Supervisor's report

The supervisor should complete the report below and then give this cover, enclosing the final version of the extended essay, to the Diploma Programme coordinator. The supervisor must sign this report; otherwise the extended essay will not be assessed and may be returned to the school.

Name of supervisor (CAPITAL letters)

Comments

If appropriate, please comment on the candidate's performance, the context in which the candidate undertook the research for the extended essay, any difficulties encountered and how these were overcome. These comments can help the examiner award a level for criterion H. Do not comment on any adverse personal circumstances that may have affected the candidate.

has approached this task very diligently. She began early and took a fair amount of time and trouble to work out the scope and structure of her essay, before settling into her final product. I think she has learnt a lot from the experience and she has been prepared to work hard to this end.

I have read the final version of the extended essay that will be submitted to the examiner.

To the best of my knowledge, the extended essay is the authentic work of the candidate.

I spent 5 hours with the candidate discussing the progress of the extended essay.

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Assessment form (for examiner use only)

Candidate session number	

		Achievement level		
		First examiner	maximum	Second examiner
General assessment criteria *Refer to the general guidelines.*	**A** Research question		2	
	B Approach		3	
	C Analysis/interpretation		4	
	D Argument/evaluation		4	
	E Conclusion		2	
	F Abstract		2	
	G Formal presentation		3	
	H Holistic judgement		4	
Subject assessment criteria *Refer to the subject guidelines. Not all of the following criteria will apply to all subjects; use only the criteria that apply to the subject of the extended essay.*	**J**			
	K			
	L			
	M			
	Total out of 36			

Name of first examiner:
(CAPITAL letters)

Examiner number:

Name of second examiner: ______________________________
(CAPITAL letters)

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Area – Mathematics

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Research Question – Does Queuing Theory Adequately Predict the Performance Measures of a Single Server Queue?

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Abstract

This essay addresses the question: "Does Queuing Theory Adequately Predict the Performance Measures of a Single Server Queue?"

Queuing Theory is an area of mathematics, which studies the behaviour of waiting lines. It is used in many fields of study to optimise the design of processing systems that involve random arrival and service (or processing) time intervals, for example, in banking, retail, airports or computer networks. One of the simplest models studied with Queuing Theory is the Single Server Queue. This model assumes that the arrival and service rates both follow the Poisson Distribution.

In this essay, two real world examples are investigated (queues at a Boost Juice Bar and at a Westpac ATM facility). The study will test how well the Queuing Theory equations predict the performance measures of the systems in these situations. However, particular emphasis is placed on the expected waiting time of the queue and the utilisation of the system. "Adequate" prediction was considered to be a prediction within approximately 10% of the measured values. In addition, the assumption that the Single Server Model had arrival and service rates following the Poisson Distribution was also investigated, as this could be essential to the accuracy of the predicted measures.

It was concluded that the equations of Queuing Theory did not accurately predict the performance measures of the analysed queues. In particular, the average waiting time was very poorly predicted. Utilisation of the system seemed better predicted, but still illustrated a significant error. The reason for this poor model prediction was evaluated to be the fact that the arrival and service rates did not follow the Poisson Distribution closely enough. Some suggestions for further work are given to study Queuing Theory models where the assumption of the Poisson Distribution is not required.

Words: 294

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To for her guidance through this essay.

And to my family, particularly my dad, for their patience and support.

1. Introduction

Queuing theory is the mathematical study of queues, or waiting lines (ATIS staff, 2001). **The topic I am researching is how well queuing theory adequately predicts the performance measures of a single server queue.** This topic was chosen due to an interest in mathematics, and the thought that it would be interesting to investigate this subject behind an everyday area, such as the routine of queuing. On studying the subject, I found the area is important and worthy of study for fields such as the design of computer software, internet and data processing systems, and of course, in the obvious retail industries.

At first, the subject seemed to be a simple situation. Little did I realise the complexity of its mathematics. Consequently, only the most basic – single server – queuing model will be explored, given the time available and this subjects required mathematical skill level. However, it should be acknowledged that there are other, more complex queuing models. Queuing theory states that a single server queue has random arrival and service rates, which follow certain probability distributions. The distribution that will be focused on is the Poisson Distribution. The main aim of this essay is to test if the performance measures of a single channel queue as stated by queuing theory are correct for two separate samples.

2. Little and Little's Law

The work of John Little is often very important in the study of Queuing Theory.

John Little was an institute professor and the Chair of Management Science at the MIT Sloan School of Management in 1961 (Various, 2007 "Little's Law"). He proved the following theorem;

"The average number of customers in a stable system over a time interval is equal to their average arrival rate, multiplied by their average time in the system". (Various, 2007 "Little's Law")

The theorem can be represented mathematically:

$N = \lambda T$

Where N is the average number of customers in the system, λ is the average arrival rate, and T is the average time spent in the system (in consistent units).

The model works very well for a single server queue.

For example, a boutique contains on average 20 customers ($N = 20$) at one time, and each customer remains in the store for approximately half an hour. ($T = 0.5$). Using the equation, we can see that

$20 = \lambda \times 0{\cdot}5$

thus $\lambda = 40$

Using Little's Law we can see that there would be approximately 40 customers arriving per hour.

3. Arrival and Service Rate Distribution

Within queuing theory there are multiple issues that have to be dealt with. Initially, there are modelling issues. In the queue, there will be random arrival rates and random service rates because different customers have different needs and therefore varying arrival times. If arrivals came at regular predictable intervals and then were also serviced at a predictable and constant rate the modelling of the queue would be very simple. Therefore, we need to consider these random rates in our model. This study is focused only on single server queues; however it may be worthwhile to investigate other issues of the queue. For example, if there were more servers, such as in a multiple server queue, waiting time would be greatly minimised, however costs would be too large than perhaps is necessary. To decide whether or not to employ another server, businesses can use the server utilisation as calculated using Queuing Theory, which may be the most useful component of Queuing Theory.

When modelling a queue, there are four major elements that must be considered. The arrivals, services, number of servers and the queue discipline (Havlicek & Domeova, 2003).

The first element, arrivals, focuses on how the customers arrive in the queue. Customers may arrive singly or in batches, they may arrive at evenly spaced time intervals or at random intervals and the population of the queue may be infinite or finite. Formulas for queues rely on the arrival rate of the customers (or the number of units arriving per time period, λ) and the service rate (or the number of units serviced per time period, μ) and the time between the arrivals. Most often, the arrival times will be randomly spaced. If not, they may be *periodic* (or have the same period length between successive arrivals) (Havlicek & Domeova, 2003). This usually only happens with a machine controlled queue, such as in a production line.

As the majority of queues have randomly spaced arrivals, and therefore, randomly spaced services, we can model the number of arrivals per time unit with the Poisson Distribution. The Poisson Distribution is used to determine the probability of obtaining a certain number of events that take place in a certain interval (Urban et. al, 2004, p.746). The equation for the Poisson distribution is as follows:

$$p(X = x) = \frac{\sigma^{x} e^{-\sigma}}{x!}$$

where σ, in this case, is the population mean and x is the possible value of, in this case, the number of people arriving in a particular minute. Thus, from the data collected from a queue, we can calculate whether or not the arrival times and/or service times are Poisson distributed.

Sometimes, a customer can balk, or reneg. The former is when the customer inspects the service facility and the queue, and then decides to leave, while the latter is when a customer joins the line and after some waiting time, they decide to leave, or reneg. (Beasley, 2004)

"A queue discipline is a priority rule or set of rules for determining the order of service to customers in a waiting line." (Havlicek & Domeova, 2003). There are many different types of Queue Disciplines, the most common being "first come, first served". This is clearly evident in a single server queue as it is clear that a customer who has arrived in the line first will be served before another who arrives later.

This may not happen in all queues however, such as in a hospital. It can have an "emergencies first" queue discipline, where the most serious emergencies will be treated first. With a queue discipline, the main problem is ensuring that the customers know the rule, and follow it (Havlicek & Domeova, 2003).

4. System Performance Measures

To answer the research question, it is necessary to state the performance measures that can be calculated using Queuing Theory. These performance measures are:

- The expected number of customers in the queue (defined as L_q)
- The expected number of customers in the system. (defined as L)
- The utilisation of the system (defined as U)
- The expected waiting time (defined as W_q)
- The total expected time in the system (defined as W)

  (Ashley, 2000)

These components of the single server queue will be investigated throughout the essay with particular emphasis on U and W_q, as these are more valuable to retail businesses (and as a result, more worthy of study).

