International Baccalaureate

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Examination session (May or November)	May	Year	2012

Diploma Programme subject in which this extended essay is registered: Mathematics

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Title of the extended essay: To what extent was the method of infinite descent conclusive in proving Fermat's Last Theorem?

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The extended essay I am submitting is my own work (apart from guidance allowed by the International Baccalaureate).

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Please comment, as appropriate, on the candidate's performance, the context in which the candidate undertook the research for the extended essay, any difficulties encountered and how these were overcome (see page 13 of the extended essay guide). The concluding interview (viva voce) may provide useful information. These comments can help the examiner award a level for criterion K (holistic judgment). Do not comment on any adverse personal circumstances that may have affected the candidate. If the amount of time spent with the candidate was zero, you must explain this, in particular how it was then possible to authenticate the essay as the candidate's own work. You may attach an additional sheet if there is insufficient space here.

The student has produced a well thought out research of Fermat's Last Theorem. We have worked together a lot, since it was my first EE supervision. The topic had to be discussed so that I can analyse what he has written.

He is an excellent mathematician who understands Maths at a level much higher than his peers and able to discuss it effectively with other.

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I spent | 10 | hours with the candidate discussing the progress of the extended essay.

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Assessment form (for examiner use only)

Candidate session number

		Achievement level		
Assessment criteria		First examiner	maximum	Second examiner
	A research question	1	2	
	B introduction	0	2	
	C investigation	4	4	
	D knowledge and understanding	2	4	
	E reasoned argument	0	4	
	F analysis and evaluation	2	4	
	G use of subject language	3	4	
	H conclusion	1	2	
	I formal presentation	3	4	
	J abstract	2	2	
	K holistic judgment	2	4	
	Total out of 36	20		

of first examiner: ______________________ Examiner number: ______
TAL letters)

of second examiner: ______________________ Examiner number: ______
TAL letters)

Mathematics Extended Essay

TO WHAT EXTENT WAS THE METHOD OF INFINITE DESCENT CONCLUSIVE IN PROVING FERMAT'S LAST THEOREM?

by

Candidate Number

December 2011

Session: May 2012

Word Count: 3,704

Essay Supervisor:

Abstract

The research question for this essay is **"To what extent was the method of infinite descent conclusive in proving Fermat's Last Theorem?"** Fermat's Last Theorem is one of the greatest problems ever encountered in mathematics, and it became a real fascination of mine after reading about it. I discovered that the method of infinite descent was a common appearance among these books, so I decided to investigate how conclusive it was in proving Fermat's Last Theorem and thus emerged my research question.

which?

?

Fermat's last theorem has not been proved so far using infinite descent so the wording of the research question does not make much sense

This essay focuses on the proofs used by Euler, Dirichlet and Kummer, and how the method of infinite descent is used in each of them. The proofs themselves are not focused on in detail, only a brief summary of how the proof works can be given seeing as how the proofs are extensive, although beautifully elegant. Euler's proof of $n = 3$ was the first looked at, as it presents the template for the method of infinite descent. Dirichlet's and Kummer's proofs were then investigated afterwards, observing how the method of infinite descent evolved to work for different exponents of Fermat's Last Theorem.

The method that Ernst Kummer used to prove Fermat's Last Theorem for specific exponents was at first made for regular primes, but was then adapted for irregular primes. This led to Fermat's Last Theorem being proved for all prime exponents up to 4 million with the aid of computers. This might seem conclusive enough, since 4 million is a relatively large number, however no one knows for sure that it might not work for prime numbers above 5 million. Therefore the conclusion is that the method of infinite descent is useful in giving an idea as to whether Fermat's Last Theorem is true or not, but it will never conclusively prove it.

How can you say that? This is a completely unjustified conclusion.

Word Count: 295

Contents

1 Introduction to Fermat's Last Theorem (FLT)

1.1 Pythagoras Theorem

Fermat's Last Theorem is known to be one of the greatest mathematical problems the world has ever encountered. Its simple look is deceiving, troubling mathematicians for 350 years until Andrew Wiles cracked it in 1995. The history of this problem starts in the sixth century B.C. with Pythagoras of Samos. During this essay I shall be referring to Fermat's Last Theorem as FLT.

