IB Diploma Extended Essay

Physics

Topic: Springs

Research Question:

"How does efficiency change when drawlength is increased?"

Examination: May 2010

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1. Abstract

The efficiencies of a recurve bow at various drawlengths were measured and presented in this essay. This essay gives an answer to the following question:

"How does the efficiency change when drawlength is increased?"

The energy transferred to the arrow at the various drawlengths were measured by finding out the speeds, which were then divided by the energy put into the bow, which was measured by using a force-gage to find out the force, for which a general formula was found. This formula was integrated to find the work exerted on the bow i.e. the energy stored by the bow.

It was found that the efficiency kept increasing until about 52 centimetres of drawlength (only 2 centimetres under my average drawlength). The efficiency decreased after that. It did not show a symmetrical parabolic shape due to a limitation in maximum drawlength due to the length of the arrow and the length of the archer's arms.

The recurve bow's efficiency does change, it increases up to about 52 centimetres of drawlength, after which it starts to decrease again. This means that although the efficiency is not the maximum at full drawlength, it does definitely show a change when drawlength is changed.

Words: 204

2. Introduction

Archery has been practiced for generations and has been one of the most important pieces of equipment for over 2000 years. It has always been assumed that the better the archer, the better the shots, while the qualities of the bows have been assumed to be as good as possible. The many companies producing both modern[1] and wooden[2] bows seem to keep on producing new kinds of bow limbs and risers[3]. But how effective are these bows? Are they a hundred percent efficient? Or could they be improved?

Of course, nothing is ever a hundred percent efficient, not even an object that has been used for centuries. Assumable is, of course, that there have been improvements from the olden wooden bent stick to the modern carbon-fibre 330fps[4] compound bow.

Although this essay is not about the history of the bow, it is related, as it is about the efficiency. Hence my research question: **"How does efficiency change when draw length is increased?"** This is all to do with the change in efficiency not between different types, shapes or ages of the bows, but instead just the change in efficiency when the string of the bow is pulled back. I predict that the efficiency will change if the drawlength is changed. I think that it will start higher and gradually decrease.

There is of course an efficiency factor: not only the arrow needs energy to move back, also the string and the bow arms need energy. This means that not all of the work that has been put into the bow will come out as velocity to the arrow. So the efficiency in this case will always be less than a hundred percent. This essay is also about finding out the optimum draw length (which has the highest efficiency factor) for my (the bow that will be experimented with) bow.

The efficiency referred to in the essay will be defined as $\frac{Kinetic\ energy\ of\ the\ arrow}{Work\ put\ into\ the\ bow}$.

[1] Compound and recurve, compound being the most advanced bow which has the highest efficiency of all sorts of bows because of the lack of movement of the bow arms, and recurve is a kind of modern bow which has the tips of the bow limbs pointing away from the archer, hence the name: re-curve

[2] A wooden bow can be any type of wood, as long as no type of metal or metal composite is used as an active part of the bow. A wooden bow can use leather, ivory, bone and horn and will still be classified as a wooden bow.

[3] Bow specific terminology is explained on the next page

[4] Fps – feet per second, the jargon for arrow speed

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3. Bow specifications

Used in experiments:

Riser: KAP Winstar II;

Limbs: KAP Winstorm Carbon. 28 lbs, 70 inch;

String: String Flex Pro, FastFlight Plus. 16 strands, 70 inch, 168 cm real length;

Button: Shibuya;

Arrow rest: Spigarelli Flipper Magnetic Rest;

Visor: Cartel K-Championship;

Clicker: Win & Win;

Arrows: Easton Aluminium/Carbon/Composite (A/C/C), 77 cm nock to tip with One Piece Easton A/C/C arrowheads (unfletched).

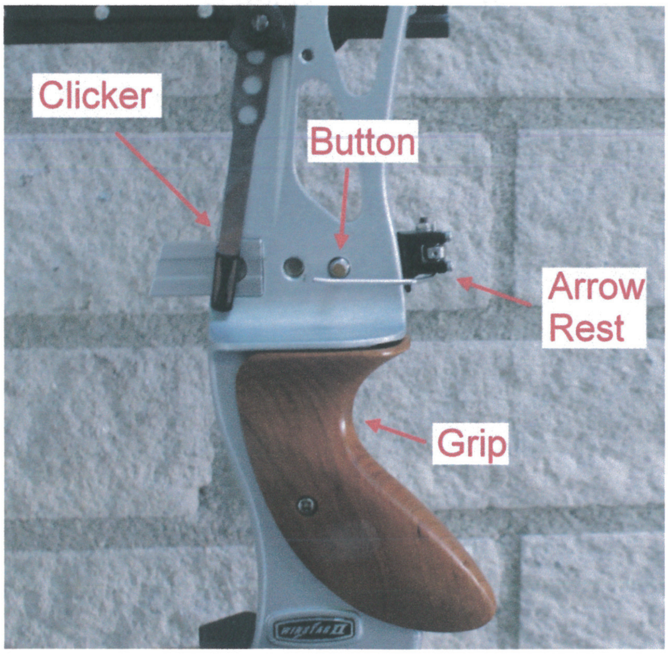
Not used in experiment, but used in regular shooting:

Black Sheep V-Bar;