Queuing Theory predicts, using mathematical analysis (which is beyond the scope of this essay), the following formulae to calculate the above performance measures. These apply to a steady state single server queue with customer arrival rate of λ and service rate of μ, both Poisson distributed:

$$U = \frac{\lambda}{\mu}$$

$$W = \frac{1}{\mu - \lambda}$$

$$W_q = W - \frac{1}{\mu}$$

$$L_q = \lambda W_q$$

$L = \lambda W$ (Note, this is also Little's Law, which applies to any queuing model)

(Ashley, 2000)

5. Data Collection

5.1 Data no. 1 - Boost Juice Bar

To test the adequacy of Queuing Theory, I went to my local juice bar to record the arrival and departure rates of the customers. The process of this queue is that the customer arrives, waits in the line, pays, waits for juice to be created, then leaves. *However, the definition of the service time was decided as the time between the* customers arriving at the cashier to the time when they have finished paying. This was done in an aim to replicate a true single channel, single phase system. To do this, I recorded the arrival time as the customer joined the line, and the departure time as the time the customer finished paying. The beginning of the service time could then be labelled as the time when the previous customer finished paying. If there was no previous customer, the arrival time would be the beginning of the service time, as they would be served immediately. In practice, the customer still needs to wait for the juice to be created, but this is not part of the queue being studied. A full table of results are given in Appendix 1.

(It should be noted that the duration of the data collection was in fact a little bit greater than 60 minutes, unintentionally. However this is relevant when calculating λ.)

5.1.1 Analysis Using Little's Law

Below are some recorded arrival times from the Boost Queue during a time interval of one hour, 60 minutes:

Person no. (arrival)	Time
1	12:40:03 PM
2	12:40:40 PM
3	12:42:03 PM
4	12:42:27 PM
5	12:42:27 PM
6	12:42:28 PM
7	12:42:51 PM
8	12:42:51 PM
9	12:43:05 PM
10	12:43:37 PM

...Refer to Appendix 1.

From this data, the mean number of customers per minute was calculated, and the arrival rate was 78 customers per hour, which is 1.29 customers per minute and λ = 1.29

To calculate the average number of customers in the juice bar at any one time, using Little's Law, I had to calculate the average time spent in the bar. I did this by calculating the time in the system for each arrival using the times between each arrival and departure.

Using customer number one, the arrival (a) and departure (d) times are as follows:

Person 1	a	12:40:03 PM	d	12:40:43 PM

By subtracting the arrival time from the departure time, we are left with a time of 00:00:40. This is the total time in the system for Person 1.

This was done with each of the arrival times, which were added together, then divided by the number of arrival times I had collected. The result was:

$$\frac{3:28:34}{78} = 00:02:40$$

And thus the average time spent in the juice bar was 2 minutes and 40 seconds or 2.66 minutes.

Therefore the average number of customers in the queue at the juice bar at any one time was calculated, using Little's Law and substituting in the appropriate numbers, (λ = 1.29 customers per hour and T =2.66 hours). I found that:

$N = \lambda T$

$N = 1{\cdot}29 \times 2{\cdot}66$

$N = 3{\cdot}44$

Thus, there was an average of 3.44 people in the juice bar at any one time.

5.1.2 Analysis of Arrival Rates

This data can also be used to check if the arrival rates could be modelled by the Poisson Distribution. The following table shows the arrival times of the customers in order:

Person no.	Arrival Time	"minute number"
1	12:40:03 PM	40
2	12:40:40 PM	40
3	12:42:03 PM	42
4	12:42:27 PM	42
5	12:42:27 PM	42
6	12:42:28 PM	42
7	12:42:51 PM	42
8	12:42:51 PM	42
9	12:43:05 PM	43
10	12:43:37 PM	43
11	12:43:39 PM	43
12	12:44:30 PM	44
13	12:44:30 PM	44

… *Refer to appendix 1*

Using these times, we can calculate how many people arrived at the line each minute i.e. during the minute from 12:40:00 PM to 12:41:00 PM ("minute 40") there are evidently 2 people who arrived at the queue during that minute. Calculating the number of customers arriving each minute, and tabulating this, we have the following outcome:

Possible values (x_i)	0	1	2	3	4	5	6
Frequencies (f_i)	24	14	11	6	2	2	1

The possible values, x_i, are the number of people arriving in one particular minute and the frequencies, f_i, are the number of minutes that displayed that particular possible value. For example, there are 14 minutes in which 1 person arrived during a minute, and zero people arrived for 24 separate minutes.

We can use the mean arrival rate, $\lambda = 1.29$, found earlier and the Poisson Distribution equation below to calculate whether or not the queue arrivals were Poisson distributed:

$$Pr(X = x) = \frac{(\lambda)^x e^{-\lambda}}{x!}$$

$$Pr(X = x) = \frac{(1{\cdot}29)^x e^{-1{\cdot}29}}{x!}$$

Therefore, the number of minutes which we would expect x customers arriving per minute in 60 minutes can be calculated using:

$$60Pr(X = x) = \frac{60e^{-1{\cdot}29}(1{\cdot}29)^x}{x!}$$

The x values are 0, 1, 2, 3, 4, 5, 6, and the Poisson frequencies are as follows:

$$60Pr(X = 0) = \frac{60e^{-1{\cdot}29}(1{\cdot}29)^0}{0!} = 16{\cdot}5$$

$$60Pr(X = 1) = \frac{60e^{-1{\cdot}29}(1{\cdot}29)^1}{1!} = 21{\cdot}3$$

$$60Pr(X = 2) = \frac{60e^{-1{\cdot}29}(1{\cdot}29)^2}{2!} = 13{\cdot}7$$

$$60Pr(X = 3) = \frac{60e^{-1{\cdot}29}(1{\cdot}29)^3}{3!} = 5{\cdot}91$$

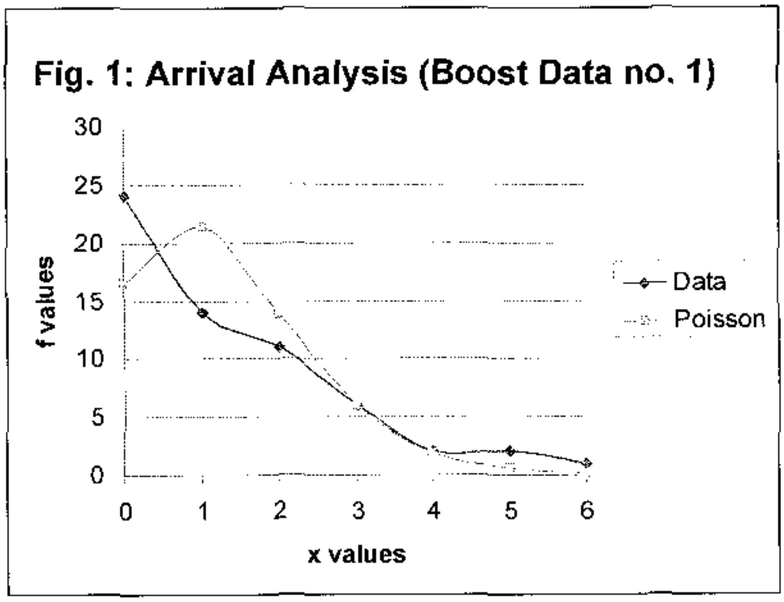
$$60Pr(X = 4) = \frac{60e^{-1{\cdot}29}(1{\cdot}29)^4}{4!} = 1{\cdot}91$$

$$60Pr(X = 5) = \frac{60e^{-1{\cdot}29}(1{\cdot}29)^5}{5!} = 0{\cdot}492$$

$$60Pr(X = 6) = \frac{60e^{-1{\cdot}29}(1{\cdot}29)^6}{6!} = 0{\cdot}106$$

Comparing these Poisson Values with the real data values in a table and a graph easily shows us whether or not the arrival times of the queue were Poisson distributed:

x	Data	Poisson
0	24	16.5
1	14	21.3
2	11	13.7
3	6	5.91
4	2	1.91
5	2	0.492
6	1	0.106



From these results it is evident that the *x* values of 3 and 4 come extremely close to the expected Poisson values. However the others, especially the *x* values 5 and 6 are very different to the expected Poisson values, and thus the service times did not accurately follow the Poisson Distribution. This may be due to the fact that only a reasonably small sample size was collected. If I had perhaps remained there all day I may have gained a more accurate population mean, and thus more accurate real data values that may have been closer to the Poisson values. There were also complications with the some of the arrivals that were in groups. This created a bit of a problem. It could have been possible to eliminate these groups and make them into one person if they arrived and were served at exactly the same time. This was not always the case, and in future, these groups may be watched more carefully to make the correct decision on making them a single arrival or not.