As Simon Singh said in his book 'Fermat's Enigma', "Usually half the difficulty in a mathematics problem is understanding the question, but in this case it was straightforward-..."[1] The problem of Fermat's Last Theorem looks very familiar to most people as it is based on Pythagoras' Theorem, a theorem engraved in millions of people's brain:

very badly presented!

$$x^2 + y^2 = z^2$$

meaningless! what are x, y, z ?

Pythagoras of Samos and his brotherhood in Croton, Italy, managed to find a very elegant proof for this, one of their biggest successes, subsequently leading to one of the greatest mathematical problem of all time.

what?

1.2 The beginning of Fermat's Last Theorem

Inevitably, this led to mathematicians asking themselves what would happen if the power in the equation was changed from '2' to '3' so that it looked like this:

$$x^3 + y^3 = z^3$$

[1] Singh 1998, p.6

No one knew that they had unleashed a monster of an equation, and although finding solutions to Pythagoras theorem, also called Pythagorean triples, was relatively easy, finding solutions to this new modified version seemed to be impossible. If the power in the equation is changed to an even higher number, finding solutions appears to be equally impossible. For centuries, mathematicians tried to find solutions to these modifications of Pythagoras' theorem with no success. This led the great seventeenth-century French mathematician Pierre de Fermat to believe that the reason nobody could find any solutions was because there were no solutions. In the margin of his copy of Diophantus' Arithmetica, he noted his observations:

meaningless! solutions to ?? a theorem..

??

what theorem? Great confusion here

Proposition 1 *It is impossible for a cube* of integers *to be written as a sum of two cubes* of integers *or a fourth power to be written as the sum of two fourth powers* of integers *or, in general, for any number which is a power greater than the second to be written as a sum of two like powers.* of integers

This gave birth to an adventure that would last 350 years to prove Fermat's Last Theorem with some successes but many failures. The first success would come with Leonhard Euler in the 18th century when he discovered a proof for the case where the power of the equation is 3.

No, Fermat himself proved it for $n=4$

2 Proofs (special cases) of Fermat's Last Theorem

2.1 Euler and the method of infinite descent

The method of infinite descent was the first method used to try and solve Fermat's Last Theorem for specific exponents, the first ones being $n = 4$ and later on $n = 3$. This method is a particular form of proof by contradiction (see further) and it is seen in Fermat's jottings in Arithmetica by Diophantus. Fermat used this method to prove the case for $n = 4$, and this is the most complete calculation by Fermat he ever committed to paper. false. He wrote on many other subjects

The method of infinite descent is very simple to understand. You begin by assuming that there is a solution to Fermat's Last Theorem for $n = 4$:

$$x = X_1 / y = Y_1 / z = Z_1$$ DO IT.

After examining the properties of this solution, you can show that if this solution does exist, then there must be a smaller solution (X_2, Y_2, Z_2). If you then examine this solution, you can find an even smaller solution (X_3, Y_3, Z_3). This can be done infinitely many times, finding infinitely many smaller solutions. However, the solutions to Fermat's Last Theorem must be whole numbers, therefore you cannot have infinitely many smaller solutions that are whole numbers so you reject the assumption that there is a ~~solution to~~ proof of Fermat's Last Theorem for $n = 4$.

"solution of a theorem" does not make sense!

Leonhard Euler saw this proof by Fermat and used this as his starting point for finding a general proof to prove all other cases of FLT. Euler started by attempting to prove FLT for $n = 3$. He adapted this method of infinite descent used by Fermat and was able to prove it. This was the first major breakthrough on FLT since Fermat himself, and it motivated more mathematicians to start working on it. In the following section I will show how Euler proved that there were no solutions to Fermat's Last Theorem for $n = 3$ using the method of infinite descent.

2.1.1 Euler's proof for $n = 3$

[2]The first thing Euler did was to assume there was a solution for Fermat's Last theorem for the case where $n = 3$.