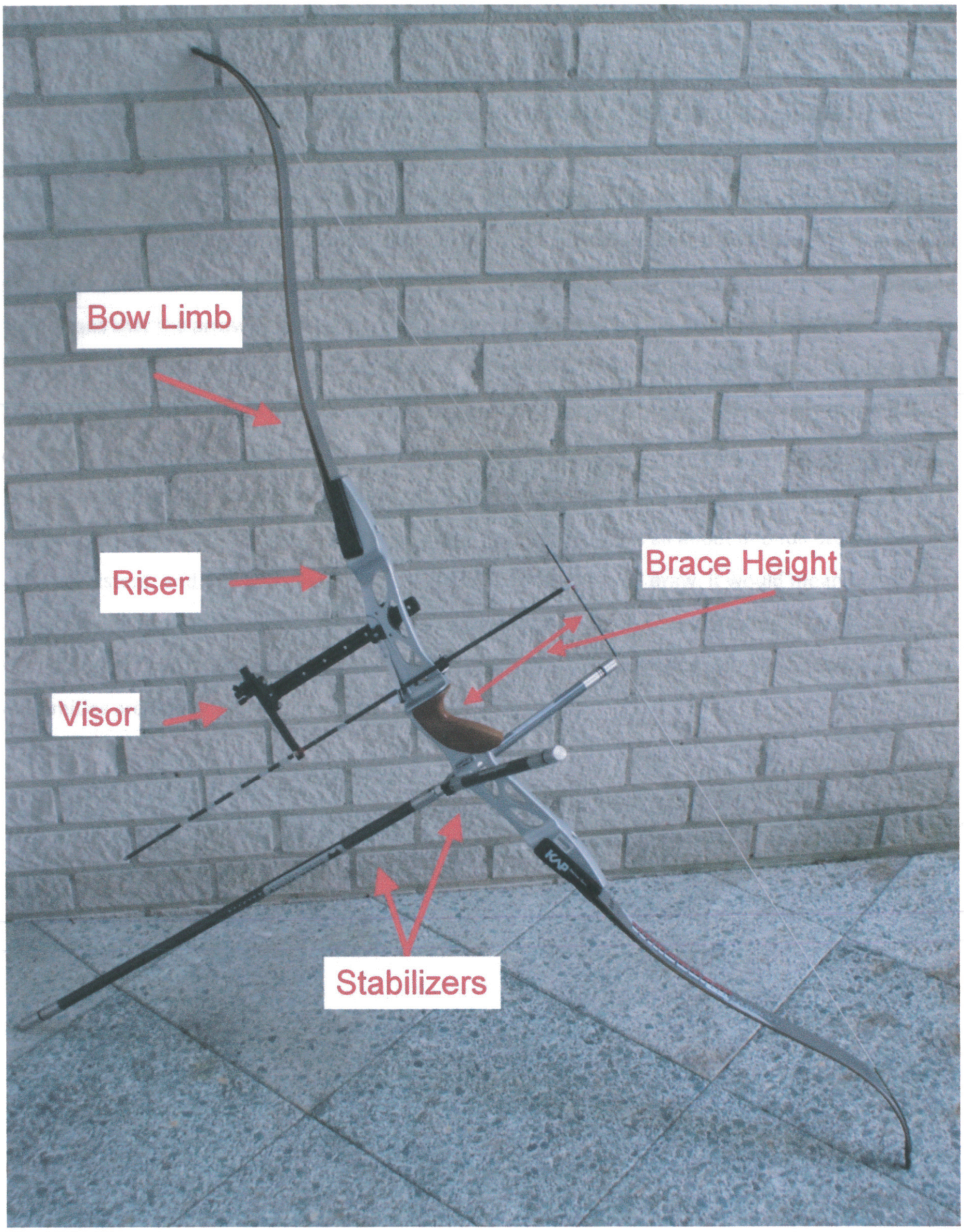
Black Sheep Stabilizer Carbon Long;

Two Black Sheep Stabilizer Carbon Short.

Here are two pictures of the bow, indicating all the parts:



Picture 3.1 Detail of the centre of the riser, see picture 3.2.



Picture 3.2

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4. Limitations

Any result obtained from the experiments will only be valid with my bow, at that time, since the specifications (drawlength, bow arms, length of the string, which is also the distance between the tips of the bow arms, so also the height of the riser and the brace height) have a significant effect on the outcome of the experiments. For instance, if the brace height is bigger than its most effective position, the arrow will travel more or less distance, while still being attached to the nocking point. This means that the arrow speed will be more or less (depending on the situation), since the string has more "time" to exert a force on the arrow. Although all recurve bows are very similar, any of the abovementioned points can make a change in efficiency, due to a difference in any part of the bow. Also, any "peak" in the efficiency will not contribute to, or change an archer's shooting style, since the current shooting style has been refined and experimented with in many different ways, and has turned out to be most effective. Since consistency[5] is what archery is all about and full draw is the most consistent, and a peak in the efficiency in not likely to be at full draw, it would mean that the archer would lose consistency to efficiency. Also, there is an enormous variation of different limbs with different draw weights, ranging from five pounds up to 60 pounds for recurve, although both extremities are barely used.

[5] In shooting style

5. Theory

Efficiency

Mechanical efficiency (which applies to bows, since they basically are springs) is defined by:

$$Efficiency = \frac{W_{ouput}}{W_{input}}$$

, where W_{output} is the work which is exerted by the bow and W_{input} is the work exerted on the bow. This will come out as a decimal, but can also be given as a percentage. In the rest of the essay I will refer to it as a percentage. Due to the law of conservation of energy, the value will never be bigger than 100%.

This formula is not really straightforward in describing the efficiency related to archery. Limiting the efficiency to the arrow, the formula can be stated as

$$Efficiency = \frac{Useful\ W_{output}}{W_{input}}$$

Since "*Useful* W_{output}" is the kinetic energy of the arrow, being equal to $\frac{1}{2}mv^2$, where *m* is the mass of the arrow and *v* is the speed of the arrow, the formula can be given as:

$$Efficiency = \frac{\frac{1}{2}mv^2}{W_{input}}$$

This means that two experiments need to take place: one to measure the work exerted on the bow, and the other to measure the speed of the arrow.

Because the work cannot be measured directly, the force F exerted on the bow needs to be determined first as a function of the drawlength *x*. The work can be calculated by integrating the formula found for the force:

$$W_{input} = \int F(x)\, dx$$

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6. Experiment

Since my research question states how efficiency will change when *drawlength is increased*, readings have to be done at various drawlengths. The selected drawlengths start at a five (5) centimetre intervals from the bow's relaxed state, and increasing by another five centimetres for each measurement for consistency.

Measuring work

Measuring work is rather straightforward: using a Newton meter to give the force exerted on the bow at a specific drawlength. Using the collected data, a graph can be plotted and the slope found. From research, this is likely to be a cubic equation After that, integrating this cubic equation and putting in the numbers will give me the amount of work in joules.

Materials

– Newton meter

– Bow and arrow

The Newton meter which I used in my experiment did not allow me to measure below 20 pounds of force, and therefore limiting my data to drawlengths higher than 20 centimetres.

Measuring speed

Measuring arrow speed will be more complex than measuring work, since there are various ways of measuring speed, while there is a low availability of measuring devices.
Possible ways to measure speed:

– Accelerometer: accelerometer will be placed on the arrow (or on the string, as it only measures acceleration), and when the arrow is shot, the accelerometer will record the acceleration the arrow (or string) underwent, allowing to be easily analyzed;

– Displacement, shooting an arrow a certain distance and measuring the time it took for a direct speed measurement;

– Doppler, using radar to track the arrow during initial stages of its flight;

– Gates, using light, magnetic or similar gates to time how long it takes for the arrow to pass through a certain displacement;

– High-speed camera, recording the initial milliseconds that the arrow travels in order to find arrow speed (regular video cameras only run at 24 fps, and since arrow speed is in excess of 40 ms^{-1}, the arrow will travel too far to be able to accurately measure the speed).

Pros

– Accelerometer: accelerometer will show the acceleration the arrow underwent in initial stages of arrow launch to minimize air resistance intervention;
– Displacement: very straightforward, easy.
– Doppler: no human intervention necessary other than shooting the arrow;
– Gates: no human intervention necessary other than shooting the arrow; and
– High-speed camera: can revisit each take to do it again, and can see where (if) it went wrong. Readily available (at school).