5.1.3 Analysis of Service Rates

Below are the collected departure times, which also indicate the beginning of the next person's service time:

Person Number	Time departed	"minute departed"
1	12:40:43 PM	40
2	12:41:45 PM	41
3	12:42:52 PM	42
4	12:43:23 PM	43
5	12:44:31 PM	44
6	12:45:12 PM	45
7	12:45:12 PM	45
8	12:45:35 PM	45
9	12:45:51 PM	45
10	12:46:24 PM	46
11	12:46:51 PM	46
12	12:47:22 PM	47
13	12:47:22 PM	47

...refer to Appendix 1

I will now calculate the same as I did for the arrival times to test the Poisson Distribution characteristics of the service times: following are the possible y values (customers capable of being served per minute) and the frequencies used to calculate the service population mean μ:

Possible values (y_i)	0	1	2	3	4	5	6
Frequencies (f_i)	17	23	11	8	1	1	0

To calculate μ, we can take the average service time, which would give us the service time per person. Therefore, its reciprocal is the number of people being served per minute, μ.

Average service time = 46 seconds or 0.77 minutes

$$\mu = \frac{1}{0.77}$$

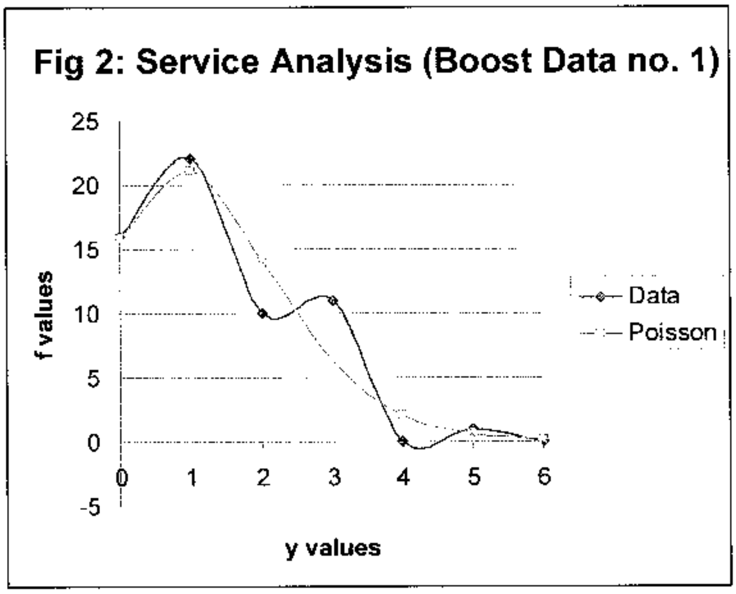
$$\mu = 1.3$$

Thus, again, using μ as our parameter, we can calculate the Poisson rates for one hour using the following, where y is the frequencies for the service time:

$$60Pr(Y = y) = \frac{60(1{\cdot}3)^{y} e^{-1{\cdot}3}}{y!}$$

The Poisson values for y = 1, 2, 3, 4, 5 and 6 are as follows, in a graph to compare the real values with the Poisson values:

y	Data	Poisson
0	17	16.4
1	23	21.3
2	11	13.8
3	8	5.99
4	1	1.95
5	1	0.506
6	0	0.11



As shown, the y values from 0, 1 and 2 seem to be closer to the real data compared with that of the y values 3, 4, 5, and 6. Hence, although they *approximately* represent a Poisson Distribution, we cannot confidently say that they do, and instead conclude that the service rates of this queue also do not follow a Poisson Distribution.

5.1.4 Performance Measures

In assessing the data from the queue at Boost, the performance measures can be calculated using the formulae on page 9 for a single server queue with Poisson distributed arrival and service times:

Using the data collected from Boost and the findings of μ and λ, we can denote that:

λ = 1.29 customers arriving per minute

μ = 1.3 customers capable of being served per minute

$$W = \frac{1}{\mu - \lambda}$$

$$W = \frac{1}{1.3 - 1.29}$$

$W = 100$ minutes

$$Wq = W - \frac{1}{\mu}$$

$$W_q = 100 - 0.769$$

$W_q = 99.2$ minutes

$$Lq = \lambda Wq$$

$$Lq = 1.29 \times 99.2$$

$Lq = 128.0$ people

$$L = \lambda W$$

$L = 129.0$ people

$$U = \frac{\lambda}{\mu} \times 100$$

$U = 99.2\%$

Above are the **expected** values according to Queuing Theory.

Using Excel and the collected data, the **actual** values have been calculated:

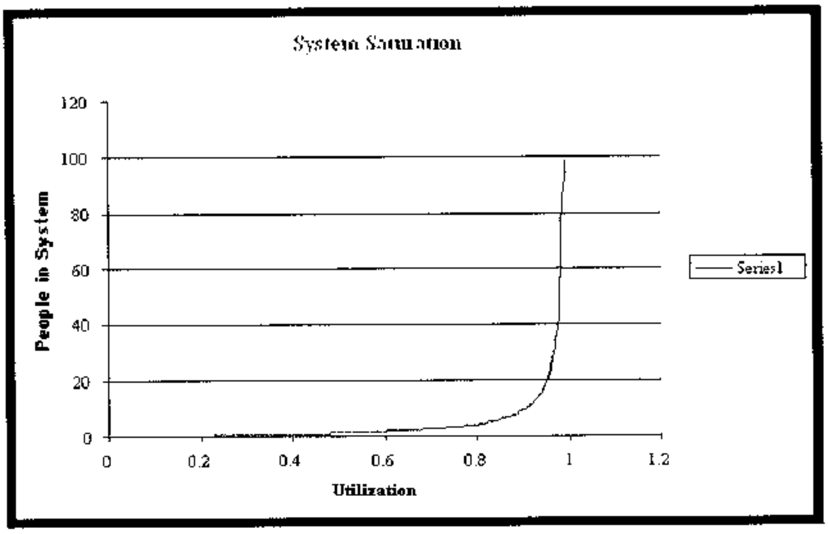
Person no.	waiting time	service time	total system time
1	0:00:40	0:00:40	0:01:20
2	0:01:05	0:01:02	0:02:07
...	...	...	...
77	00:01:41	00:01:18	00:02:58
78	00:00:25	00:01:08	00:01:33
AVERAGE	**0:01:56**	**0:00:53**	**0:02:40**

...refer to appendix 1

5.1.5 Summary of Solutions

	Predicted values	Measured values
L_q	128 customers	2.33 customers
L	129 customers	3.44 customers
W_q	100 minutes	1 minute 56 seconds
W	99.2 minutes	2 minutes 40 seconds
U	99.2%	99.5%

As clearly shown above, the predicted lengths and waiting times were much larger than the measured lengths and waiting times. The explanation for this is that the arrival rates and service rates of the queue both did not follow a Poisson Distribution. However, the question that arises is why the measured values did not reach the predicted values. The very high predicted waiting time and queue length arise when utilisation is high as it was in this experiment. As shown in the below graph, as the utilisation approaches 100%, the number of people in the system should reach infinity.



(Ashley, 2000)

However, the measured queue lengths and waiting times did not rise as high as the theoretical formula suggests, and the queue did not reach a very large number. This might be explained because when potential customers see that the queue is very long, they may balk. The result of this is that the length of the queue is limited, demonstrating that the assumption that the probability of arrivals is independent of the queue length does not apply to this queue. This could be detrimental to Boost as they may be losing customers. Upon learning of this, they may decide to employ another server. This decision could also be aided by the calculated utilisation of the queue, as it was calculated that the server was almost fully utilised. However, Boost may also realise that their customers are postponing their purchase for a time when the queue is shorter. This would mean that Boost might not need to employ another server. Resolving these issues is beyond the scope of this essay.

Utilisation was predicted to within 10% of the measured value; however this was due to its extremely high value. Overall it was concluded that the model did not adequately predict the measured performance values.