An important part of the whole proof is showing that different num-

[2]Edwards 2000

two by two

bers are coprime[3]. The first time he does this is when he shows that x, y, z are coprime. To do this he proved that if an integer d divides 2 numbers in Fermat's Last Theorem, then d^n divides the nth power of the third. After that he proved that if d^n divides x^n then d divides that number x. By doing this he showed that if 2 of the numbers in FLT have a greatest common divisor larger than 1, then this number also divides the 3rd number. Therefore you can divide all of them by that number and keep doing this until they are coprime.

Euler then went on to show that if x, y, z are coprime, then there exist two integers p, q such that:

1. $\gcd(p, q) = 1$
2. p, q are positive
3. p, q have opposite parity (one is odd, one is even)
4. $2p(p^2 + 3q^2)$ is a cube

too sketchy
Not a proof

He later proved that the greatest common divisor of $2p$, $p^2 + 3q^2$ can only be 1 or 3. Euler did this by showing that the greatest common divisor cannot be 2 because $p^2 + 3q^2$ is odd and it can't be any prime larger than 3 by showing that it would divide both p and q, going against p and q being coprime.

By doing this, Euler can show that $(2a)(a - 3b)(a + 3b)$ is a cube because $(2a)(a-3b)(a+3b) = 2a^3 - 18ab^2 = 2p$ ($2p$ is a cube). Again he shows that $2a, a - 3b, a + 3b$ are coprime so that each of them is a cube.

And thus he found a new solution to Fermat's Last Theorem for $n = 3$ since $A^3 = 2a = (a+3b) + (a-3b) = B^2 + C^2$. He then showed that this new solution is smaller than the previous solution. This argument can be done infinitely many times and so there is a case of infinite descent. Since the solutions to Fermat's Last Theorem must be whole numbers, this is contradictory so he rejects the initial assumption that there exists a solution to Fermat's Last Theorem for $n = 3$.

Inadequate as a proof

[3] Coprime: Two integers a, b are said to be coprime if their greatest common divisor is 1 (they have no common positive divisor other than 1)

In this proof we see the fundamental principles of the method of infinite descent. Euler assumed a solution existed, and through some heavy number theory, (greatest common divisors, and proving two numbers are coprime) Euler was able to prove that if a solution did exist, then another solution must exist. This solution $2a, a - 3b, a + 3b$ is shown to be smaller than the first solutions and this process can be repeated infinitely many times, which would make no sense. Therefore there is no solution to FLT for $n = 3$. The original proof for $n = 3$ is much longer than the condensed version I have presented, as I have only shown the most important parts of the argument leading to the final conclusion. This method was adapted from Pierre de Fermat's proof of $n = 4$, and it was the first major breakthrough of Fermat's Last Theorem after Pierre de Fermat himself. Mathematicians went back to work, and the method of infinite descent seemed promising to give a final proof for all exponents of Fermat's Last Theorem.

??

indeed!

2.2 Dirichlet, Sophie Germain's Theorem, proof by contradiction

Proof by contradiction is a very popular form of proof in the world of mathematics, and can be seen in many cases, for example proving the irrationality of $\sqrt{2}$. It is very similar to the method of infinite descent seen previously, as that itself is a type of proof by contradiction. To explain how proof by contradiction works, I will use the example of proving the irrationality of $\sqrt{2}$.

You start by assuming that something is true, in this case you assume that $\sqrt{2}$ is rational. If $\sqrt{2}$ is rational, then it can be written as a fraction $\frac{p}{q}$. By doing some calculations we can then find that this fraction can be simplified:

not needed Irrelevant here

1. $\sqrt{2} = \frac{p}{q}$
2. Square both sides: $2 = \frac{p^2}{q^2}$

3. Multiply by q^2: $2q^2 = p^2$

4. From this we can see that p^2 must be even, so p must also be even. Therefore, we can substitute $2m$ for p: $2q^2 = (2m)^2 = 4m^2$

5. Divide both sides by 2: $q^2 = 2m^2$

6. Here we have the same situation as before, q^2 is even therefore q must be even. We can then say that $q = 2n$. From this we have found that $\sqrt{2} = \frac{2m}{2n} = \frac{m}{n}$