Cons

– Accelerometer: expensive (non-rentable), needs to be wireless and small, and difficult to attach to the arrow since it influences the weight, causing a systematic error, and besides that will also affect the behaviour of the arrow;
– Displacement: high random error probability to take into account: wind velocity, air friction, not always the same angle of firing. Is overall not useful as many other factors could affect the final arrow displacement;
– Doppler: uses radar to reflect off of the arrow, which needs to have a sufficient amount of visible surface area, low availability;
– Gates: will require a room where it is safe to shoot arrows somewhere in the vicinity, low availability;
– High-speed camera: camera will only make a recording which needs to be analyzed. Analyzing is done by placing data points (markers) in the film, frame by frame, in order to find the displacement. Time is obtained by knowing how much frames per second the camera has filmed at, and how much frames were used to find the displacement, not practical to be concise.

I have opted for the high-speed camera, as there was a low availability of a Doppler radar or of any of the gates. Since the displacement option would generate big uncertainty values due to the timing issue and the air resistance intervention and would also not give a real answer to my question, and since it only gives the average speed, instead of the speed which required all the work, I have chosen not to use the displacement as a factor of measuring speed. The accelerometer was also not an option, since the higher-end types (which would be able to handle the forces on the arrow, up to 55

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m/s in under 1/30th of a second, about *150 g*[6]) that were small enough to fit on an arrow or on the string, *and* wireless, would be very expensive. I haven't been able to find any yet, but the ones I have tracked down which range to about 100 g range from about $90 and more[1], and that is the bulky, wired one. This will probably mean that the accelerometers which can be used for my experiment would cost a lot more. Also, a high-speed camera was available through school, which was definitely an advantage.

Independent variable

Drawlength

Dependent variable

Arrow speed

Control variables:

– Weather and air friction (which I cannot control, although I can anticipate);
– Shooting technique
– Arrow mass (mass of stickers which indicate proper drawlength);
– Angle θ (which is the angle between the direction of the arrow and the vision of the camera).

Setup

I had the high-speed camera, which is a Casio Exilim EX-F1, set on a tripod, about 1.70 m high, eye height. The camera's line of sight will have to form a 90° angle with the shooting line of my bow, as the camera won't register any movement along that line. There also was a "pink screen" (same as a blue- or green-screen, but it's pink) next to me, to make analysing the films a lot easier. I shot a distance of 10 metres, to minimize walking distance. I shot outside at my local archery club, Handboogschutterij Almere (HBSA). Outside because the camera will detect the 50 Hertz frequency of the AC light, which is in use inside the club.
I have a picture of the setup in my fourth Appendix for clarity.

Analyzing

I used the program Logger Pro (which is graphing software with a tool to analyze movies frame by frame by setting data points on the area of interest, which, in this case, is the nock of the arrow to

[6] $g = 9.81\ m/s^2$

show the displacement).

I have a sample movie of four frames which I would typically use to analyze the speeds in the third Appendix, showing the arrow as a complete blur.

As analysing frame by frame will give errors, I will analyse each film three times to find the uncertainty in my measurement, and thus, in arrow speed and in the kinetic energy in the arrow.

7. Results

7.1 Work Experiment

The results of the work experiment taken from four trials (the whole data set can be found in Appendix one):

	Draw (cm)	Average Force (lbs)	Average Force (N)
Uncertainty	0,5 cm	0,4 lbs	1,78 N
	20,0	19,7	87,60
	25,0	21,8	97,06
	30,0	24,1	107,35
	35,0	26,4	117,36
	40,0	29,1	129,32
	45,0	31,8	141,28
	50,0	35,3	156,85

Table 7.1.1

The uncertainty in drawlength was determined by taking half of a 1 pound interval on the scale (which was one centimetres in width).

The uncertainty in the Average force (lbs) was found by standard deviation. The uncertainty in Average force (N) was found by multiplying this number by 0.453 (the amount of kilograms in a pound).

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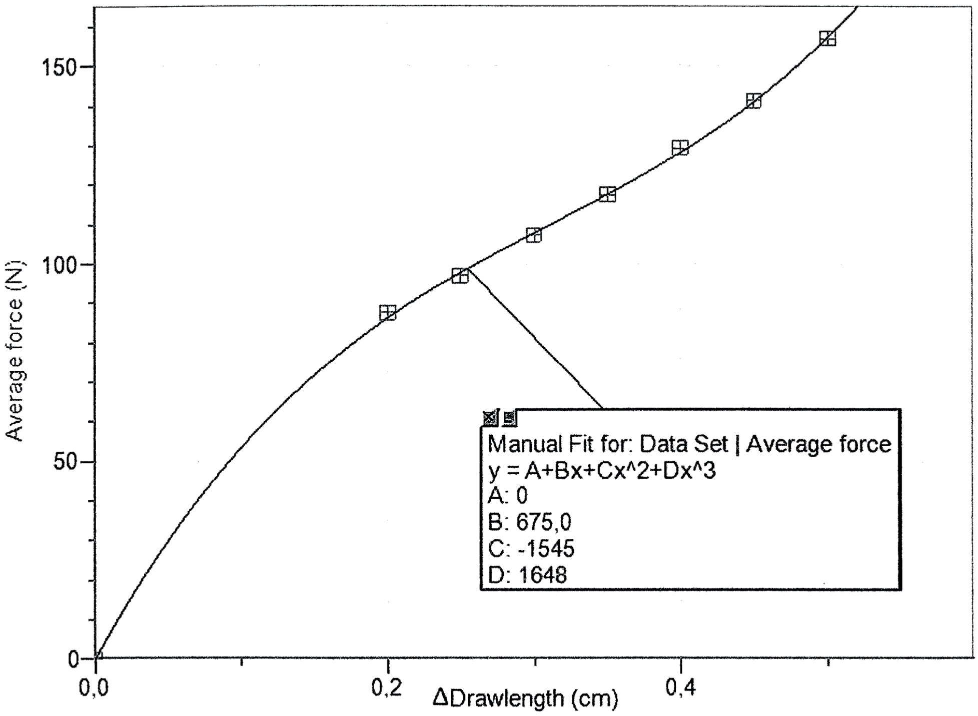
The Average Force (N) versus the Drawlength:



Graph 7.1.1

This graph clearly shows a non-linear relationship in the draw-work. Thus my bow is not the type of spring which behaves according to Hooke's law, since a spring that would, would have a linear extension-force relationship. In Bob Kooi's article[2] a similar curve is presented to, confirming the validity of my experiment. This also means that Hooke's law could not be used to find the work exerted on the bow, but instead the formula for the line must be integrated to find the work exerted on the bow.

