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Extended essay cover

✓

Candidates must complete this page and then give this cover and their final version of the extended essay to their supervisor.

Candidate session number	[redacted]		
Candidate name	[redacted]		
School number	[redacted]		
School name	[redacted]		
Examination session (May or November)	May	Year	2012

Diploma Programme subject in which this extended essay is registered: Physics

(For an extended essay in the area of languages, state the language and whether it is group 1 or group 2.)

Title of the extended essay: Resonance in Plastic Balls Containing a Hole

Candidate's declaration

This declaration must be signed by the candidate; otherwise a grade may not be issued.

The extended essay I am submitting is my own work (apart from guidance allowed by the International Baccalaureate).

I have acknowledged each use of the words, graphics or ideas of another person, whether written, oral or visual.

I am aware that the word limit for all extended essays is 4000 words and that examiners are not required to read beyond this limit.

This is the final version of my extended essay.

Candidate's signature: [redacted] Date: 2/2/12

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Supervisor's report and declaration

The supervisor must complete this report, sign the declaration and then give the final version of the extended essay, with this cover attached, to the Diploma Programme coordinator.

Name of supervisor (CAPITAL letters) ______________________

Please comment, as appropriate, on the candidate's performance, the context in which the candidate undertook the research for the extended essay, any difficulties encountered and how these were overcome (see page 13 of the extended essay guide). The concluding interview (viva voce) may provide useful information. These comments can help the examiner award a level for criterion K (holistic judgment). Do not comment on any adverse personal circumstances that may have affected the candidate. If the amount of time spent with the candidate was zero, you must explain this, in particular how it was then possible to authenticate the essay as the candidate's own work. You may attach an additional sheet if there is insufficient space here.

did a good job taking a seemingly simple phenomenon and going deeply into the complex mathematical theory (spherical harmonics) behind it. He independently found outside PhD's in Physics who could help him master the theory, and then had to figure out how to simplify his level of presentation to make it appropriate for an IB Extended Essay. I was impressed.

This declaration must be signed by the supervisor; otherwise a grade may not be issued.

I have read the final version of the extended essay that will be submitted to the examiner.

To the best of my knowledge, the extended essay is the authentic work of the candidate.

I spent 4 hours with the candidate discussing the progress of the extended essay.

Supervisor's signature: ______________________ Date: 2 March 2012

Assessment form (for examiner use only)

Candidate session number	[redacted]

Achievement level

Criteria	Examiner 1	maximum	Examiner 2	maximum	Examiner 3
A research question	2	2	2	2	
B introduction	2	2	2	2	
C investigation	3	4	3	4	
D knowledge and understanding	3	4	3	4	
E reasoned argument	3	4	3	4	
F analysis and evaluation	3	4	3	4	
G use of subject language	3	4	3	4	
H conclusion	2	2	2	2	
I formal presentation	2	4	1	4	
J abstract	1	2	2	2	
K holistic judgment	3	4	3	4	
Total out of 36	27		27		

e of examiner 1: [redacted] Examiner number: [redacted]
PITAL letters)

e of examiner 2: [redacted] Examiner number: [redacted]
PITAL letters)

e of examiner 3: ______ Examiner number: ______
PITAL letters)

IB Cardiff use only: **B:** ✓

IB Cardiff use only: **A:** 104833 **Date:** 21/05

Abstract

Resonance occurs when a whistle is blown, a Helmholtz resonator is used, or a ball is struck. This essay investigates how the size of a hole in a spherical cavity affects the resonance frequencies present (created), as well as how it would affect the different harmonics.

RA

RQ

Air was blown across different sized holes in a ball with a high velocity. This sound was recorded by a mic connected to loggerpro. From the recorded data a FFT graph was produced showing the frequencies present and their respective relative amplitudes. The frequencies present with the highest amplitude for each hole size was compared to the predicted frequencies produced by the Helmholtz resonance theory, and the spherical harmonics theory.

how inv. done

Limited Scope.

After concluding that Helmholtz resonance was not the correct model for this phenomenon, the peak frequencies from the FFT graphs of the different size holes were compared with one another and with those frequencies predicted by spherical harmonics. It was seen that the hole size did not seem to greatly affect the frequency produced by the ball, although there did seem to be a very slight increase in the frequencies as the hole diameter increased. Due to the apparent lack of effect the hole size had on the frequency, the recorded frequencies for select peaks were averaged and compared to the predicted values from spherical harmonics. The recorded results closely matched the predicted values indicating that spherical harmonics is probably the correct model, however not all the predicted frequencies were prominent. Thus the hole size does not greatly affect the frequencies produced due to spherical harmonics, however it could preclude some of the harmonics.

concl.

Conc

1

Word Count: 270

2nd element ("how investigation undertaken") clear enough, classic procedure (FFT) followed + relevant info re models proposed

abstract: 2/2

Table of Contents

vague generic

Introduction:

When wind blows across the top of a chimney there is the potential for a noise to be made. This can be seen in many other situations with airflow across a narrow opening and gives rise to the expression of "howling winds." Humans reproduce these phenomena when blowing on glass bottles or blowing on whistles. I shall examine how this corresponds to air being blown across a hole drilled in a hollow rigid plastic ball. This research will investigate how the size of a hole in the spherical cavity affects the resonance frequencies present, as well as how it would affect the different harmonics. There are two important models to note in reference to sound produced by a flow of air. They are Helmholtz resonance, which governs the phenomena of the bottle, and Spherical harmonics, which refers to modes of harmonic resonance of an elastic medium in a spherical cavity. Helmholtz resonance was named after, and describes the device Hermann von Helmholtz produced to identify particular frequencies out of music and other complex sounds made from a multitude of frequencies ("Helmholtz Resonator"). Spherical harmonics are very important because they represent the most accurate and precise way to measure the speed of sound in a gas, as well as the universal gas constant (Russell).

RQ: 2/2

NR

intro: signif. worthiness: ✓ — Refs!

2/2 physics principles: incomplete in intro "per se" (1st paragraph) more under "competing models", fine;

Competing Models

Helmholtz resonance explains that the sound created by the ball, is caused by resonance of oscillating air. The air that was blown pushes the air in the hole downwards which can be thought of as a block of air. Thus the air in the cavity is pressurized, and due to the increase pressure the air in the cavity pushes the air from the hole back outwards. However, due to the mass of the "block" of air, the air's momentum carries it past its original position. This causes lower air pressure inside the cavity than outside the cavity. Thus the "block" of air is pulled back into the cavity with the air's momentum carrying it past the equilibrium point again. The air is pushed back out again and the oscillations continue, similar to a spring. The created sound is caused by the oscillations of the air in the neck of the cavity (Wolfe).