7. We now have a fraction that is simpler than $\frac{p}{q}$ which is $\frac{m}{n}$

8. This argument can be repeated over and over again to find simpler fractions. However we know that fractions cannot be simplified forever therefore we must reject our assumption that $\sqrt{2}$ is rational.

not necessary if $\frac{p}{q}$ is irreducible

We can see that this proof by contradiction, is actually another case of the method of infinite descent. Johann Dirichlet used this method to attempt to prove the case for $n = 5$. Dirichlet completed part of the proof for $n = 5$, and the whole proof was then completed by Adrien-Marie Legendre. This proof uses Sophie Germain's Theorem, named after Sophie Germain that deals with the divisibility of the solutions of FLT, which I will explain in the proof for $n = 5$ in the following section.

NO

2.2.1 Dirichlet's proof for $n = 5$

[4]Just like Euler did, Dirichlet started by assuming there was a solution to Fermat's Last Theorem and proving that the solutions x, y, z were coprime. Dirichlet then made the assumption that x, y are odd and z is even, because there can only be at most one even number since they are all coprime but there must be at least one even because odd+odd≠odd.

Dirichlet used Sophie Germain's Theorem to help him prove this case of Fermat's Last Theorem. Sophie Germain's theorem said that if Fermat's Last Theorem is true for any prime $n \geq 3$ and if $2n + 1$ is a prime, then n must divide the product xyz. He used this theorem to show that

[4]Edwards 2000

not the same n!
Very poorly stated

either 5 divides z or it divides x, y. Not clear how!

Dirichlet then assumed that 5 divides z and showed a case of infinite descent by showing that if there is a solution, there must be a smaller solution. Therefore, if 5 divides z, there are no integer solutions. He also showed that if 5 divides x or y (doesn't matter which since they are symmetric) then there are also no integer solutions by using the method of infinite descent once again.

The proof that Dirichlet used to prove Fermat's Last Theorem for $n = 5$ showed lots of similarities to the previous proof by Euler for $n = 3$, the main one being that they both used the method of infinite descent to prove that there couldn't be any solutions. A considerable amount of number theory was again used, the concept of coprime numbers and greatest common divisors being used constantly to develop the argument further. The method of infinite descent was the key to proving Fermat's Last Theorem for $n = 5$, however it needed an extra bit of help from Sophie Germain's theorem. For $n = 3, 4$ the method of infinite descent alone was enough to prove Fermat's Last Theorem, but for this case something else was needed to come to the final conclusion. Doubts about the method of infinite descent being used to prove Fermat's Last Theorem completely started appearing, but people had faith in it, and so continued to use it to carry on proving specific exponents.

not heavy number theory!

Again too sketchy. Not a proof

2.3 Lamé and Kummer: Cyclotomic Integers to prove FLT

In 1847 the French academy of sciences set up an award and offered prizes, of which one was a gold medal and 3,000 francs, to whoever could prove Fermat's Last Theorem once and for all. Mathematicians were given an extra motivation to go and prove FLT, as apart from the personal satisfaction of proving it, there was a respectable sum of money involved as well. Various rumours were running around France as to who was using which methods and how close people were to actually proving it. The big shock came on the 1st of March 1847, in the hands of Gabriel Lamé.

2.3.1 Lamé's idea of a final proof

Gabriel Lamé had proved FLT for the case $n = 7$ and was now stepping up in front of the meeting of the French academy of sciences and made it known that he was on the verge of proving Fermat's Last Theorem. Lamé's idea was very simple and could potentially work if it were not for the flaw in his logic that Liouville and Kummer pointed out later. Lamé realized that in the previous proofs for the cases $n = 3, 4, 5, 7$, a lot depended on an algebraic factorization of some sort. An example would be in the case for $n = 3$, where $x^3 + y^3$ is factorized into $(x + y)(x^2 - xy + y^2)$. Lamé noted that as n becomes very large, it becomes harder to factorize as the degree of the polynomial becomes very large. Therefore, Lamé thought of using complex numbers to factorize $x^n + y^n$ completely into linear factors. The only way this can be done is by inputting a complex number α such that $\alpha^n = 1$ where $\alpha \neq \pm 1$. The equation would then look like this:

not enough!