On the basis of Bob Kooi's article [2], in which he did measure the drawlength for values from no draw up to full draw, in which showed that the curve passes through the origin, I conclude that I can manually fit mine through the origin as well.

The equation for the line was analyzed by Logger Pro, and was found to be:

$$y = 1648x^3 - 1545x^2 + 675x$$

The y-intercept is 0, since there is no force exerted on the string when it is not drawn. So for the efficiency not to change, the slope of the energy exerted on the bow must be parallel (the same

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exact line in this case, since both the work and kinetic energy curves go through the origin) to the slope of the kinetic energy in the arrow.

To find this energy, integrating the formula would do the trick. Here is the integration:

$$y = 1648x^3 - 1545x^2 + 675x$$

$$W = \int_0^{drawlength} y\,dx$$

$$W = \int_0^{drawlength} 1648x^3 - 1545x^2 + 675x\,dx$$

$$W = [412x^4 - 0515x^3 + 337.5x^2]_0^{drawlength}$$

Where "W " is the energy put into the bow. So to find the efficiency, the measured drawlengths must be substituted as *"drawlength"* in the integral.

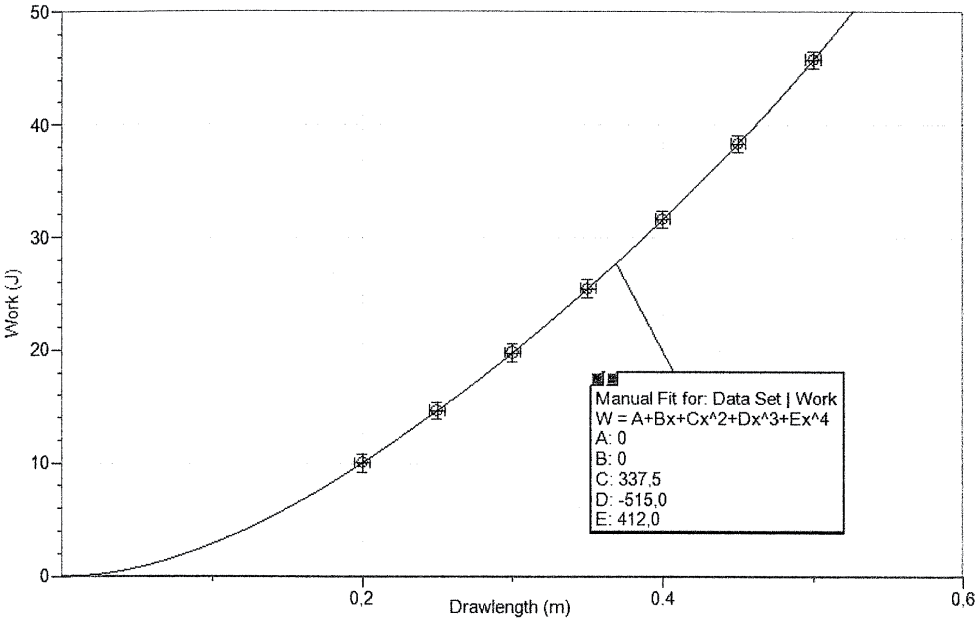
After substituting each drawlength:

	Drawlength (m)	Work (J)
Uncertainty	0,05 m	0,78 J
	0,20	10,04
	0,25	14,66
	0,30	19,81
	0,35	25,45
	0,40	31,59
	0,45	38,31
	0,50	45,75

Table 7.1.2

The uncertainty in the Work was found by substituting the uncertainty of the drawlength into the integral.

Work (J) versus Drawlength:



Graph 7.1.2

This diagram has shows the relation between the work and the drawlength. It is derived from the data of the average force versus the drawlength by integrating the equation (graph 7.1.1).

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7.2 Speed Experiment

The results from the speed experiment, with at least five trials per drawlength, each trial analyzed three times (the whole data set can be found in Appendix two) are:

	Drawlength (cm)	Average speed (cm/s)	Average Speed (m/s)
Uncertainty	0,5 cm	300,0 cm/s	0,30 m/s
	20,0	2497,682	24,98
	25,0	2967,949	29,68
	30,0	3624,443	36,24
	35,0	4021,378	40,21
	40,0	4641,488	46,41
	45,0	5165,749	51,66
	50,0	5541,519	55,42

Table 7.2.1

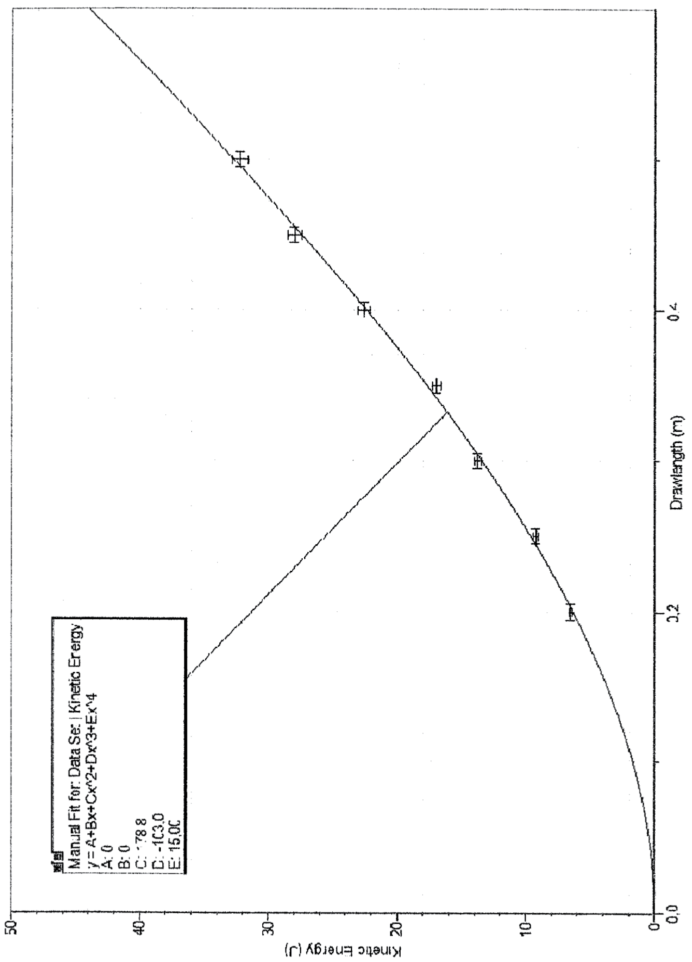
Uncertainties were found using standard deviation. A discussion is included in paragraph 7.4.