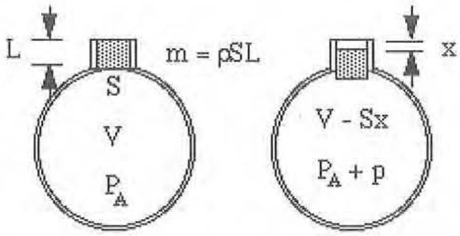


Figure 1 This figure shows the ball with the air in the neck before being blown and the ball with the air oscillating in the neck. V is the volume of the ball, P_A is the pressure, x is the distance that the air moved, m is the mass of the air, S is the cross sectional area of the neck, ρ is the density of the air, L is the neck length and finally p is the change in pressure. (Wolfe) Refs!

According to Helmholtz:

$$f = \frac{c}{2\pi}\sqrt{\frac{S}{V\,(h + 1.5r)}}$$ [Eq. 1]

(Raichel)

Where c is the speed of sound, S is the cross sectional area of the hole, V is the volume of the ball, h is the wall thickness of the ball, r is the radius of the ball, and f is the frequency of sound produced. Assuming that the thickness of the ball is zero the equation can be rewritten as:

$$f^2 = \frac{c^2}{4\pi^2}\frac{S}{V\,(1.5r)}$$ [Eq. 2]

Since the cross sectional area of the hole can be rewritten in terms of the radius of the hole, this equation can be simplified to:

$$f^2 = \frac{c^2}{4\pi^2} \times \frac{\pi r}{1.5V}$$ [Eq. 3]

fine

Thus from equation 3 it can be seen that if Helmholtz Resonance is the correct model for the studied phenomenon, the frequency squared is expected to be proportional to the radius of the hole. Thus to test the validity of the Helmholtz Resonance model for this situation it shall be seen if the frequency squared is proportional to the radius of the hole as predicted.

The alternate theory being considered is Spherical Harmonics. Spherical harmonics explains the noise as a standing wave within the instrument. The "white" noise of the air being blown into the hole creates a standing sound wave inside the cavity with the frequencies present depending on the dimensions of the cavity. This happens due to the cavity causing most frequencies to destructively interfere, while allowing a few to constructively interfere and resonate.

Analyzing spherical resonators "requires the use of Legendre polynomials and spherical Bessel functions and necessitates a computational approach to visualize the mode shapes" (Russell). However, for this experiment, in order to simplify the calculations, the equations for finding the speed of sound can be used with a known sound velocity to calculate the frequency. The simplified equation for finding the speed of sound in a ball if the volume is known and the frequency is found, is:

$$f_{ln} = z_{ln}\left(\frac{c}{2\pi r}\right)$$ [Eq.4] (Raichel)

$f_{ln} \propto r^{-1}$

Where f_{ln} is the frequency, z_{ln} represents discrete harmonics created by 3 dimensional harmonic shapes, c is the speed of sound, and r is the radius of the spherical cavity. The harmonic recorded is determined by the vector coordinates, and number of modes in the sphere as can be seen by *figure 2.*

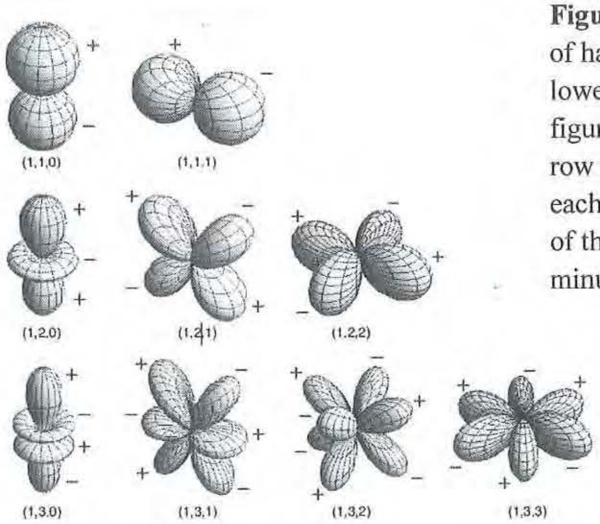


Figure 2 This figure shows the different shapes of harmonics made depending on the three lowest mode indices indicated below the figures in (n, l, m) for a spherical cavity. Each row represents a different harmonic, and thus each row has a different frequency. The phase of the oscillation is indicated by the plus or minus signs (Russell).

z_{ln}: elaborate

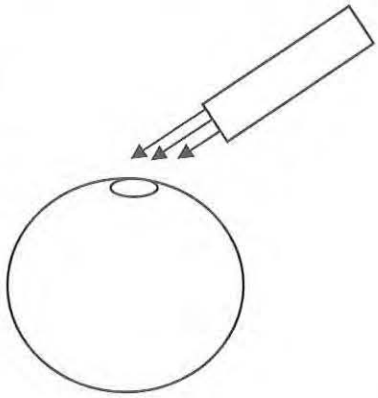
From this equation it can be seen that if a hole is in the wall of the ball, its size should not greatly affect the frequencies produced. However, there is the potential for the positioning of the hole to cause a node to become an antinode, disrupting the standing wave produced. Thus certain frequencies may not be produced. Therefore, the effect of the hole on the frequencies produces can help to determine which organization of essay: the intro should be self-contained and the formal theory developed in a separate section or chapter;

RQ ok 2
Intro — ok 2

theory correctly models the situation present. The theoretical frequencies predicted by the two models can be calculated, which may also aid in determining the correct model for this phenomenon.

Set Up:

Figure 3 This photo shows air being blown across the hole in the ball. Note that the angle, the distance from the ball, and power of the air coming from the straw should be constant for each trial of a certain ball.



Variables

The independent variable is the size of the hole in the ball producing the sound. The dependent variables are the frequency of the sound created when air is blown across the hole of the ball, as well as the harmonics produced. The size of the hole will be altered from 5mm diameter to 13mm diameter by drilling into identical balls with different size drill bits. The holes will then be measured with a Vernir caliper after being shaved with a box cutter to ensure that there are no ridges or rough edges around the hole. The dependant variables will be measured with a microphone connected to loggerpro. A FFT graph will be produced that shows the frequencies of the sound produced.

some details

a word to define & explain Fourier series

Controlled Factors

- The volume of the balls
- Temperature of?
- Harmonic of the sound with the highest amplitude
- Neck length ...