$$x^n + y^n = (x + y)(x + \alpha y)(x + \alpha^2 y)...(x + \alpha^{n-1} y)$$

Once Lamé had this equation, all that was left for him to do was prove that all the linear factors are coprime, i.e. their greatest common divisor is 1. This would mean, as seen in the other proofs earlier, that each linear factor is an nth power and from this he would then demonstrate a case of infinite descent which would prove FLT. It seemed as though with the help of complex numbers, the method of infinite descent would prove Fermat's Last Theorem once and for all. It had been used for $n = 3, 4, 5, 7$, and had worked perfectly so people were becoming more convinced that this would finally give the solution everyone was looking for.

However, Lamé missed out a minor detail, but a detail that would ultimately make all his work up to then useless. After Lamé's presentation, Liouville came up on the podium, and showed everyone Lamé's unfortunate flaw in his proof.

2.3.2 Liouville's discovery of Lamé's flaw

We all know that integers can only be fully factorized in one way, for example the number 76 is factorized to $76 = 2^2 * 19$ and it can't be factorized in any other way. In other words, "there is only one possible combination of primes that will multiply together to give any particular integer greater than 1"[5]. Lamé's proposed proof depended on this theorem, however he had failed to consider if complex numbers could also be factorized uniquely and Liouville was there to point this out. This didn't stop Lamé though, as he realized that the law for integers also worked for complex numbers when $n = 5$. He was determined to carry on with his work.

However, later on, Liouville read a letter from Ernst Kummer, a German mathematician, stating that Liouville was correct when he was questioning Lamé's use of unique factorization on complex numbers. Apparently, Kummer had proved this in a memoir he had published three years earlier. After this, Lamé deserted his attempts to prove Fermat's Last Theorem and Kummer continued this work, trying to find an alternative.

2.3.3 Kummer and cyclotomic integers

The problem which Kummer posed was the breaking up of numbers built up from α by repeated addition, multiplication and subtraction into prime factors. The numbers look like this: ??

[5] Singh 1998, p.114

$$\alpha_0 + \alpha_1\alpha + \alpha_2\alpha^2 + ... + \alpha_{\lambda-1}\alpha^{\lambda-1}$$ $\alpha \neq 1$ enough in this case

In this number, $\alpha_1, \alpha_2, ..., \alpha_{\lambda-1}$ are integers. Kummer used the letter λ to represent a prime number and say that $\alpha^\lambda = 1, (\alpha \neq (\pm 1))$. These complex numbers are known as cyclotomic integers. Since $\alpha^\lambda = 1$, Kummer reduced all the powers of the equation by saying that $\alpha^{\lambda+1} = \alpha$, $\alpha^{\lambda+2} = \alpha^2$ and so on. All this is pretty trivial. Kummer's work was deeper!

An interesting property of cyclotomic integers that will be needed later to prove another property is that "representations of cyclotomic integers in the form as seen above are not unique"[6]. An example of this would be that $1 + \alpha + \alpha^2 + ... + \alpha^{\lambda-1} = \alpha^\lambda + \alpha + \alpha^2 + ... + \alpha^{\lambda-1} = \alpha(1+\alpha+\alpha^2+...+\alpha^{\lambda-1})$. This implies that either $1+\alpha+\alpha^2+...+\alpha^{\lambda-1} = 0$ or $\alpha = 1$. Kummer had already assumed that $\alpha \neq 1$ therefore the former must be true. So? Incomplete reasoning! Not understood!

[illegible]

Another property of cyclotomic integers that was necessary for Kummer to try to prove Fermat's Last Theorem is the norm of a cyclotomic integer. The norm of a cyclotomic integer $f(\alpha)$ would be written as $Nf(\alpha)$ and it is defined as "the product of $\lambda - 1$ conjugates of $f(\alpha)$"[7]: what is f?

what is that?

$$Nf(\alpha) = f(\alpha)f(\alpha^2)...f(\alpha^{\lambda-1})$$?? Copied without understanding!