Some difficulties with the experiment, which may have skewed data, are summarised below:

- Finding a single server queue – most queues are now multiple server with multiple waiting lines (supermarket checkouts) or single lines (banks)
- Recording the data –laptop was used with excel and a macro to record time on spreadsheet.
- Keeping track of balkers
- Determining if people in the queue were together or being served separately

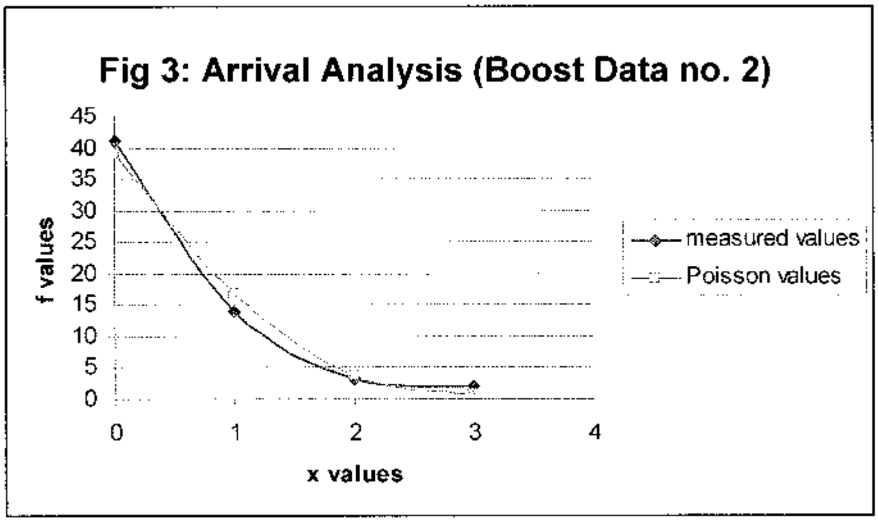
5.2 Boost Data No. 2

Because the arrival and service rates of the earlier Boost Juice Bar queue did not follow a Poisson Distribution very closely, data from the Boost queue on another day was collected. Upon this occasion, the server was not utilised to the same capacity. This was to investigate the possibility that a queue with lower server utilisation had arrival and service rates which better followed a Poisson Distribution. The same method was used as before, and the results are shown in Appendix 2.

5.2.1 Analysis of Arrival Rates

The following arrival rates in a particular minute (b_i) and their frequencies (f_i) were calculated using $\lambda = 0.433$ customers arriving per minute. These were tabulated and then graphed to show the differences in the values.

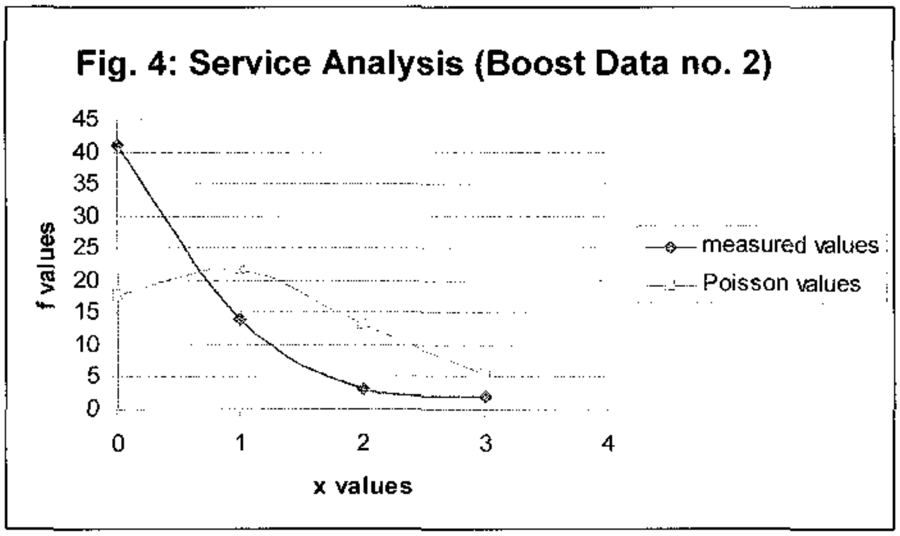
Possible values (b_i)	0	1	2	3
Frequencies (f_i)	41	14	3	2
Poisson	38.9	16.8	3.65	0.527



5.2.2 Analysis of Service Rates

The same analysis was employed for the service rates, where μ = 1.22.

Possible values (b_i)	0	1	2	3
Frequencies (f_i)	37	20	3	0
Poisson	17.7	21.6	13.2	5.36



As shown above, the Service rates departed considerably from the Poisson Distribution. Therefore, it can be concluded that although arrival rates followed the Poisson Distribution closely, the service rates did not.

5.2.3 Summary of Performance measures.

A summary of the Predicted and Measured performance measures are below:

	Predicted values	Measured values
L_q	0.195 customers	0.154 customers
L	0.55 customers	0.615 customers
W_q	27 seconds	20 seconds
W	1 minute 16 seconds	1 minute 9 seconds
U	35.5%	48.2%

From the table, it can be said that although the performance measures were quite close, the predicted and measured utilisations were more than 10% apart. Therefore this queue's performance measures were not accurately predicted.

5.3 Westpac Bank ATM

To further investigate the essay question, I decided to collect data from another single server queue, a Westpac ATM., as this queue may have differences from the Boost queue, and thus may follow Queuing Theory. After learning from the initial data collection, I decided to also collect the beginning time of the service to make it easier to calculate the service time. Results can be found in Appendix 3.

5.3.1 Analysis using Little's Law

Using the same method as previously, I calculated the average number of customers per minute:

There were 62 customers that arrived within the hour, thus $\lambda = 1.033$ customers per minute.

Then, using Little's Law to calculate the average number of customers in the system at any one time, I calculated the average waiting time. The result was 55 seconds, or 0.917 minutes.

Thus, we get $\lambda = 1.033$ customers arriving per minute and the time, T, is 0.917 minutes, and the average number of customers waiting is:

$$N = \lambda T$$
$$N = 1.033 \times 0.917$$
$$N = 0.947$$

We therefore have an average of 0.947 customers in the system at any one time.

5.3.2 Analysis of Arrival Rates

Again, using the same method as earlier, the number of customers that arrived in each particular minute was calculated:

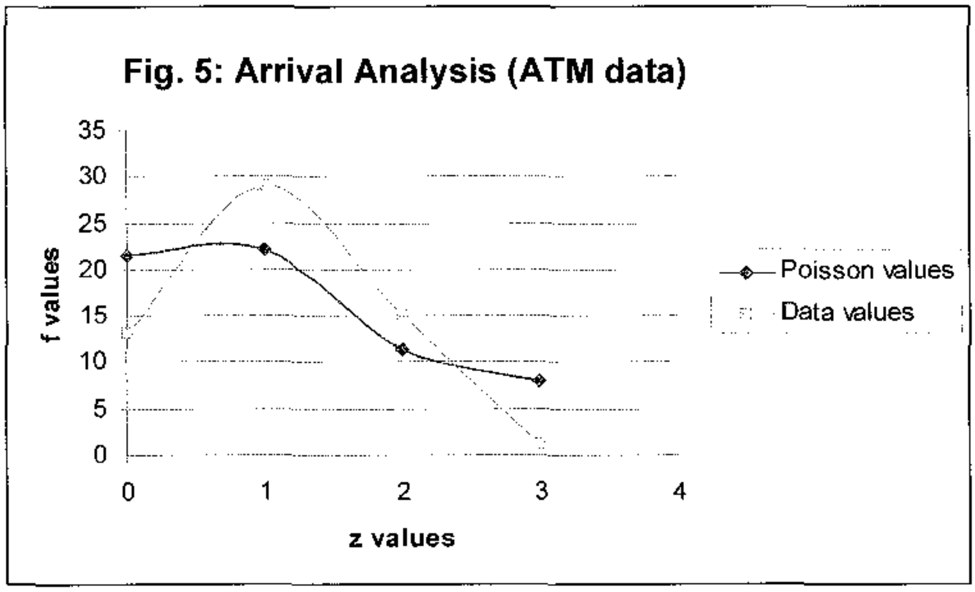
Possible values (z_i)	0	1	2	3
Frequencies (f_i)	13	29	15	1

Using these values, and the arrival rate in minutes ($\lambda = 1.033$), we can see if arrivals rates of this queue follow the Poisson Distribution.
The calculations followed the same method as earlier in this essay, and these gave the following Poisson values in contrast to the measured data collected, where z is the frequency of the arrival times in a particular minute:

$$60P(Z = z) = \frac{60e^{-1\cdot033}(1\cdot033)^z}{z!} \text{ etc.}$$

z	Data	Poisson
0	13	21.4
1	29	22.1
2	15	11.4
3	1	3.92

Graphically represented:



With these values, there are not many Data values that are accurate, but the most accurate would the z-value of 2.

We can see from figure 5 that the values are very different from the expected values. However, looking at the graph, it seems that the data values have a similar pattern to the Poisson values, but they have been recorded in a shorter period of time with less z-values. However, it did have a significant variation from the Poisson Distribution.