Procedure

hollow dia? thickness How measured?

Six **identical rigid plastic balls** with a volume of 220ml ± 10, a **drill**, and six **drill bits** of different diameter were obtained. A hole 5.0 mm ± 0.5 in diameter was drilled in a ball and a **box cutter** was used to clean the edges of the hole and ensure that it was smooth without jagged edges. A **Vernier Caliper** was used to measure the final hole diameter that was smoothed with a box knife. A **Vernir microphone** was hooked up to **loggerpro** on the **computer** and set to a sample rate of 100000 samples per second for 0.5 seconds. **Air** was blown through a **straw** onto the hole in the ball at an angle such that a sound was made, and with an airflow such that a lower frequency could be obtained by blowing softer. . If air was blown softly across the hole a quiet, and very low sound was produced, however this was not used as it was hard to distinguish between the sound produced and other background noises. If air was blown harder, then a higher and louder frequency was heard. The same thing occurred if air was blown harder still, in discrete increments. The frequency studied in this experiment was the first one after the very low

- repeat
- hold later should not be used

mics: some details...

constant: how to make sure?

and quiet resonant frequency. As soon as the sound was produced the microphone was started and data collected. This was repeated six times for each of the six balls with hole diameters ranging from 5.0 mm ± 0.5 to 13.0 mm ± 0.5.

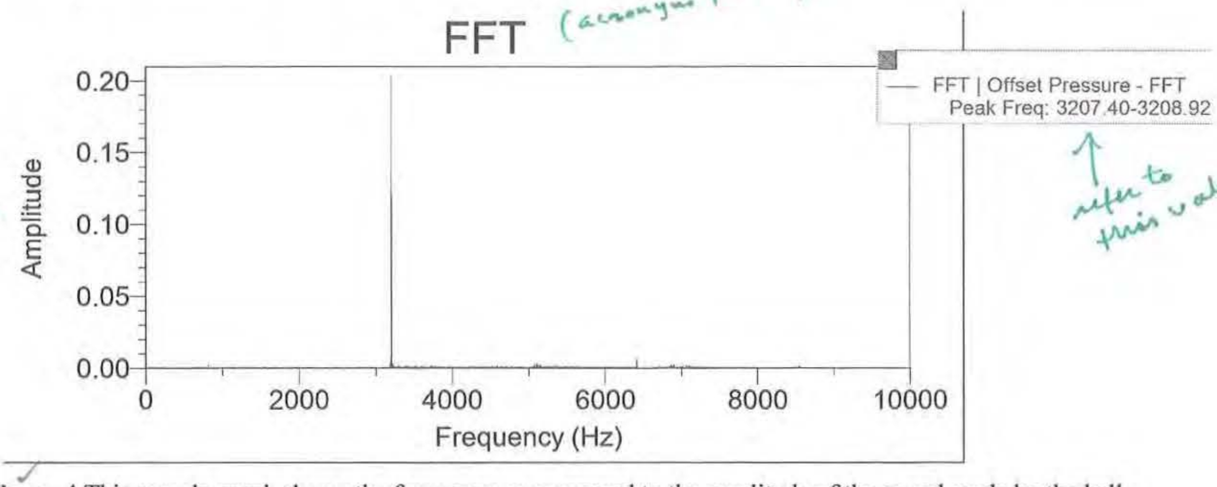
Data:

Controlled Factors

The controlled factors were measured before the investigation and kept constant during all trials. The temperature of the air outside of the ball was 27 °C±1 and was measured with a thermometer. The temperature is important because it affects the speed of sound in the medium. However, the air temperature outside the ball is not the most important temperature as the air inside the ball is the oscillating medium. The temperature inside the ball was 32 °C ± 2, and was measured by placing a thermometer inside the ball after being blown on numerous times. Although it was assumed that the thickness of the ball was negligible, the validity of that claim was still unknown so the measurements were still taken. The wall thickness was 0.00035m ± 0.00005 and was measured by cutting a piece of the ball's wall out and using a micrometer. The volume of the ball was 220 ml ± 10, and was measured by filling the ball with water, and measuring the volume of water the ball contained with a 1 liter graduated cylinder. The speed of sound in the ball was 350 m/s ± 3 according to engineeringtoolbox.com where the temperature inside the ball was used to calculate the speed of sound. The neck length (as seen *on figure 1*) for the balls was kept at zero by not having a neck on any of the balls.

OK

After the data was recorded on loggerpro the "examine" function was used to find the frequency with the highest amplitude on the FFT graph created by loggerpro.



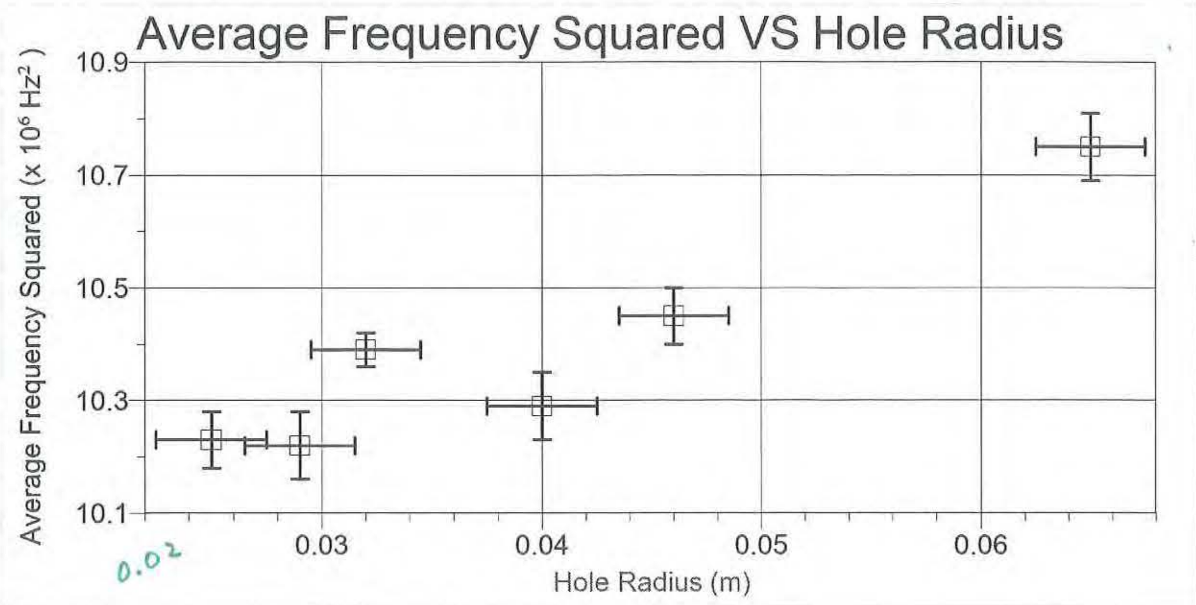
refer to this value

Figure 4 This sample graph shows the frequency as compared to the amplitude of the sound made by the ball with the smallest hole diameter for its first trial. The frequency with the largest amplitude was taken for every trial, for all the balls.