He then established that the norm of any cyclotomic integer is an integer itself. The proof for this is fairly simple. We must first note that if we convert $\alpha \rightarrow \alpha^j (j = 1, 2, ..., \lambda - 1)$ this only rearranges the factors of $Nf(\alpha)$ but does not change the norm. Thus, we have that $Nf(\alpha) = c_0 + c_1\alpha + c_2\alpha^2 + ... + c_{\lambda-1}\alpha^{\lambda-1} = c_0 + c_1\alpha^j + c_2\alpha^{2j} + ... + c_{\lambda-1}\alpha^{(\lambda-1)j}$. From this we can say that $c_0 - c_0 = c_j - c_1 = 0$. Therefore $c_j = c_1, (j = 1, 2, 3, ..., \lambda-1)$ and $Nf(\alpha) = c_0 + c_1(\alpha+\alpha^2+...+\alpha^{\lambda-1})$.From earlier, we know that $1 + \alpha + \alpha^2 + ... + \alpha^{\lambda-1} = 0$ and so we know that

completely inadequate Copied without understanding

[6]Edwards 2000, p.82

[7]Edwards 2000, p.83

why?

$\alpha+\alpha^2+...+\alpha^{\lambda-1} = -1$. Hence $Nf(\alpha) = c_0+c_1(\alpha+\alpha^2+...+\alpha^{\lambda-1}) = c_0-c_1$ which is an integer.

If a cyclotomic integer is found to have norm 1, this integer is then called a unit. A cyclotomic integer $h(\alpha)$ which is irreducible (it cannot be factored into two primes) only has the factorizations $h(\alpha) = f(\alpha)g(\alpha)$ where either of them is a unit. If one was talking about ordinary integers here, they would be tempted to then call $h(\alpha)$ prime. However, when talking about cyclotomic integers, just because it is irreducible, does not mean it is prime. Another factor that has to be taken into account for a cyclotomic integer to be prime is that "there must exist cyclotomic integers that it does not divide and if the product of any two cyclotomic integers it does not divide, is itself a cyclotomic integer it does not divide"[8]. The fact that a cyclotomic integer can be irreducible but not prime is the main problem that causes the failure of unique factorization for cyclotomic integers.

?? not clearly stated.

Now Kummer had to apply this to try and prove Fermat's Last Theorem. His problem now was that he had to factor binomials of the form $x + \alpha^j y$ and also find all possible prime factors to those binomials. This would then show if $(x + y)(x + \alpha y)(x + \alpha^2 y)...(x + \alpha^{\lambda-1}y)$ were relatively prime so that he could prove FLT. Kummer was aware that not all cyclotomic integers could be factorized in only one way and so he introduced the concept of ideal numbers. Kummer's discovery was that "the set of all complex integers defined by an nth root of unity could be so enlarged by the introduction of ideal numbers that unique factorization into primes would prevail in the enlarged set"[9]. An example of how ideal numbers can help make unique factorization possible is shown below:

not defined! just copied!

1. 25 can be factorized into $5 * 5$ or it can also be factorized into $(4 - \theta)(4 + \theta)$ where $\theta = 3i$

[8] Edwards 2000, p.84

[9] Dickson 1917, p.170

2. This problem of unique factorization can be solved by the introduction of α,β, γ ideal prime numbers such that:

$$\begin{aligned} 5 &= \alpha\beta \\ 4-\theta &= \alpha^2 \\ 4+\theta &= \beta^2 \end{aligned}$$

Now each factor of 25, breaks down further into ideal prime numbers such that $25 = \alpha^2\beta^2$ so that now there is only one way to factorize 25. ??

The introduction of ideal numbers helped to save unique factorization for complex numbers and Kummer was able to prove Fermat's Last Theorem for all regular primes, but not for the irregular primes which occur around 39% of the time. Kummer's method was later extended to irregular primes in the 20th century and his method was then inputted into computers so that the computers could carry on proving cases for Fermat's Last Theorem. By 1993, Fermat's Last Theorem had been proved for all prime numbers up to 4 million. Fermat's Last Theorem only needs to be proved for prime numbers as every other number is built up from prime numbers therefore it could be re-written with the exponent as one of the primes. Thanks to Ernst Kummer, the method of infinite descent prevailed and was used right up until the final proof of Fermat's Last Theorem to prove specific exponents. Some people might say that Fermat's Last Theorem is true simply because so many cases have been proved, and it seems as though this will carry on working for all other exponents. However, mathematicians are never satisfied with the finite, only with the infinite. It could be that Fermat's Last Theorem is false when the exponent is 5 million. Only when Andrew Wiles proved Fermat's Last Theorem for all possible exponents, were mathematicians satisified that it was true.

not defined!