5.3.3 Analysis of Service Rates

Again, using the same method, the following values were collected for the end of the service times:

Possible values (a_i)	0	1	2	3
Frequencies (f_i)	9	37	14	0

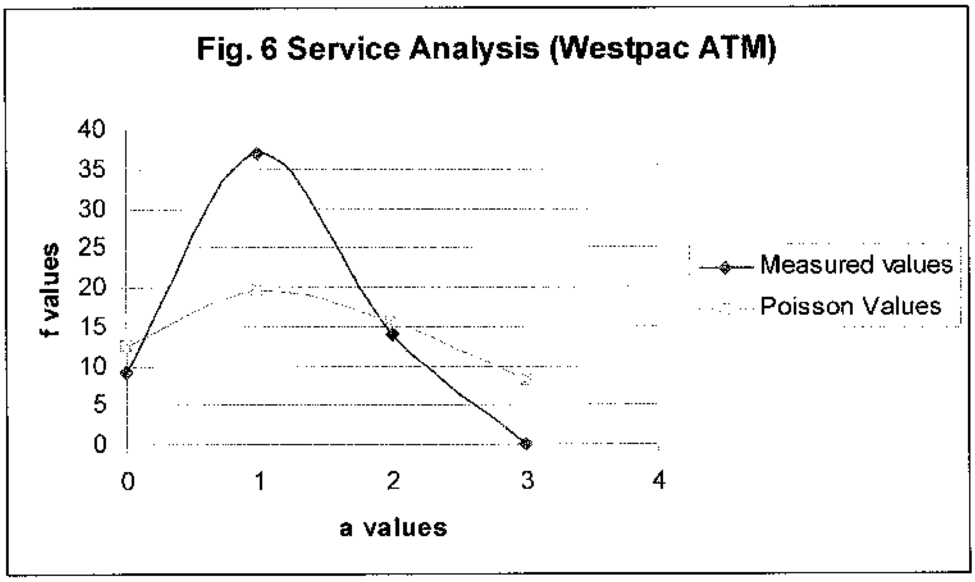
(from Appendix 2…)

μ was calculated using the reciprocal of the average service time (0.633 minutes) of each customer, and was found to be 1.58. The Poisson values were calculated, where a is the frequency of the service rates in this queue:

$$60P(A = a) = \frac{60e^{-1\cdot58}(1\cdot58)^a}{a!} \quad \text{etc.}$$

a	Data	Poisson
0	9	12.4
1	37	19.5
2	14	15.4
3	0	8.12

Graphically represented, this gives:



Again, this shows us that the most accurate data value was the a-value of 2.

The same occurrence happened here as with figure 5, and the Poisson Values were quite different to the measured values.

5.3.4 Performance Measures and Summary of Solutions

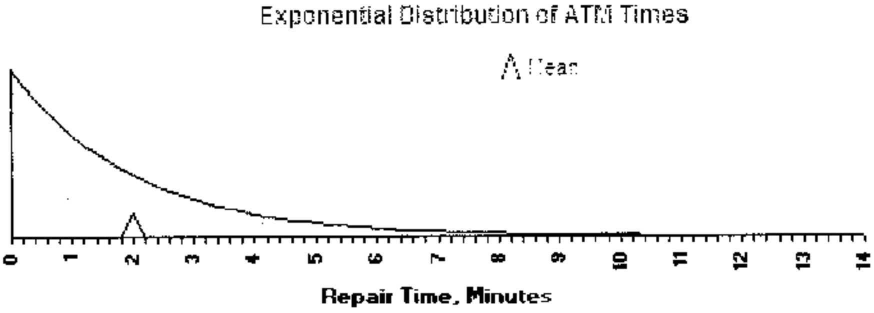
Using the same method and formulae as with the first experiment (refer to page 17), using the parameters μ and λ the following solutions were obtained:

Where:
$\lambda = 1.033$
$\mu = 1.58$

	Predicted values	Measured values
L_q	1.24 customers	0.17 customers
L	1.89 customers	0.947 customers
W_q	1 minute 12 seconds	18 seconds
W	1 minute 50 seconds	55 seconds
U	65.4%	71.5%

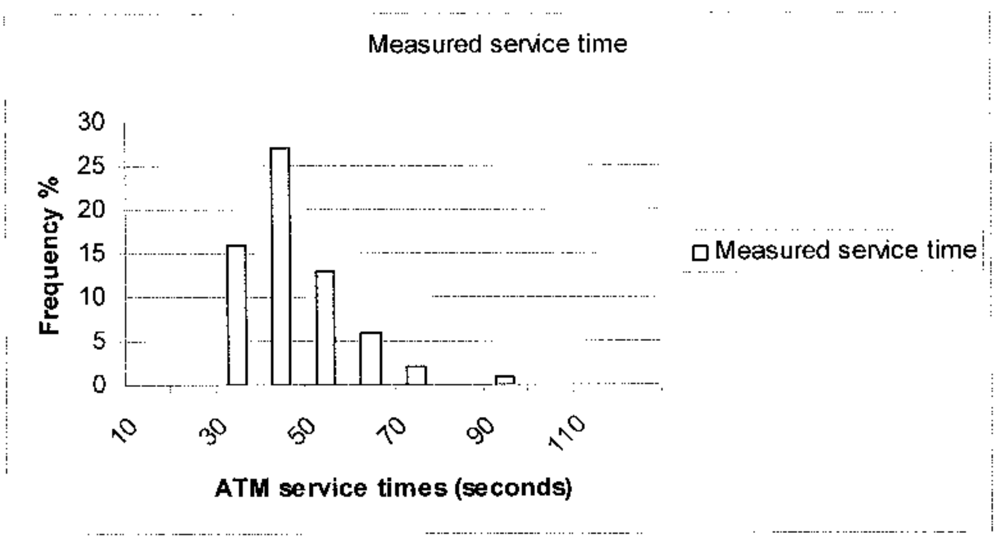
As shown by comparison with the above table, the predicted values and the measured values were significantly different. The predicted length of the line was much larger than the measured value, as were the predicted waiting times. However, the utilisation of the server was in reasonable agreement. This is possibly the most useful performance measure component as described by Queuing Theory for businesses as it can allow them to decide if their server is being utilised to the maximum or not, and if they need another server to improve the queue capacity. Therefore, we may say that Queuing Theory *adequately* predicts performance measure for this single server queue, as Westpac may not need to know precisely the waiting time of expected queue length, but they may want to know server utilisation. This would of course depend on how many customers they are annoying by making them wait in long queues and how much money they would lose as a result.

Although the data does not correspond entirely with Queuing Theory, this is not a rare finding. Ashley (2000) identifies that the Poisson Distribution often does not apply in practice, particularly for service times. This is because the Poisson Distribution is derived from the Exponential Distribution (which models the times between events) The Exponential Distribution implies that "shorter times are always more likely than longer times" (Ashley, 2000), as shown in the below graph, (a graph of the repair time for ATMs).



(Ashley, 2000)

In practice, this is not usually true. For example, for the ATM data, the times between services were definitely not more likely to be a nanosecond compared to 2 minutes as humans cannot physically withdraw money in that length of time. The graph below shows the distribution of ATM service times in the form of a frequency histogram. The distribution is of a very different form to the exponential distribution above. This is another way of stating that service rates do not follow the Poisson Distribution. This is probably the main reason as to why the sample values were not the same as the expected values, because the queues were not perfect single server queues. The use of the Poisson Distribution in these cases often overestimates the expected queue length and waiting time (Ashley, 2000), as I have found.



6. Conclusion

The performance measures of three different queues were not adequately predicted by the application of Queuing Theory equations for a single server Poisson distributed queue. Only the arrival rates for the queue of Boost the second time could be described as following the Poisson Distribution. This was the main reason Queuing Theory failed to predict the performance measures.

The main unresolved question from this investigation was which distributions the queues did follow. Therefore, to significantly improve the predictions, a more advanced model, other than one that followed the Poisson Distribution, but that more closely modelled the distribution of the actual arrival and service times, could be used.

Other areas of further investigation might involve looking at the same queues over a whole day as arrival times vary throughout the day, or extending the work to multiple line queues. Some explanation or model for the behaviour of balkers when queue lengths increase would also be interesting and might improve the model.