5.0 mm

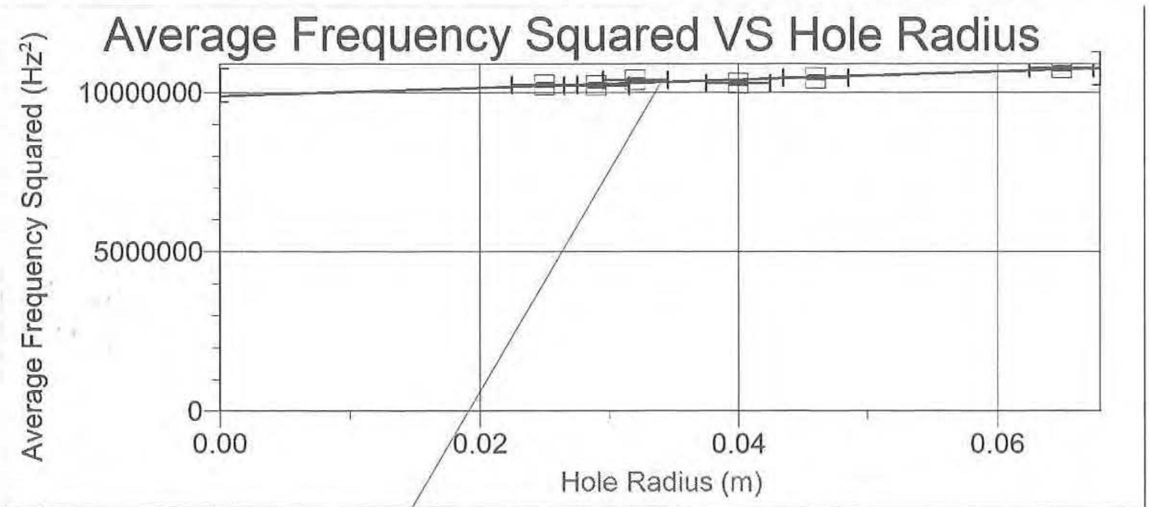
Compare to Helmholtz Resonance

The data recorded (appendix 1) for the frequencies produced with the highest amplitudes was used to calculate (appendix 3) the frequencies squared as compared to the hole radius.



from data of Table 1 in appdx.

Figure 5 This graph shows the Average frequency of sound produced squared as compared to the radius of the hole in the ball at which the sound was produced.



why this graph?

Linear Fit for: Data Set | Average Frequency Squared
y = mx+b
m (Slope): 1.286E+007 Hz^2/m
b (Y-Intercept): 9.880E+006 Hz^2
Correlation: 0.9374
RMSE: 7.809E+004

??

(why not?) ?.. [illegible]

Figure 6 This graph shows the average frequency of sound produced squared as compared to the radius of the hole in the ball at which the sound was produced. The linear fit suggests that the frequency squared is not proportional to the hole radius as is predicted by Helmholtz for a ball with negligible neck length.

?

The data appears pretty random.

Fig. 5 no clear trend

From this graph it can easily be seen that the data collected does not seem to support the theory of Helmholtz resonance as the correct model for this situation. This is clearly shown by the incredibly high y-intercept, as a proportional fit must pass through the origin. Also it can be seen that there does not seem to be much increase in the frequencies recorded as the hole size was increased by 260% with only a 5% increase in frequency. This indicates that the hole size does not significantly affect the frequency produced, again evidence that Helmholtz is not the correct model. However, as the slope is a rather large number, conclusions cannot yet be made as to the effect of the hole size. Also, as the frequency squared idea assumed that the ball had a negligible wall thickness, and the validity of that assumption is unknown, Helmholtz might still relate to the data. Thus the recorded frequencies will be compared to the frequencies predicted by Helmholtz Resonance. The full list and derivation of predicted frequencies can be found in appendix 4. — Again – why not here? (I)

(actually 3)

using h+1.5r 0.35mm + 1.5(2.5) 0.35 + 3.75 no... see app. 3!

but app. 4 is almost identical: wall ...

