits could be calculated using derivatives as backbones from which graphs can be generated and interpreted. The application of derivatives in physics is very apparent as the motion of particles is changes over increasing or decreasing distances. Here, speed, velocity, and acceleration, especially instantaneous, can be calculated using both the first and second derivatives.

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