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Appendices

Appendix 1: Boost Data no. 1

person no.	arrival time	departure time	waiting time (in queue)	service time	total system time
1	12:40:03 PM	12:40:43 PM	00:00:00	00:00:41	00:00:41
2	12:40:40 PM	12:41:45 PM	00:00:03	00:01:01	00:01:05
3	12:42:03 PM	12:42:52 PM	00:00:00	00:00:49	00:00:49
4	12:42:27 PM	12:43:23 PM	00:00:25	00:00:31	00:00:56
5	12:42:27 PM	12:44:31 PM	00:00:56	00:01:07	00:02:03
6	12:42:28 PM	12:45:12 PM	00:02:03	00:00:42	00:02:44
7	12:42:51 PM	12:45:12 PM	00:02:21	00:00:12	00:02:21
8	12:42:51 PM	12:45:35 PM	00:02:21	00:00:22	00:02:43
9	12:43:05 PM	12:45:51 PM	00:02:29	00:00:16	00:02:46
10	12:43:37 PM	12:46:24 PM	00:02:14	00:00:33	00:02:47
11	12:43:39 PM	12:46:51 PM	00:02:45	00:00:27	00:03:12
12	12:44:30 PM	12:47:22 PM	00:02:21	00:00:30	00:02:52
13	12:44:30 PM	12:47:22 PM	00:02:52	00:00:10	00:02:52
14	12:46:33 PM	12:47:44 PM	00:00:49	00:00:22	00:01:11
15	12:46:33 PM	12:48:35 PM	00:01:11	00:00:52	00:02:03
16	12:46:52 PM	12:49:17 PM	00:01:44	00:00:42	00:02:26
17	12:46:59 PM	12:51:09 PM	00:02:18	00:01:52	00:04:10
18	12:50:32 PM	12:54:45 PM	00:00:38	00:03:36	00:04:13
19	12:54:04 PM	12:55:17 PM	00:00:41	00:00:32	00:01:13
20	12:54:17 PM	12:59:36 PM	00:01:00	00:04:19	00:05:19
21	12:56:17 PM	1:00:22 PM	00:03:19	00:00:46	00:04:05
22	12:57:11 PM	1:00:48 PM	00:03:11	00:00:26	00:03:37
23	12:57:52 PM	1:01:21 PM	00:02:56	00:00:33	00:03:28
24	12:58:06 PM	1:01:21 PM	00:03:15	00:00:10	00:03:15
25	12:58:06 PM	1:01:25 PM	00:03:15	00:00:04	00:03:19
26	12:58:08 PM	1:01:49 PM	00:03:17	00:00:24	00:03:41
27	12:58:55 PM	1:01:49 PM	00:02:53	00:00:09	00:02:53
28	12:59:33 PM	1:02:18 PM	00:02:16	00:00:30	00:02:45
29	12:59:37 PM	1:02:49 PM	00:02:41	00:00:31	00:03:12
30	1:00:08 PM	1:03:12 PM	00:02:41	00:00:23	00:03:04
31	1:01:54 PM	1:03:36 PM	00:01:18	00:00:25	00:01:43
32	1:01:54 PM	1:03:37 PM	00:01:42	00:00:12	00:01:43
33	1:02:16 PM	1:04:02 PM	00:01:21	00:00:26	00:01:47
34	1:02:16 PM	1:04:59 PM	00:01:46	00:00:57	00:02:44
35	1:02:41 PM	1:05:45 PM	00:02:18	00:00:46	00:03:04
36	1:03:00 PM	1:06:16 PM	00:02:45	00:00:31	00:03:16
37	1:05:21 PM	1:06:23 PM	00:00:55	00:00:07	00:01:02
38	1:05:44 PM	1:06:34 PM	00:00:39	00:00:11	00:00:49
39	1:05:45 PM	1:07:10 PM	00:00:49	00:00:36	00:01:25
40	1:05:45 PM	1:07:15 PM	00:01:25	00:00:05	00:01:30
41	1:06:20 PM	1:07:50 PM	00:00:55	00:00:35	00:01:30
42	1:06:20 PM	1:10:06 PM	00:01:30	00:02:16	00:03:46
43	1:06:41 PM	1:10:43 PM	00:03:26	00:00:37	00:04:03
44	1:08:56 PM	1:11:31 PM	00:01:47	00:00:47	00:02:34
45	1:09:14 PM	1:13:40 PM	00:02:17	00:02:10	00:04:27
46	1:10:55 PM	1:13:40 PM	00:02:45	00:00:11	00:02:45
47	1:12:47 PM	1:14:00 PM	00:00:53	00:00:19	00:01:12
48	1:12:47 PM	1:18:14 PM	00:01:12	00:04:15	00:05:27
49	1:13:13 PM	1:18:56 PM	00:05:01	00:00:42	00:05:43
50	1:16:24 PM	1:19:17 PM	00:02:32	00:00:20	00:02:53

51	1:16:37 PM	1:20:11 PM	00:02:39	00:00:54	00:03:34
52	1:16:37 PM	1:20:39 PM	00:03:34	00:00:28	00:04:02
53	1:19:03 PM	1:21:15 PM	00:01:37	00:00:35	00:02:12
54	1:19:12 PM	1:23:37 PM	00:02:02	00:02:22	00:04:25
55	1:19:21 PM	1:24:25 PM	00:04:15	00:00:48	00:05:03
56	1:21:56 PM	1:25:03 PM	00:02:28	00:00:38	00:03:06
57	1:23:22 PM	1:25:50 PM	00:01:41	00:00:48	00:02:29
58	1:23:29 PM	1:25:51 PM	00:02:21	00:00:17	00:02:21
59	1:25:16 PM	1:26:14 PM	00:00:35	00:00:24	00:00:59
60	1:25:16 PM	1:26:48 PM	00:00:58	00:00:33	00:01:32
61	1:25:16 PM	1:26:48 PM	00:01:31	00:00:13	00:01:31
62	1:25:17 PM	1:27:12 PM	00:01:30	00:00:24	00:01:54
63	1:25:18 PM	1:29:03 PM	00:01:54	00:01:52	00:03:45
64	1:26:49 PM	1:29:23 PM	00:02:14	00:00:20	00:02:34
65	1:28:06 PM	1:29:44 PM	00:01:16	00:00:21	00:01:37
66	1:28:10 PM	1:30:17 PM	00:01:34	00:00:33	00:02:07
67	1:28:41 PM	1:30:27 PM	00:01:35	00:00:11	00:01:46
68	1:28:41 PM	1:30:59 PM	00:01:46	00:00:32	00:02:18
69	1:28:41 PM	1:32:15 PM	00:02:18	00:01:16	00:03:34
70	1:30:26 PM	1:34:07 PM	00:01:49	00:01:52	00:03:40
71	1:31:52 PM	1:34:07 PM	00:02:14	00:00:01	00:02:15
72	1:31:54 PM	1:35:04 PM	00:02:13	00:00:57	00:03:10
73	1:32:27 PM	1:35:21 PM	00:02:38	00:00:16	00:02:54
74	1:32:27 PM	1:35:53 PM	00:02:54	00:00:32	00:03:26
75	1:34:24 PM	1:36:16 PM	00:01:28	00:00:24	00:01:52
76	1:34:48 PM	1:36:44 PM	00:01:28	00:00:28	00:01:56
77	1:35:04 PM	1:38:02 PM	00:01:41	00:01:18	00:02:58
78	1:37:37 PM	1:39:10 PM	00:00:25	00:01:08	00:01:33
		average	**00:01:56**	**00:00:53**	**00:02:40**