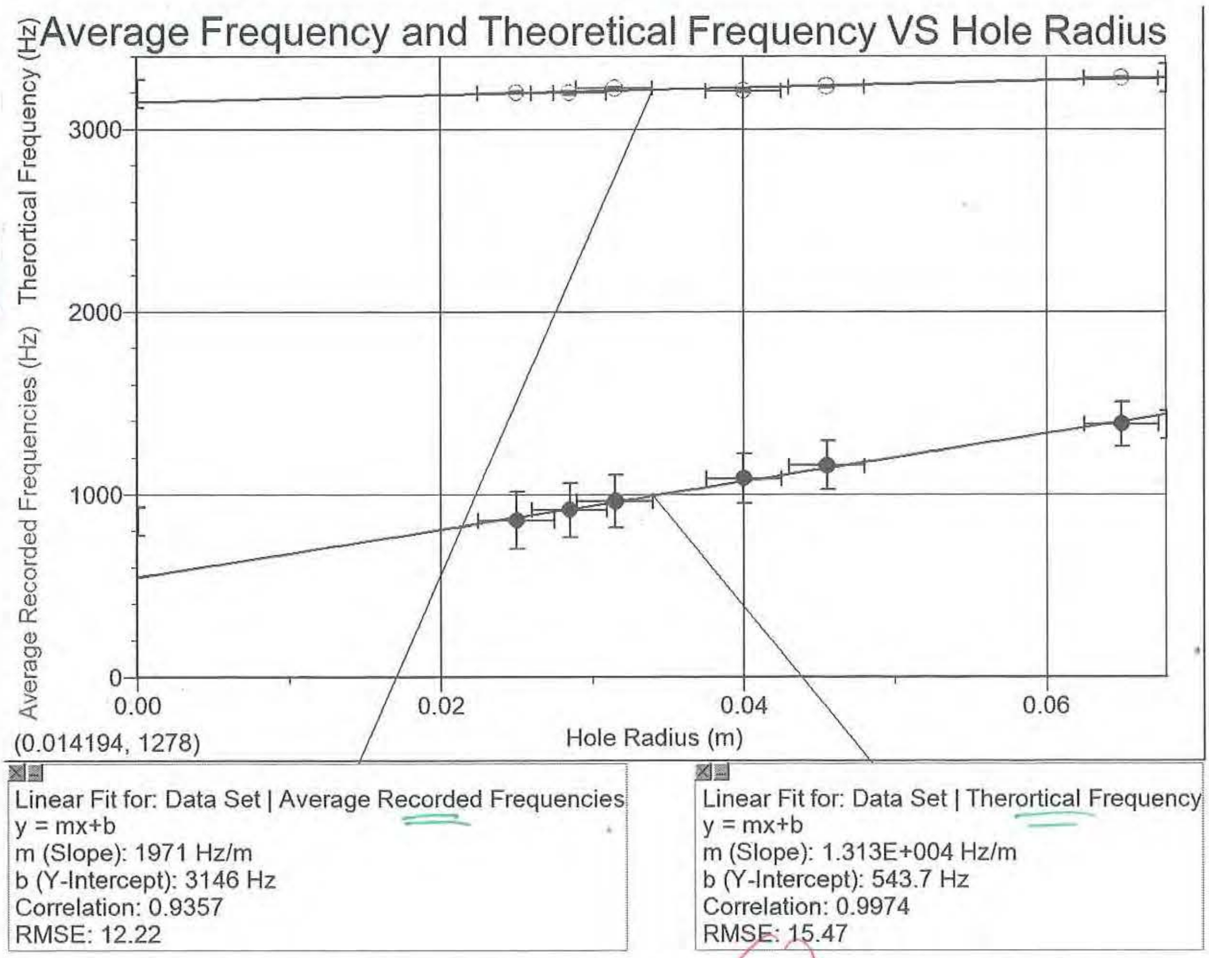


Figure 7 This graph shows the frequencies obtained experimentally (red) and the theoretical frequencies predicted by Helmholtz resonance (blue) as compared to the different hole sizes. The hole radii were used along with the equation for Helmholtz resonance (including the wall thickness) to find the predicted frequencies. It can easily be seen that Helmholtz does not seem to describe what is occurring.

(I) appendix 3

?

From this graph it can be seen that the assumption for negligible wall distance was false because the predicted values are not proportionally related. Also, there is a large difference between the frequencies received, and the theoretical frequencies predicted by Helmholtz Resonance. This shows that even though the assumption of negligible wall thickness might have been false, Helmholtz does not seem to explain this data. Aside from the difference in frequencies produced, the slope of the linear fit regarding the predicted values is 666% of the slope for the recorded frequencies. This also suggests that said model is not correct, and that the hole size may not greatly affect the frequency produced. Now the question remains: Does Spherical Harmonics model the phenomenon correctly?

Compare to Spherical Harmonics ?

The frequencies predicted by Spherical Harmonics were found through Russell's investigation. Assuming that spherical harmonics is the correct model for this research, the hole size would not greatly affect the frequencies produced; an idea supported by the data. Thus to test the validity of the spherical harmonics model the average frequencies produced by the different hole sizes will all be averaged to create a mean frequency for this harmonic.

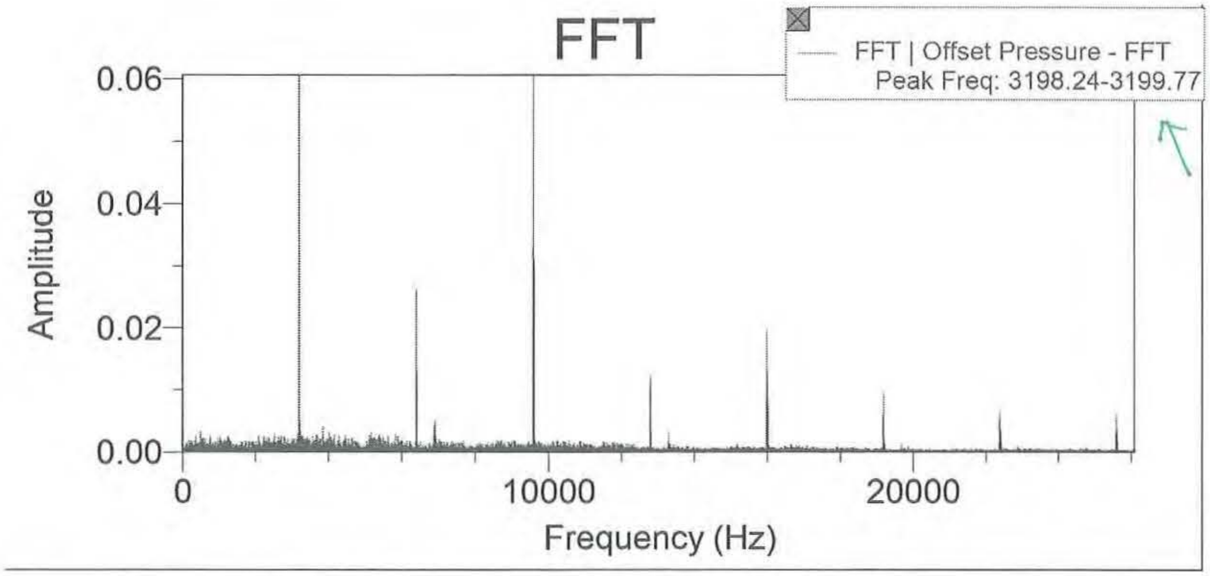
This average frequency produced from the ball (the average of the averages) is only 6% higher than the theoretical value predicted by spherical harmonics. However, that is only one data point, and as such this investigation will be extended to other harmonics besides the one previously tested, which is the first harmonic according to spherical harmonics.

value? data? E

Other Harmonics

The FFT graphs originally recorded for the earlier data can be revaluated to find the other harmonics. As can be seen by figure 8, there are multiple peak frequencies aside from the one looked at previously, however they have lower amplitudes. These peaks can be found and cross referenced with the predicted frequencies from spherical harmonics to see if the data supports theory. Since these peaks are much smaller in amplitude than the first harmonic, they are not always picked up by the microphone over other background noises. This means that some graphs did not have certain harmonics represented while other graphs did. As such, only the most prevalent harmonics will be discussed; however ten of the first thirteen predicted harmonics were found in multiple graphs.

arbitrary units?