From this, the kinetic energy per drawlength can be calculated using the mass of 21 grams (with 0.25 grams as uncertainty) as "m":

	Drawlength (m)	Average Speed (m/s)	Kinetic Energy (J)
Uncertainty	0,005 m	0,30 m/s	1,93%
	0,20	24,98	6,55
	0,25	29,68	9,25
	0,30	36,24	13,79
	0,35	40,21	16,98
	0,40	46,42	22,62
	0,45	51,66	28,02
	0,50	55,42	32,24

Table 7.2.2

Kinetic energy versus Drawlength:



Graph 7.2.1

As we can see here, the kinetic energy has a fluctuation (which stays in the uncertainty range) due to a fluctuation in the speed.

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7.3 Efficiency

Now for the efficiency. The formula for the efficiency is:

$$Efficiency = \frac{\frac{1}{2}mv^2}{W}$$

For the different drawlengths, these are the calculated efficiencies:

	Drawlength (m)	Work (J)	Kinetic Energy	Efficiency (%)
Uncertainty	0,005 m	0,78 J	1,93%	5,28%
	0,20	10,04	6,55	65,24
	0,25	14,66	9,25	63,09
	0,30	19,81	13,79	69,63
	0,35	25,45	16,98	66,72
	0,40	31,59	22,62	71,61
	0,45	38,31	28,02	73,14
	0,50	45,75	32,24	70,48

Table 7.3.1

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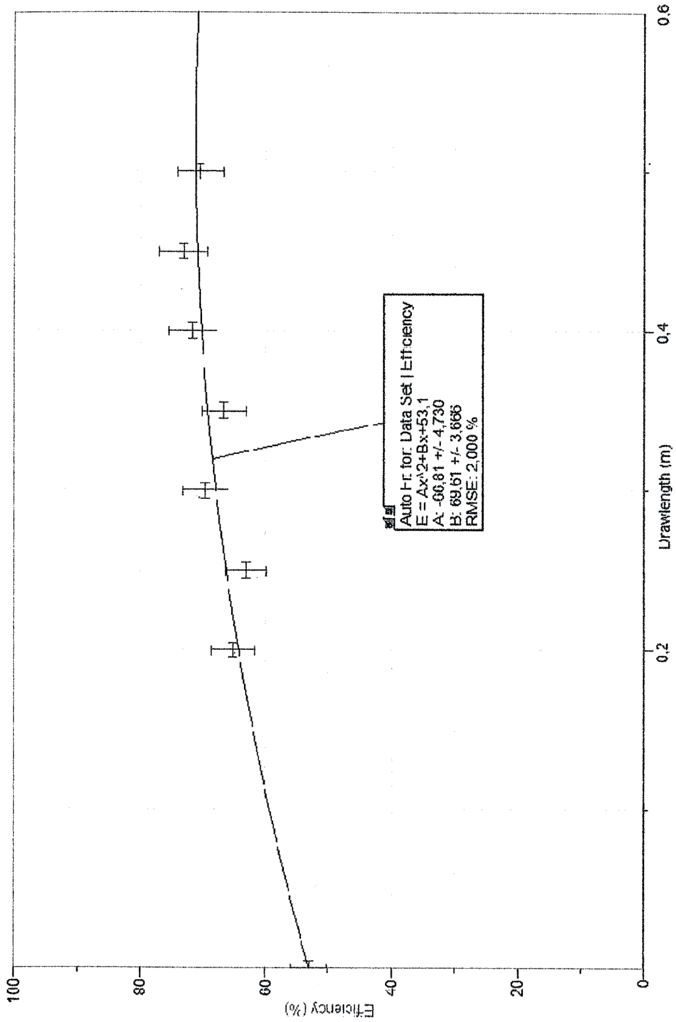
The results can be shown in the following graph:



Graph 7.3.1

The efficiency will be 53% when the drawlength is 1 mm. This was calculated by using the derived equations for the kinetic energy and work.

Due to the fluctuation in the kinetic energy, the efficiency will also fluctuate.

Using the above graph, a peak in the efficiency can be found (optimal drawlength) by differentiating the formula and equating to zero. Here is how the local maxima and minima are found:

$$y = -66.81x^2 + 69.61x + 53.1$$

$$\frac{dy}{dx} = -133.62x + 69.61 = 0$$

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acceleration of the arrow, the arrow will travel less fast due to this change.

Human factors

– When the bow has been drawn to a certain length, the arrow will not have a measurable acceleration due to the fact that there is something countering the force (me, in this case) which is exerted by the bow. When the arrow is about to be released, my muscles relax too slowly for the string to leave the position where its velocity will be unaffected: the string will slide down the tips of my fingers at a slower rate than it would have if wouldn't have slid down my fingers. This will increase the uncertainty of the results. Also the effective draw length will have decreased, as the tips of my fingers will still exert a counter force (although with a lesser magnitude) on the bowstring. Therefore, the measurements of the arrow speed will not be in sync with the work measurements. This will be a systematic error, as it will slide down the same length every time.

This could have been solved by using a rig which would hold the bow and the string. However, this was neither practical nor easy work.

– When selecting proper data points in Logger Pro, it was not feasible to pick the same points on the arrow on each successive frame. This was because of the blur. To minimize the effects, I have measured each recording three times.

– Pulling out my bow to a certain afore-defined drawlength (these 5 cm intervals have been marked off on my arrow), I was never able to *exactly* hit that mark. Thus the arrow speed changed accordingly. Since the camera only filmed from the belly of the riser (that's the part near the string) to the string, it was not possible to spot the exact drawlength. This was minimized by repeated trials.

Environmental factor

– Wind influences the speed of the arrow, since it controls the amount of drag. Changes of the wind velocity affect the consistency of the experiment. As stated before, it was not possible to do the experiment inside, and thus eliminate the wind factor.

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8. Conclusion

This essay treats the following research question:

"How does efficiency change when drawlength is increased?"

There is a definite change in efficiency when drawing the bow: it is low at low draw, increases, has a peak just before full draw at 52 centimetres, and starts decreasing from there. The graph is not a perfect parabola, because it does not show symmetry after the peak.

In Bob Kooi's article [2], a similar graphical shape for the work (W_{input}) is presented. On the basis of this I conclude that my measurements for the work are valid.
Unfortunately, no other experiments for determining the speed of the arrow in relation to the drawlength have been conducted and are available in literature.

An explanation for the efficiency changes can be found in the fact that the energy that does not go into the arrow has gone into other parts of the bow, including, but not limited to, the bow limbs, the string and the movement of the riser and into the archer holding the bow. Some of these efficiency decreasing factors could have been eliminated by making a sturdy rig to hold the bow and release the string. Also, some of the energy goes into the bending of the arrow, which does not contribute to the final velocity [3].

However, I am not able to confirm any change in efficiency, as I am the first to concentrate only on efficiency *change*: other people such as Kooi [2][4] have worked with efficiency depending on the material and types of bow, but only at full draw, not at intervals in the drawlength.
Also, in contradiction to experiments conducted by other people [2][3][4], I measured the speed of the arrow directly after it had left the string. This leads to more valid data.