Appendix 2

Boost Data no. 2						
Person no.	arrival time	begin serivce time	departure time	waiting time (in queue)	service time	total system time
1	3:55:49 PM	3:55:53 PM	3:56:31 PM	0:00:04	0:00:37	0:00:41
2	3:55:57 PM	3:56:31 PM	3:57:18 PM	0:00:34	0:00:46	0:01:20
3	3:55:59 PM	3:57:18 PM	3:57:54 PM	0:01:19	0:00:36	0:01:54
4	3:56:13 PM	3:57:54 PM	3:58:34 PM	0:01:41	0:00:40	0:02:21
5	3:58:25 PM	3:58:35 PM	3:59:13 PM	0:00:09	0:00:38	0:00:47
6	3:58:45 PM	3:59:13 PM	4:00:55 PM	0:00:28	0:01:42	0:02:10
7	4:03:20 PM	4:03:20 PM	4:03:38 PM	0:00:00	0:00:18	0:00:19
8	4:04:50 PM	4:04:51 PM	4:05:46 PM	0:00:01	0:00:56	0:00:56
9	4:04:53 PM	4:05:46 PM	4:07:07 PM	0:00:54	0:01:20	0:02:14
10	4:13:43 PM	4:13:43 PM	4:14:30 PM	0:00:00	0:00:47	0:00:47
11	4:20:58 PM	4:20:58 PM	4:21:38 PM	0:00:00	0:00:40	0:00:40
12	4:22:11 PM	4:22:11 PM	4:23:04 PM	0:00:00	0:00:53	0:00:53
13	4:24:40 PM	4:24:41 PM	4:26:16 PM	0:00:00	0:01:35	0:01:36
14	4:25:24 PM	4:26:16 PM	4:26:54 PM	0:00:52	0:00:38	0:01:30
15	4:26:02 PM	4:26:54 PM	4:27:25 PM	0:00:53	0:00:31	0:01:24
16	4:27:58 PM	4:27:58 PM	4:28:39 PM	0:00:00	0:00:41	0:00:41
17	4:29:33 PM	4:29:33 PM	4:30:30 PM	0:00:00	0:00:57	0:00:57
18	4:31:02 PM	4:31:02 PM	4:31:42 PM	0:00:00	0:00:39	0:00:39
19	4:31:15 PM	4:31:42 PM	4:32:21 PM	0:00:27	0:00:39	0:01:06
20	4:33:37 PM	4:33:37 PM	4:34:12 PM	0:00:00	0:00:35	0:00:35
21	4:33:40 PM	4:34:12 PM	4:35:01 PM	0:00:32	0:00:49	0:01:21
22	4:35:35 PM	4:35:35 PM	4:36:50 PM	0:00:00	0:01:15	0:01:15
23	4:36:25 PM	4:36:50 PM	4:37:11 PM	0:00:25	0:00:21	0:00:46
24	4:37:51 PM	4:37:52 PM	4:38:53 PM	0:00:01	0:01:01	0:01:01
25	4:38:33 PM	4:38:53 PM	4:39:59 PM	0:00:20	0:01:06	0:01:26
26	4:45:40 PM	4:45:41 PM	4:46:23 PM	0:00:00	0:00:43	0:00:43
			average	**0:00:20**	**0:00:49**	**0:01:09**

Appendix 3

Westpac ATM Data						
(queue began with 3 people already in the queue)						
person no.	arrival time	begin service time	departure time	waiting time (in queue)	service time	total system time
1		1:14:11 PM	1:14:53 PM		00:00:43	
2		1:14:56 PM	1:15:31 PM		00:00:34	
3		1:15:35 PM	1:16:14 PM		00:00:39	
4	1:16:35 PM	1:16:44 PM	1:17:19 PM	00:00:09	00:00:35	00:00:44
5	1:17:43 PM	1:17:44 PM	1:18:26 PM	00:00:01	00:00:42	00:00:44
6	1:18:16 PM	1:18:30 PM	1:18:54 PM	00:00:14	00:00:25	00:00:39
7	1:20:29 PM	1:20:30 PM	1:21:26 PM	00:00:00	00:00:56	00:00:57
8	1:20:43 PM	1:21:28 PM	1:22:02 PM	00:00:45	00:00:34	00:01:19
9	1:21:28 PM	1:22:03 PM	1:22:36 PM	00:00:35	00:00:33	00:01:08
10	1:21:57 PM	1:22:41 PM	1:23:10 PM	00:00:44	00:00:28	00:01:13
11	1:22:42 PM	1:23:12 PM	1:23:39 PM	00:00:30	00:00:28	00:00:58
12	1:23:12 PM	1:23:41 PM	1:24:21 PM	00:00:29	00:00:40	00:01:09
13	1:24:29 PM	1:24:30 PM	1:25:07 PM	00:00:01	00:00:37	00:00:38
14	1:24:50 PM	1:25:12 PM	1:26:03 PM	00:00:22	00:00:51	00:01:13
15	1:26:18 PM	1:26:19 PM	1:26:52 PM	00:00:01	00:00:33	00:00:34
16	1:27:32 PM	1:27:37 PM	1:28:17 PM	00:00:05	00:00:40	00:00:45
17	1:27:59 PM	1:28:18 PM	1:28:58 PM	00:00:20	00:00:40	00:01:00
18	1:31:42 PM	1:31:44 PM	1:32:17 PM	00:00:02	00:00:33	00:00:35
19	1:34:09 PM	1:34:10 PM	1:34:44 PM	00:00:00	00:00:35	00:00:35
20	1:34:33 PM	1:34:46 PM	1:35:23 PM	00:00:12	00:00:37	00:00:50
21	1:36:43 PM	1:36:44 PM	1:37:14 PM	00:00:01	00:00:30	00:00:31
22	1:37:22 PM	1:37:22 PM	1:38:13 PM	00:00:00	00:00:51	00:00:51
23	1:39:29 PM	1:39:29 PM	1:39:54 PM	00:00:00	00:00:25	00:00:25
24	1:39:42 PM	1:39:56 PM	1:40:27 PM	00:00:14	00:00:30	00:00:44
25	1:40:45 PM	1:40:46 PM	1:41:24 PM	00:00:00	00:00:38	00:00:38
26	1:41:16 PM	1:41:29 PM	1:41:56 PM	00:00:12	00:00:28	00:00:40
27	1:41:16 PM	1:42:00 PM	1:43:28 PM	00:00:44	00:01:27	00:02:11
28	1:42:52 PM	1:43:29 PM	1:44:04 PM	00:00:38	00:00:35	00:01:13
29	1:43:40 PM	1:44:06 PM	1:44:50 PM	00:00:26	00:00:44	00:01:11
30	1:43:45 PM	1:44:52 PM	1:45:28 PM	00:01:08	00:00:35	00:01:43
31	1:44:28 PM	1:45:32 PM	1:46:04 PM	00:01:04	00:00:32	00:01:36
32	1:45:13 PM	1:46:05 PM	1:46:39 PM	00:00:52	00:00:34	00:01:26
33	1:46:44 PM	1:46:47 PM	1:47:50 PM	00:00:03	00:01:03	00:01:07
34	1:49:38 PM	1:49:39 PM	1:50:28 PM	00:00:01	00:00:49	00:00:50
35	1:50:21 PM	1:50:30 PM	1:50:55 PM	00:00:10	00:00:25	00:00:35
36	1:50:53 PM	1:50:59 PM	1:51:30 PM	00:00:06	00:00:31	00:00:38
37	1:51:38 PM	1:51:41 PM	1:52:09 PM	00:00:03	00:00:28	00:00:32
38	1:51:56 PM	1:52:11 PM	1:52:53 PM	00:00:14	00:00:43	00:00:57
39	1:52:36 PM	1:52:55 PM	1:53:31 PM	00:00:19	00:00:37	00:00:56
40	1:52:53 PM	1:53:32 PM	1:54:08 PM	00:00:39	00:00:36	00:01:15
41	1:53:20 PM	1:54:17 PM	1:54:57 PM	00:00:56	00:00:40	00:01:37
42	1:54:20 PM	1:55:00 PM	1:55:42 PM	00:00:39	00:00:42	00:01:21
43	1:55:27 PM	1:55:45 PM	1:56:09 PM	00:00:17	00:00:25	00:00:42
44	1:55:50 PM	1:56:10 PM	1:56:39 PM	00:00:20	00:00:29	00:00:49
45	1:56:52 PM	1:56:52 PM	1:57:35 PM	00:00:00	00:00:43	00:00:43

46	1:57:53 PM	1:57:53 PM	1:58:27 PM	00:00:00	00:00:33	00:00:34
47	1:58:07 PM	1:58:28 PM	1:59:25 PM	00:00:21	00:00:58	00:01:18
48	1:59:28 PM	1:59:28 PM	2:00:12 PM	00:00:00	00:00:43	00:00:44
49	2:00:21 PM	2:00:21 PM	2:00:45 PM	00:00:00	00:00:25	00:00:25
50	2:01:52 PM	2:01:52 PM	2:02:23 PM	00:00:00	00:00:31	00:00:32
51	2:02:21 PM	2:02:26 PM	2:03:30 PM	00:00:05	00:01:04	00:01:09
52	2:02:28 PM	2:03:31 PM	2:04:03 PM	00:01:03	00:00:32	00:01:34
53	2:02:40 PM	2:04:04 PM	2:04:31 PM	00:01:24	00:00:27	00:01:51
54	2:04:48 PM	2:04:48 PM	2:05:41 PM	00:00:00	00:00:54	00:00:54
55	2:05:34 PM	2:05:43 PM	2:06:34 PM	00:00:09	00:00:51	00:00:59
56	2:06:32 PM	2:06:38 PM	2:07:15 PM	00:00:06	00:00:37	00:00:43
57	2:07:05 PM	2:07:17 PM	2:07:57 PM	00:00:12	00:00:40	00:00:52
58	2:07:36 PM	2:07:58 PM	2:08:26 PM	00:00:23	00:00:28	00:00:50
59	2:08:08 PM	2:08:28 PM	2:08:52 PM	00:00:21	00:00:24	00:00:44
60	2:09:20 PM	2:09:20 PM	2:10:01 PM	00:00:00	00:00:41	00:00:41
61	2:10:52 PM	2:10:52 PM	2:11:32 PM	00:00:00	00:00:40	00:00:40
62	2:10:54 PM	2:11:33 PM	2:12:07 PM	00:00:38	00:00:35	00:01:13
63	2:12:11 PM	2:12:12 PM	2:12:40 PM	00:00:01	00:00:29	00:00:29
64	2:12:41 PM	2:12:42 PM	2:13:11 PM	00:00:01	00:00:30	00:00:30
65	2:13:50 PM	2:13:50 PM	2:14:11 PM	00:00:00	00:00:20	00:00:21
			average	**00:00:18**	**00:00:38**	**00:00:55**