Hz

Figure 8 This sample graph shows the frequency as compared to the amplitude of the sound made by the ball with the 4mm radius hole diameter for its first trial. The scale has been set to clearly show the different harmonics reached.

Here please.

Table 2 in appendix 1 shows the frequencies found when each of the FFT graphs were analyzed for the second major peak. There was one other easily visible peak before this one, however it only consistently occurred for balls with the largest and the smallest hole sizes.

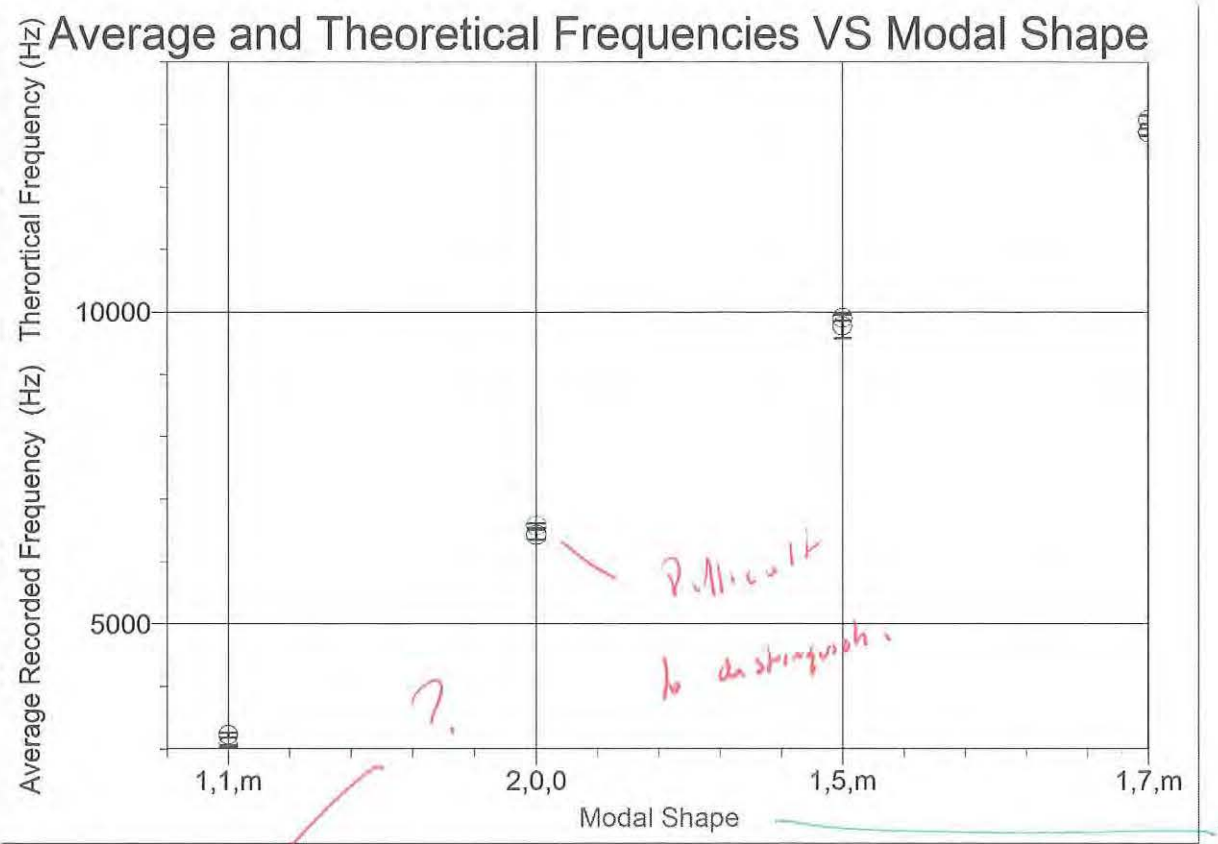
Again, the data supports that the hole size does not significantly affect the frequency, as there is less than 2.83% difference between the average frequencies of the different hole radii. The FFT graphs were again reanalyzed for any other prominent frequencies. Two other modal patterns were also frequently noticeable on the FFT graphs, with the higher frequencies having less trials containing adequate data. When evaluated, it was seen that the higher the frequency, the lower the percentage change due to changing hole sizes. As all the data supports that hole size does not significantly impact frequency, the assumption that an average frequency from the different hole sizes represents the harmonic frequency, seems valid. The average frequencies were compared to calculated theoretical frequencies, as predicted by spherical harmonics.

statements must be supported with extracted data (E)

(confusing organization of data in app. 1 where only considering H's mode!) ...

no reference made to appendix 4 and appendix 5 (I)

?



?

Difficult to distinguish.

Figure 9 this graph visually shows the variation between the recorded value and the theoretical values of the frequencies. The blue dots represent the theoretical values, and the black circles symbolize the recorded values. The x values represent the modal shapes that produce the frequencies graphed, and show the different harmonics.

elaborate not well explained in context of essay

From figure 9 it can be seen that the average values are very close to the theoretical values predicted for those harmonics with the assumed modal shapes. Also, the small error bars further supports the idea of hole size not significantly affecting the frequency as the range of average frequency values was used to find the uncertainty. Frequencies predicted by Helmholtz resonance did occur, and were found in some of the graphs. However the number of graphs with a clear peak at Helmholtz resonance frequency was sparse. Also Helmhotz resonance seemed to break down after the first three hole sizes as peaks did occur but with a lower frequency than predicted Helmholtz values.