9. Appendix One

All of the measurements which were taken with the work experiment:

	Set One		Set Two		Set Three		Set Four	
Draw (cm)	Force (lbs	Force (N)	Force (lbs	Force (N)	Force (lbs	Force (N)	Force (lbs	Force (N)
20	20	88,99482	19,5	86,76995	19,5	86,76995	19,75	87,88239
25	21,5	95,66943	22	97,89431	21,75	96,78187	22	97,89431
30	24	106,7938	24	106,7938	24,5	109,0187	24	106,7938
35	26	115,6933	26,5	117,9181	26,5	117,9181	26,5	117,9181
40	29	129,0425	28,75	127,9301	29,5	131,2674	29	129,0425
45	31,5	140,1668	31,5	140,1668	32	142,3917	32	142,3917
50	35	155,7409	35,5	157,9658	35,25	156,8534	35,25	156,8534

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10. Appendix Two

All of the measurements which were taken during the speed experiment. The data point values are x-axis values; displacements in the film. The drawlengths are in groups of three, because that is just one film.

Drawlength	Data point 1	Data point 2	Data point 3	Data point 4	Distance	Speed
20 cm	17,89444423	15,91042412	14,06811973	11,83609711	6,058347119	2423,339
20 cm	17,68960386	15,89036367	13,84891807	11,7728717	5,91673216	2366,693
20 cm	17,82775932	16,00306845	13,88046887	11,7206307	6,107128618	2442,851
20 cm	19,05122751	17,12494624	15,0763614	12,99720067	6,054026847	2421,611
20 cm	17,42578697	15,47374362	13,2134829	11,12445406	6,301332911	2520,533
20 cm	19,07719605	17,08491519	15,01884615	13,10035347	5,976842579	2390,737
20 cm	16,43357842	14,55741688	12,48166368	10,4458288	5,98774962	2395,1
20 cm	16,62931459	14,8230298	12,60622574	10,47152554	6,157789049	2463,116
20 cm	16,73490397	14,63865792	12,58052543	10,33182511	6,403078864	2561,232
20 cm	20,37525346	18,34887321	15,96063934	13,78951764	6,585735829	2634,294
20 cm	17,37430181	15,46495559	13,25610409	11,23444339	6,139858422	2455,943
20 cm	17,26626426	15,21807158	13,0601543	10,82908728	6,437176983	2574,871
20 cm	17,80812783	15,35615294	13,24368226	11,0180435	6,790084328	2716,034
20 cm	15,18971273	13,03044727	10,90716958	8,855867392	6,333845339	2533,538
20 cm	15,64284597	13,66425698	11,7197816	9,229488562	6,41335741	2565,343

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Drawlength	Data point 1	Data point 2	Data point 3	Data point 4	Distance	Speed
25 cm	20,61724037	18,10248842	15,64982912	13,1661235	7,451116865	2980,447
25 cm	24,16567411	21,71975568	19,3275937	16,61289302	7,552781093	3021,112
25 cm	20,47460623	18,00332002	15,58819941	13,1169132	7,357693028	2943,077
25 cm	21,05051605	18,61146501	16,2627492	14,00436861	7,046147445	2818,459
25 cm	22,11655651	19,44050258	17,38199955	15,05883185	7,057724661	2823,09
25 cm	23,64564204	21,46838341	19,07646548	16,74587877	6,899763266	2759,905
25 cm	18,1406523	15,5781294	13,0156065	10,5535747	7,587077601	3034,831
25 cm	18,85103448	16,09241379	13,43724138	10,88551724	7,965517241	3186,207
25 cm	21,7214008	19,19992118	16,39190979	14,15696194	7,564438862	3025,776
25 cm	20,71638965	18,41197205	15,84790176	13,05663537	7,659754271	3063,902
25 cm	20,70960326	17,97717852	15,54502024	13,02278202	7,686821237	3074,728
25 cm	20,43944821	17,83998786	15,42620325	12,82674289	7,612705315	3045,082
25 cm	18,79487843	16,35389827	13,91291812	11,4050618	7,389816632	2955,927
25 cm	18,91755929	16,40809815	13,76371975	11,33520897	7,582350317	3032,94
25 cm	15,5194732	13,12657808	10,5084693	7,974815645	7,544657558	3017,863
25 cm	17,81324121	15,38173136	10,3363484	10,3363484	7,47689281	2990,757
25 cm	17,81247423	15,31795534	12,79661367	10,27527199	7,537202231	3014,881
25 cm	20,95226419	18,68614598	16,181489	13,72453979	7,227724402	2891,09
25 cm	19,14932894	16,56110484	14,00229238	11,47289155	7,676437386	3070,575
25 cm	19,10191404	16,42692608	13,95193722	11,50194824	7,5999658	3039,986
25 cm	18,79771285	16,25374319	13,75917101	11,3880925	7,409620347	2963,848
25 cm	17,80775049	15,30797269	12,90818599	10,50839929	7,299351198	2919,74
25 cm	17,72215563	15,23976511	12,94223346	10,6182934	7,10386223	2841,545
25 cm	17,68320465	15,35606781	12,87716116	10,67649914	7,006705512	2802,682
25 cm	23,19083883	20,62935797	18,30380298	15,8771369	7,313701934	2925,481
25 cm	23,42925324	21,04046628	18,38078595	16,13975901	7,289494232	2915,798
25 cm	26,78083597	24,20438514	21,54570715	19,35298303	7,427852937	2971,141
25 cm	21,39506815	18,96184079	16,49528155	13,86206291	7,533005236	3013,202
25 cm	20,97306398	18,70301788	16,1627282	13,62243852	7,350625453	2940,25
25 cm	21,12521361	18,69785922	16,19303607	13,73985876	7,385354846	2954,142

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Drawlength	Data point 1	Data point 2	Data point 3	Data point 4	Distance	Speed
30 cm	51,72320422	48,53613697	45,6388031	42,66903588	9,05416834	3621,667
30 cm	48,64347182	45,45047358	42,25747534	39,43575596	9,207715861	3683,086
30 cm	49,50886715	46,51411813	43,27655161	40,20086342	9,308003732	3723,201
30 cm	53,39817587	50,38560293	47,2448354	44,42455435	8,973621528	3589,449
30 cm	54,76348909	51,58141535	48,73080762	46,0127863	8,750702789	3500,281
30 cm	54,1878968	50,93287972	48,42380406	45,50785126	8,68004554	3472,018
30 cm	51,69840919	48,89124336	46,01561007	42,9345744	8,763834794	3505,534
30 cm	52,24326947	49,04735829	45,90955459	43,2366107	9,00665877	3602,664
30 cm	53,1149814	49,84155723	46,50132849	43,96275464	9,15222676	3660,891
30 cm	51,29633395	48,5156541	45,8379624	42,33636556	8,959968386	3583,987
30 cm	51,80306568	48,60647312	46,06224639	42,86565383	8,937411845	3574,965
30 cm	54,74088765	51,66800253	48,53109897	45,33017697	9,410710675	3764,284
30 cm	16,17104054	13,14948418	10,16041768	7,366290301	8,804750238	3521,9
30 cm	15,83285437	12,76790092	9,702947471	6,606396565	9,226457801	3690,583
30 cm	20,30125435	16,91815857	13,87002277	10,62091098	9,680343365	3872,137