Appendix 3

Westpac ATM Data						
(queue began with 3 people already in the queue)						
person no.	arrival time	begin service time	departure time	waiting time (in queue)	service time	total system time
1		1:14:11 PM	1:14:53 PM		00:00:43	
2		1:14:56 PM	1:15:31 PM		00:00:34	
3		1:15:35 PM	1:16:14 PM		00:00:39	
4	1:16:35 PM	1:16:44 PM	1:17:19 PM	00:00:09	00:00:35	00:00:44
5	1:17:43 PM	1:17:44 PM	1:18:26 PM	00:00:01	00:00:42	00:00:44
6	1:18:16 PM	1:18:30 PM	1:18:54 PM	00:00:14	00:00:25	00:00:39
7	1:20:29 PM	1:20:30 PM	1:21:26 PM	00:00:00	00:00:56	00:00:57
8	1:20:43 PM	1:21:28 PM	1:22:02 PM	00:00:45	00:00:34	00:01:19
9	1:21:28 PM	1:22:03 PM	1:22:36 PM	00:00:35	00:00:33	00:01:08
10	1:21:57 PM	1:22:41 PM	1:23:10 PM	00:00:44	00:00:28	00:01:13
11	1:22:42 PM	1:23:12 PM	1:23:39 PM	00:00:30	00:00:28	00:00:58
12	1:23:12 PM	1:23:41 PM	1:24:21 PM	00:00:29	00:00:40	00:01:09
13	1:24:29 PM	1:24:30 PM	1:25:07 PM	00:00:01	00:00:37	00:00:38
14	1:24:50 PM	1:25:12 PM	1:26:03 PM	00:00:22	00:00:51	00:01:13
15	1:26:18 PM	1:26:19 PM	1:26:52 PM	00:00:01	00:00:33	00:00:34
16	1:27:32 PM	1:27:37 PM	1:28:17 PM	00:00:05	00:00:40	00:00:45
17	1:27:59 PM	1:28:18 PM	1:28:58 PM	00:00:20	00:00:40	00:01:00
18	1:31:42 PM	1:31:44 PM	1:32:17 PM	00:00:02	00:00:33	00:00:35
19	1:34:09 PM	1:34:10 PM	1:34:44 PM	00:00:00	00:00:35	00:00:35
20	1:34:33 PM	1:34:46 PM	1:35:23 PM	00:00:12	00:00:37	00:00:50
21	1:36:43 PM	1:36:44 PM	1:37:14 PM	00:00:01	00:00:30	00:00:31
22	1:37:22 PM	1:37:22 PM	1:38:13 PM	00:00:00	00:00:51	00:00:51
23	1:39:29 PM	1:39:29 PM	1:39:54 PM	00:00:00	00:00:25	00:00:25
24	1:39:42 PM	1:39:56 PM	1:40:27 PM	00:00:14	00:00:30	00:00:44
25	1:40:45 PM	1:40:46 PM	1:41:24 PM	00:00:00	00:00:38	00:00:38
26	1:41:16 PM	1:41:29 PM	1:41:56 PM	00:00:12	00:00:28	00:00:40
27	1:41:16 PM	1:42:00 PM	1:43:28 PM	00:00:44	00:01:27	00:02:11
28	1:42:52 PM	1:43:29 PM	1:44:04 PM	00:00:38	00:00:35	00:01:13
29	1:43:40 PM	1:44:06 PM	1:44:50 PM	00:00:26	00:00:44	00:01:11
30	1:43:45 PM	1:44:52 PM	1:45:28 PM	00:01:08	00:00:35	00:01:43
31	1:44:28 PM	1:45:32 PM	1:46:04 PM	00:01:04	00:00:32	00:01:36
32	1:45:13 PM	1:46:05 PM	1:46:39 PM	00:00:52	00:00:34	00:01:26
33	1:46:44 PM	1:46:47 PM	1:47:50 PM	00:00:03	00:01:03	00:01:07
34	1:49:38 PM	1:49:39 PM	1:50:28 PM	00:00:01	00:00:49	00:00:50
35	1:50:21 PM	1:50:30 PM	1:50:55 PM	00:00:10	00:00:25	00:00:35
36	1:50:53 PM	1:50:59 PM	1:51:30 PM	00:00:06	00:00:31	00:00:38
37	1:51:38 PM	1:51:41 PM	1:52:09 PM	00:00:03	00:00:28	00:00:32
38	1:51:56 PM	1:52:11 PM	1:52:53 PM	00:00:14	00:00:43	00:00:57
39	1:52:36 PM	1:52:55 PM	1:53:31 PM	00:00:19	00:00:37	00:00:56
40	1:52:53 PM	1:53:32 PM	1:54:08 PM	00:00:39	00:00:36	00:01:15
41	1:53:20 PM	1:54:17 PM	1:54:57 PM	00:00:56	00:00:40	00:01:37
42	1:54:20 PM	1:55:00 PM	1:55:42 PM	00:00:39	00:00:42	00:01:21
43	1:55:27 PM	1:55:45 PM	1:56:09 PM	00:00:17	00:00:25	00:00:42
44	1:55:50 PM	1:56:10 PM	1:56:39 PM	00:00:20	00:00:29	00:00:49
45	1:56:52 PM	1:56:52 PM	1:57:35 PM	00:00:00	00:00:43	00:00:43

46	1:57:53 PM	1:57:53 PM	1:58:27 PM	00:00:00	00:00:33	00:00:34
47	1:58:07 PM	1:58:28 PM	1:59:25 PM	00:00:21	00:00:58	00:01:18
48	1:59:28 PM	1:59:28 PM	2:00:12 PM	00:00:00	00:00:43	00:00:44
49	2:00:21 PM	2:00:21 PM	2:00:45 PM	00:00:00	00:00:25	00:00:25
50	2:01:52 PM	2:01:52 PM	2:02:23 PM	00:00:00	00:00:31	00:00:32
51	2:02:21 PM	2:02:26 PM	2:03:30 PM	00:00:05	00:01:04	00:01:09
52	2:02:28 PM	2:03:31 PM	2:04:03 PM	00:01:03	00:00:32	00:01:34
53	2:02:40 PM	2:04:04 PM	2:04:31 PM	00:01:24	00:00:27	00:01:51
54	2:04:48 PM	2:04:48 PM	2:05:41 PM	00:00:00	00:00:54	00:00:54
55	2:05:34 PM	2:05:43 PM	2:06:34 PM	00:00:09	00:00:51	00:00:59
56	2:06:32 PM	2:06:38 PM	2:07:15 PM	00:00:06	00:00:37	00:00:43
57	2:07:05 PM	2:07:17 PM	2:07:57 PM	00:00:12	00:00:40	00:00:52
58	2:07:36 PM	2:07:58 PM	2:08:26 PM	00:00:23	00:00:28	00:00:50
59	2:08:08 PM	2:08:28 PM	2:08:52 PM	00:00:21	00:00:24	00:00:44
60	2:09:20 PM	2:09:20 PM	2:10:01 PM	00:00:00	00:00:41	00:00:41
61	2:10:52 PM	2:10:52 PM	2:11:32 PM	00:00:00	00:00:40	00:00:40
62	2:10:54 PM	2:11:33 PM	2:12:07 PM	00:00:38	00:00:35	00:01:13
63	2:12:11 PM	2:12:12 PM	2:12:40 PM	00:00:01	00:00:29	00:00:29
64	2:12:41 PM	2:12:42 PM	2:13:11 PM	00:00:01	00:00:30	00:00:30
65	2:13:50 PM	2:13:50 PM	2:14:11 PM	00:00:00	00:00:20	00:00:21
			average	**00:00:18**	**00:00:38**	**00:00:55**