Conclusion:

From figure 9 it can easily be seen that the spherical harmonics theory seems to support the results much more accurately than Helmholtz resonance theory as the theoretical frequencies for spherical harrmonics are almost the same as the recorded frequencies. However, as frequencies matching Helmholtz Resonance were occasionally found, and a lower frequency could have been produced by the balls, Helmholtz may still govern some aspect in this situation. Helmholtz resonance could have governed the low quiet sound produced when the air was blown softly, and might be a useful further study. This indicates that the velocity of the airflow could determine which of the constructively interfering waves has the highest amplitude and thus the prominent modal pattern. It may even determine which model explains the resonance present. The first harmonic predicted by spherical harmonics was the lowest frequency analyzed, however peak frequencies were occasionally found lower than that; leading to the conclusion that Helmholtz resonance could have been occurring, just not producing sound with a high enough amplitude to compete with the sound produced by the spherical harmonics. As all frequencies measured were lower than the theoretical, except for the first harmonic, it appears that the hole causes the frequency to be slightly lower than theoretical predictions. However, as the size of the hole increases it seems that the frequency increases as well, bringing the frequency produced closer to the theoretical values for all but the first harmonic. It must be noted that the predicted frequencies were calculated for air in a sealed ball being tapped with a metal rod, while the investigation undertaken in this research utilized balls with holes (Russell). This changes the basic design for the research; however the model of spherical harmonics governs both. In the situation discussed above the sound created results from a standing wave in the ball due to a stream of air blown through the hole; whereas in the theoretical situation the sound is created by the oscillations of the sides of a rubber ball after being tapped with a metal rod. The hole in the ball may also affect the possible modes created. If the point of the wall with the hole was a nodal point in a in a certain standing wave modal pattern, that modal shape may not be possible in the holey ball as it is in the complete ball. Even though the hole may preclude some modal shapes, it does not necessarily preclude the formation of certain predicted frequencies as there can be different modal patterns of the same frequency. This is due to the *m* component as can be seen in *figure 2*. Therefore, in conclusion, spherical harmonics better predictes the frequencies produced in the investigation, yet provides little insight into the effect of the hole size.

Good Conclusion relevant to Rq

Evaluation:

One of the biggest sources of error was that some of the holes in the balls contained ridges and were not smooth. As the holes got larger the holes tended to become rougher, with the ball with the largest hole containing a hole not circular with slightly jagged edges. Although a box cutter was used in an attempt to smooth the edges of the hole, it normally would cut into the edge of the hole reducing the circularity of the hole. This made it difficult to measure the diameter, and thus the size of the hole. This could have caused unwanted turbulence around the hole, disrupting the resonance occurring in the ball. This could have been reduced if another method had been used to try and smooth the holes. A dental tool, such as the Turbo Carver, could be used with sandpaper in the end to smooth the edge of the hole.

Another weakness of this lab is that though it was attempted to maintain a constant speed of the airflow and angle of the straw, there was some variance in the way that the air was applied to the hole in the ball. This variance in the speed and angle of the air could have changed the frequency as different strengths produces different harmonics and the angle of application can change the frequency. This could be counteracted by using a machine, rather than a human, to apply the airflow. An air pump could be hooked up to the straw and secured in place. The ball could then be introduced to the stream of air in such a way that sound is produced and then also secured in place. In this way neither the ball, nor the air supply/power, nor the straw would be moved during the trials; increasing consistency and precision, and reducing random error.

A third source of error in the evaluation of the data is that sometime it was difficult to decide the correct modal shape that the frequencies represented. For example the (1,7,m) modal shape prescribed to the last set of frequencies has two other modal frequencies within 4% difference of the predicted frequency of (1,7,m). Although the average recorded frequency for that peak is only 1.9% different from the theoretical value, there is the possibility that the recorded peak does belong to a different peak and the presence of a hole just changed it. Even though it seems as if the size of a hole does not affect the frequency much, the presences of a hole could affect the frequency as no balls were tested in which there was no hole. This is one of the hardest problems to fix, and most likely will only realistically be able to be accomplished for the lower frequencies. The strength of the airflow on the hole of the ball decides which modal pattern will have the highest amplitude. If air was blown harder on some of the balls then the (1,2,m) modal frequency had the highest amplitude. Thus the strength of the airflow could be increased so that data is recorded when each modal shape's frequency has the highest amplitude. In this way the modal shapes could be moved up one by one. However, as the hole sizes increased the force with which air was need to be blown increased as well. Meaning that for the ball with the largest hole it was very difficult to induce resonance, as a strong stream of air was needed. Thus after the first couple harmonics it will be very difficult to increase the harmonics for the hole sizes in this investigation, even the small ones.

An interesting follow up for this experiment would be to see if the hole shape affects the frequency produced, even though the size did not seem to. This might tell us more about the nature of the resonance pattern in the balls.

Too much important information is
relegated to the Appendix.
Quite an interesting EE with a valid conclusion.

- presentation: organization of the essay re appendices is poor; the appendix is not part of the EE per se (number of words ~ 4000); sample of all data should in core; some appendix referred to with wrong number; no reference to appendix 4 + 5 (spherical harmonics); half of sources listed in biblio. not cited in core of essay;
- language: uncertainties without units; acronyms not defined; some terms not clearly / fully defined;

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Appendix 1

This table shows the raw data for the frequencies with the highest amplitudes over the six trials for each of the six hole sizes.

Hole Diameter (mm) ±0.5 mm	Highest Frequency (Hz)							
	Trial1	trial2	trial3	Trial4	Trial5	Trial6	Average	Uncertainty
5.0	3207	3198	3192	3192	3209	3192	3207	±8
5.7	3209	3194	3192	3189	3203	3192	3209	±10
6.3	3220	3229	3223	3221	3221	3224	3220	±5
8.0	3198	3210	3198	3209	3210	3218	3198	±10
9.1	3239	3233	3233	3238	3223	3232	3239	±8
13.0	3279	3290	3281	3279	3271	3275	3279	±9

Table 1 This table shows the recorded frequencies of sound as compared to the diameter of the hole in a ball which produced the sound. The uncertainty was found by halving the range of the frequency values.

Theses tables show the raw data after the FFT graphs were reanalyzed for the most prominent frequencies. elaborate

Hole Diameter (mm) ±0.5	Highest Frequency (Hz)							
	Trial1	trial2	trial3	Trial4	Trial5	Trial6	Average	uncertainty
5.0	6420	6410	6360	6380		6380	6390	±30
5.7	6411	6336			6401	6372	6380	±40
6.3	6410	6460	6430	6420	6420	6410	6420	±20
8.0	6330	6370	6408	6410	6410	6430	6390	±50
9.1	6470	6460	6470	6490	6440	6450	6460	±30
13.0	6550	6580	6570	6570	6550	6550	6560	±10

Table 2 This table shows the second peak set of frequencies recorded for the different size holes. The uncertainty was found by halving the range of the frequency values. The absent values indicate that there were no recognizable peaks for those trials.

As can be seen, the above harmonic appears in almost every trial. However for the next modal patterns presented, the data for certain hole radii was not abundant enough for analysis. Those hole sizes are not shown in the following tables.