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Drawlength	Data point 1	Data point 2	Data point 3	Data point 4	Distance	Speed
35 cm	17,55692178	14,38449373	11,07106889	7,863391644	9,693530134	3877,412
35 cm	21,90093034	18,67260924	15,12145603	11,86623226	10,03469809	4013,879
35 cm	21,9106836	18,67049059	15,20597653	11,94085896	9,969824635	3987,93
35 cm	17,1410916	13,86916166	10,41545783	6,987721699	10,15336991	4061,348
35 cm	21,29040621	17,93949316	14,44778544	10,89975986	10,39064635	4156,259
35 cm	17,13376727	13,73951313	10,37198541	6,951004855	10,18276242	4073,105
35 cm	18,69148302	15,54060831	12,14260618	8,96084054	9,730642478	3892,257
35 cm	18,60076292	15,29961972	12,1163745	8,96260377	9,638159145	3855,264
35 cm	22,65987193	19,39410409	16,20428434	13,03978062	9,620091315	3848,037
35 cm	17,98175441	14,29598619	10,87990833	7,823417614	10,1583368	4063,335
35 cm	18,06801902	14,60615539	10,88247014	7,856975874	10,21104314	4084,417
35 cm	26,77234945	22,47846822	18,58713836	14,25971119	12,51263826	5005,055
35 cm	17,79415628	14,27625934	10,86259637	7,318640933	10,47551535	4190,206
35 cm	18,02866607	14,43592919	10,84319231	7,500384955	10,52828111	4211,312
35 cm	22,20000734	19,01723009	15,18157546	11,88998531	10,31002204	4124,009
35 cm	14,91474228	11,63867778	8,310196248	4,981714718	9,933027557	3973,211
35 cm	15,0104959	11,68683226	8,207371892	4,961606622	10,04888928	4019,556
35 cm	19,16629011	16,17666774	12,52268485	9,405300851	9,760989255	3904,396
35 cm	17,68560549	14,30648297	10,96016747	7,548237943	10,13736754	4054,947
35 cm	17,9562674	14,69657155	11,04881668	7,763250231	10,19301717	4077,207
35 cm	22,06658747	18,67994878	14,87307026	11,83251143	10,23407603	4093,63
35 cm	16,16701934	12,84467897	9,49039303	6,104161506	10,06285783	4025,143
35 cm	16,19930877	12,76020062	9,57070516	6,187066498	10,01224227	4004,897
35 cm	20,43059027	17,26238887	13,91676819	10,41907385	10,01151642	4004,607
35 cm	18,12323	14,63341992	11,42796474	7,886453769	10,23677623	4094,71
35 cm	18,14934242	14,65948196	11,2988756	7,783164332	10,36617808	4146,471
35 cm	22,50830931	18,87911597	15,6676715	12,29957121	10,2087381	4083,495
35 cm	15,26783906	11,97051339	8,642656937	5,314800479	9,953038581	3981,215
35 cm	15,31511698	11,94151098	8,85782425	5,378793064	9,936323918	3974,53
35 cm	19,29026954	15,98220761	12,62078984	9,419439578	9,870829964	3948,332

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Drawlength	Data point 1	Data point 2	Data point 3	Data point 4	Distance	Speed
40 cm	53,18372678	49,28508389	45,46441386	41,64374384	11,53998294	4615,993
40 cm	52,59000441	48,50145991	44,66451814	40,63887433	11,95113008	4780,452
40 cm	52,87770426	48,83377427	44,91055861	41,2287716	11,64893266	4659,573
40 cm	51,94048367	47,91108171	43,65782408	40,150752	11,78973167	4715,893
40 cm	49,71613678	46,31385642	42,53354491	38,24919186	11,46694492	4586,778
40 cm	53,81376984	49,89390756	46,80553123	42,29175043	11,52201941	4608,808
40 cm	54,35422077	50,32248709	46,53265744	42,90409712	11,45012365	4580,049
40 cm	54,06228932	50,17798901	46,41320563	42,88745612	11,1748332	4469,933
40 cm	54,17920625	50,15022903	46,61712593	42,52616445	11,6530418	4661,217
40 cm	54,41208123	50,12811515	46,48674398	42,77397337	11,63810786	4655,243
40 cm	54,80056581	50,78807701	47,15175903	42,88848968	11,91207613	4764,83
40 cm	54,27716581	50,98396819	46,8345392	42,81683811	11,4603277	4584,131
40 cm	56,1003841	52,2961085	48,26805199	44,38918276	11,71120134	4684,481
40 cm	54,89442186	51,33618162	47,31668801	43,42898108	11,46544078	4586,176
40 cm	54,68840827	51,32135618	47,26842311	43,21549004	11,47291823	4589,167
40 cm	23,29932656	19,67455465	15,78455553	11,80614734	11,49317922	4597,272
40 cm	19,05021885	15,33730491	11,29022871	7,61444391	11,43577494	4574,31
40 cm	23,57596844	19,27005543	15,58448583	11,75295306	11,82301538	4729,206