Examples do NOT prove a theorem!

3 Conclusion

This wasn't the end of the history of proving Fermat's Last Theorem. Goro Shimura and Yutaka Taniyama would create the Taniyama-Shimura conjecture. Frey would then link this to Fermat's Last Theorem and whoever could prove the Taniyama-Shimura conjecture would then subsequently prove Fermat's Last Theorem and that someone was Andrew Wiles.[10]

The method of infinite descent was the key to the first proofs of Fermat's Last Theorem. The pure method created by Pierre de Fermat himself, is seen in both his and Euler's proof of FLT for $n = 3, 4$ and it is both simple yet solid. In the proof for $n = 5$ by Dirichlet, you can see the method of infinite descent being used along with Sophie Germain's theorem about the divisibility of solutions to Fermat's Last Theorem to show that there are no solutions and we can also see many aspects that are similar between Dirichlet's proof and Euler's proof.

Later on it seemed that the method of infinite descent would be the key to proving Fermat's Last Theorem completely. Lamé proposed a way to do this, by factorizing using roots of unity and cyclotomic integers, and then using the method of infinite descent to show that there couldn't possibly be a solution in the first place. Liouville and Kummer however crushed this proposed proof because a cyclotomic integer does not abide to the laws of ordinary integers when talking about unique factorization into prime numbers.

All this is purely descriptive, copied from various textbooks No mathematical activity involved!

Kummer however, picked this method up again and by introducing the concept of ideal numbers, he was able to prove Fermat's Last Theorem for regular primes using this method of factorizing and then proving a case of infinite descent. Kummer's method then lived on and was adapted to be able to prove Fermat's Last Theorem for irregular primes as well and computers were able to prove it for all primes up to 4 million

[10] Singh 1998

by 1993.

The method of infinite descent was a beautiful and elegant way to prove Fermat's Last theorem and it is usual for mathematicians to strongly believe in something elegant. The method works perfectly for specific exponents, with a few adjustments along the way, the main ones being Sophie Germain's theorem, the introduction of ideal numbers, and also an adaption of this method to work for irregular primes. However, how conclusive was the method of infinite descent in proving Fermat's
meaning? Last Theorem? From a normal person's perspective, it is perfectly acceptable, as proving a theorem for all prime numbers up to 4 million seems to be solid evidence. However, from a mathematician's perspective, the method of infinite descent was nowhere near conclusive in prov-
so far! ing Fermat's Last Theorem. The actual proof is very different, using much more complex mathematics concerning elliptic curves and modular forms. The method of infinite descent helped to motivate mathematicians to carry on working on Fermat's Last Theorem, and other proofs emerged from it as well such as proving the irrationality of $\sqrt{2}$. Over- nonsense!
all the method of infinite descent was a great discovery in the world of mathematics, but was only partly successful in proving Fermat's Last Theorem from a mathematicians perspective.

4 Bibliography

Works cited in this essay:

References

[1] Dickson, L.E. "Fermat's Last Theorem and the origin and nature of the theory of algebraic numbers". The Annals of Mathematics, Second Series, Vol. 18, No. 4 (June 1917), pp. 161-187, Annals of Mathematics, 1917.

[2] Edwards, Harold M. "Fermat's Last Theorem: A genetic introduction to algebraic number theory". Graduate Texts in Mathematics, Springer, 2000

[3] Singh, Simon. "Fermat's Enigma". Anchor Books, 1998

This essay is purely descriptive: just a juxtaposition of text copied from various textbooks, with heavily reduced mathematical content (just ~~sketch~~ succinct sketches of proof) and no mathematical input from the candidate showing more than a very superficial understanding

An EE in mathematics must contain some mathematical activity (such as a proof) and must not be purely descriptive