(Continues on next page)

Hole Diameter (mm)	Highest Frequency (Hz)							
±0.5	Trial1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Average	Uncertainty
5.0	10200	9600	10200			10200	10000	±300
6.3	9630	9650	9620	9650	9650	9650	9640	±10
8.0	9560	9570	9600	9630	9640	9650	9610	±50
9.1	9720	9690	9690	9750	9670	9710	9700	±40
13.0	9830	9870	9830	9830	9820	9830	9830	±30

Table 3 This table shows the raw data for the third frequency recorded for the different size holes. The uncertainty was found by halving the range of the frequency values.

Hole Diameter (mm)	Highest Frequency (Hz)							
±0.5	Trial1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Average	Uncertainty
6.3	12860	12880	12860	12880	12850	12890	12870	±20
8.0	12780	12830	12780	12850	12850	12880	12830	±50
9.1	12990	12940	12940	12960	12880	12940	12940	±50

Table 4 This table shows the raw data for the fourth frequency recorded for the different size holes. The uncertainty was found by halving the range of the frequency values.

radius... units of data of table 1

Appendix 2

Here is the calculated data used in creating figures 5 and 6 and the method in which this data was calculated.

which f is squared? of app. 1 tables...

Radius (mm) ±0.25	Average Frequency Squared $(Hz)^2$	Uncertainty units
2.50	10230000 *	±50000
2.85	10220000	±60000
3.15	10390000	±30000
4.00	10290000	±60000
4.55	10450000	±50000
6.50	10750000	±60000

Table 5 This table shows the measured frequency squared as compared to the radius of the hole in of the ball

* $\sqrt{10230000}$ = 3198 Hz → for radius of 4.0 mm see table 1 of appendix ??

at 2.5 mm $\bar{f}$ = 3207 Hz not 3198 Hz ... ?

Sample Calculation: Finding the uncertainty of the Frequency squared for the hole with the largest diameter

13 mm dia

$$f^2 = f \times f$$
$$f^2 = (3279 \pm 9) \times (3279 \pm 9)$$
$$\text{Actual: } 10751841$$
$$\text{High: } 10810944$$
$$\text{Low: } 10692900$$
$$\text{Final: } 10750000 \pm 60000$$

Appendix 3

The values for the frequencies predicted by Helmholtz Resonance are calculated and shown in tale 6.

Sample Calculation: Theoretical Value for Frequency Produced by the Ball with the Smallest Hole Including the Ball Wall Thickness

$$f = \frac{c}{2\pi}\sqrt{\frac{S}{V\,(h + 1.5r)}}$$

$$f = \frac{350 \pm 3}{2\pi}\sqrt{\frac{\pi(.0250 \pm .0025)^2}{(.00022 \pm .00001)\,((.00035 \pm .00005) + 1.5(.0250 \pm .0025))}}$$

$$\text{Actual: } 855.38$$
$$\text{High: } 1023.41$$
$$\text{Low: } 711.98$$
$$\text{Final: } 900 \pm 200$$

Radius (m) ±.0025	Theoretical Frequency (Hz) ± 200
0.0250	900
0.0285	900
0.0315	1000
0.0400	1100
0.0455	1200
0.0650	1400

Table 6 This table shows the Frequencies predicted by the equation for Helmholtz resonance as compared to the radius of the hole in the ball

Appendix 4

The average values for the frequencies of each modal pattern are calculated and displayed in table 7.

Sample calculation: Average Frequency (Hz) for all hole sizes for the first Harmonic

$$\text{Average} = \frac{f_1 + f_2 + f_3 + f_4 + f_5 + f_6}{6}$$

$$\text{Average} = \frac{3207 \pm 8 + 3209 \pm 10 + 3220 \pm 5 + 3198 \pm 10 + 3239 \pm 8 + 3279 \pm 9}{6}$$

$$\text{Actual: } 3222.91$$
$$\text{High: } 3233.6667$$
$$\text{Low: } 3217$$
$$\text{Final: } 3223 \text{ Hz} \pm 8$$

Probable Mode Shape	Theoretical Frequency (Hz)	Average Frequency (Hz)	Average Frequency Uncertainty	Percent Difference from Theoretical
1,1,m	3046	3223	± 40	5.8%
2,0,0	6575	6430	± 80	-2.2%
1,5,m	9898	9770	± 200	-1.3%
1,7,m	13078	12880	± 60	-1.5%

Table 7 This table shows the theoretical values of the frequency as predicted by spherical harmonics and how they compare to the average frequencies collected assuming that the hole size does not have a significant impact on the frequency emitted. It also shows the percentage by which the average frequencies differ from the theoretical values (Russell). The uncertainties were found by halving the range of the average frequencies as that uncertainty was always larger than the uncertainty as calculated by the previous sample calculation.

not referred to

Appendix 5

The calculated frequencies predicted by spherical harmonics are displayed in table 8.

elaborate

Modal Shape	Predicted frequency (Hz)	Uncertainty
1,1,m	3050	± 20
1,2,m	4890	± 30
2,0,0	6580	± 30
1,3,m	6600	± 30
1,4,m	8270	± 40
2,1,m	8700	± 50
1,5,m	9900	± 50
2,2,m	10680	± 60
3,0,0	11320	± 60
1,6,m	11500	± 60
2,3,m	12570	± 60
1,7,m	13080	± 70
3,1,m	13490	± 70
2,4,m	14410	± 70
1,8,m	14660	± 80
3,2,m	15540	± 80
4,0,0	15960	± 80
2,5,m	16210	± 80
1,9,m	16230	± 80
3,3,m	17530	± 90
2,6,m	17980	± 90
4,1,m	18160	± 90
3,4,m	19500	± 100
2,7,m	19700	± 100

Table 8 This table shows the predicted Spherical Harmonics frequencies and their corresponding mode shapes

The theoretical frequencies according to spherical harmonics were calculated in Russel's investigation, and since the frequencies are inversely proportional to the radius of the hollow ball, the frequencies predicted for this research can be found using proportions.

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Sample Calculation: Finding the First Theoretical Harmonic for Spherical Harmonics

$$f_1 \times r_1 = f_2 \times r_2$$
$$f_2 = \frac{f_1 \times r_1}{r_2}$$
$$f_2 = \frac{979 \times .116}{.037278 \pm .0002}$$
$$Actual: 3046.4$$
$$High: 3061.02$$
$$Low: 3028.37$$
$$Final: 3050 \pm 20$$

Hz