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Drawlength	Data point 1	Data point 2	Data point 3	Data point 4	Distance	Speed
45 cm	13,25096781	8,907943768	4,455431724	0,221895682	13,02907213	5211,629
45 cm	17,71818096	14,01454781	9,323279165	4,570283298	13,14789766	5259,159
45 cm	17,98766278	14,17830495	9,588341008	5,466740731	12,52092205	5008,369
45 cm	14,07098249	9,940205635	5,809428781	1,558919265	12,51206322	5004,825
45 cm	14,58565984	10,0884037	5,789555918	1,490708138	13,0949517	5237,981
45 cm	14,74531031	10,12883803	5,959121125	1,372432534	13,37287778	5349,151
45 cm	15,06590928	10,38100903	6,435829876	1,230385155	13,83552413	5534,21
45 cm	15,10120833	10,88283118	6,635361784	2,213338848	12,88786948	5155,148
45 cm	15,12107354	10,9907693	6,652464127	2,135872438	12,98520111	5194,08
45 cm	13,19853524	9,07860484	4,774199944	0,40830355	12,79023169	5116,093
45 cm	18,05294252	14,14477063	9,646155496	5,766099945	12,28684258	4914,737
45 cm	17,87756928	14,18811123	10,01244553	5,779578934	12,09799035	4839,196
45 cm	15,20416062	10,70393982	6,615324576	2,334626742	12,86953387	5147,814
45 cm	20,12270128	15,54735555	11,82644185	7,223533793	12,89916749	5159,667
45 cm	20,03258597	15,52350571	11,7199278	7,3335436	12,69904237	5079,617
25 cm	14,66216991	10,31040916	6,203434952	1,8516742	12,81049571	5124,198
25 cm	19,81303056	15,56535845	11,58316585	6,657046119	13,15598444	5262,394
25 cm	19,90307845	15,82164846	11,23376365	6,914001986	12,98907646	5195,631
45 cm	16,99472936	12,492862	7,990994641	3,759916294	13,23481307	5293,925
45 cm	17,08426427	12,71943521	8,543209871	4,313097755	12,77116652	5108,467
45 cm	17,0531295	12,59706024	8,54327501	4,365710078	12,68741942	5074,968
45 cm	14,74130642	10,26328021	5,785253995	1,841032233	12,90027418	5160,11
45 cm	14,51747463	10,26155581	6,3156708	1,806087937	12,7113867	5084,555
45 cm	14,63808699	10,39816736	6,252468157	1,824107648	12,81397935	5125,592
45 cm	15,69731997	11,6064125	7,889836439	2,970052296	12,72726767	5090,907
45 cm	16,21459498	11,77568717	8,021979893	3,880985359	12,33360962	4933,444
45 cm	16,24118599	11,88297919	8,062085561	3,912833884	12,32835211	4931,341
45 cm	16,89403928	12,63742244	8,482153623	4,090406088	12,8036332	5121,453
45 cm	16,84669116	12,65071554	8,394366178	3,98708244	12,85960872	5143,843
45 cm	16,61898567	12,12268821	8,324573584	3,967912691	12,65107298	5060,429
45 cm	18,88237421	14,78345072	10,34295027	6,337184133	12,54519007	5018,076
45 cm	18,91164451	14,78766698	10,57204551	5,745464398	13,16618011	5266,472
45 cm	18,93854982	14,76184157	10,41580731	6,154436051	12,78411377	5113,646

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Drawlength	Data point 1	Data point 2	Data point 3	Data point 4	Distance	Speed
50 cm	19,09880943	14,30262724	9,712416062	5,181053745	13,91775569	5567,102
50 cm	14,2469872	9,967511128	5,535196624	0,308122277	13,93886492	5575,546
50 cm	19,11249619	14,19523603	9,836755436	5,170946081	13,94155011	5576,62
50 cm	22,70724736	17,5971876	8,248101	8,248101	14,45914636	5783,659
50 cm	17,7000169	12,59665329	8,239395472	2,897278008	14,80273889	5921,096
50 cm	17,62041634	12,60026475	8,464165018	3,633453112	13,98696323	5594,785
50 cm	17,53768656	13,03150859	8,555172854	3,899783691	13,63790287	5455,161
50 cm	22,2723356	18,01398288	13,23772241	9,26709623	13,00523937	5202,096
50 cm	17,50695529	12,98866441	8,596759302	3,920486233	13,58646905	5434,588
50 cm	16,43126769	11,86100588	7,425163532	2,828017828	13,60324986	5441,3
50 cm	16,49284242	11,90153589	7,446066827	2,229907929	14,26293449	5705,174
50 cm	16,53916073	11,80591018	7,368487792	2,783151323	13,75600941	5502,404
50 cm	17,70449987	12,93209832	8,187443299	3,969972168	13,7345277	5493,811
50 cm	17,96152464	13,21587072	8,749372919	4,09677104	13,8647536	5545,901
50 cm	17,6969729	13,02121678	8,589944641	4,066991009	13,62998189	5451,993
50 cm	16,73194707	11,6563327	7,431379962	3,007939509	13,72400756	5489,603
50 cm	16,65252371	12,37947222	7,664380913	2,94928961	13,7032341	5481,294
50 cm	16,55035631	11,45863529	7,045810403	2,887571567	13,66278475	5465,114
50 cm	21,67578502	16,73488689	11,90081899	7,547487128	14,12829789	5651,319
50 cm	21,60362603	16,6874812	11,77133636	7,570267144	14,03335888	5613,344
50 cm	21,40927713	16,60067724	11,73414242	7,475924448	13,93335268	5573,341
50 cm	18,16127909	13,57285034	8,428248401	4,145714899	14,01556419	5606,226
50 cm	18,5174857	13,53629834	8,523782753	4,231816283	14,28566942	5714,268
50 cm	18,22015736	13,45285977	8,515301554	4,145278762	14,0748786	5629,951
50 cm	18,27044776	13,61667224	8,537246528	4,167237812	14,10320995	5641,284
50 cm	17,82882498	12,88207107	8,719558645	4,014109812	13,81471516	5525,886
50 cm	17,69154313	13,2528215	8,67538981	3,57085993	14,1206832	5648,273
50 cm	15,93073389	11,76439123	7,371308834	2,836514104	13,09421978	5237,688
50 cm	16,69426229	11,91411467	7,243855491	2,903261671	13,79100062	5516,4
50 cm	16,13531971	11,57474319	7,185188298	2,653115387	13,48220432	5392,882
50 cm	17,4281552	12,78860067	8,231895324	3,454258814	13,97389638	5589,559
50 cm	17,5839418	12,91999119	8,171241473	3,50729086	14,07665094	5630,66
50 cm	17,51994763	12,97943151	8,292447124	3,371113519	14,14883411	5659,534

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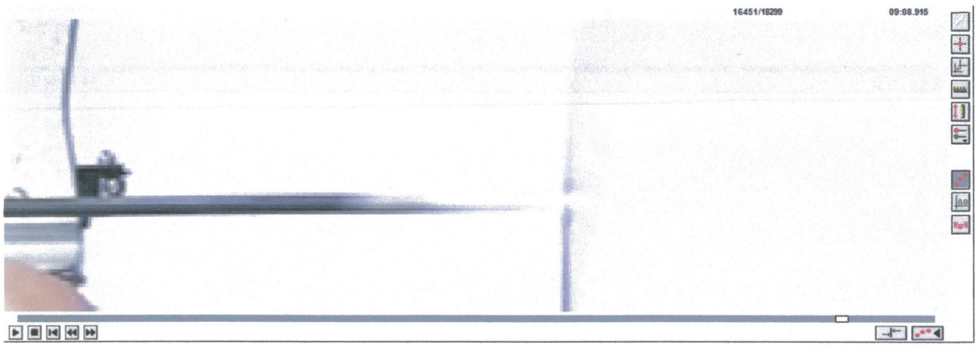
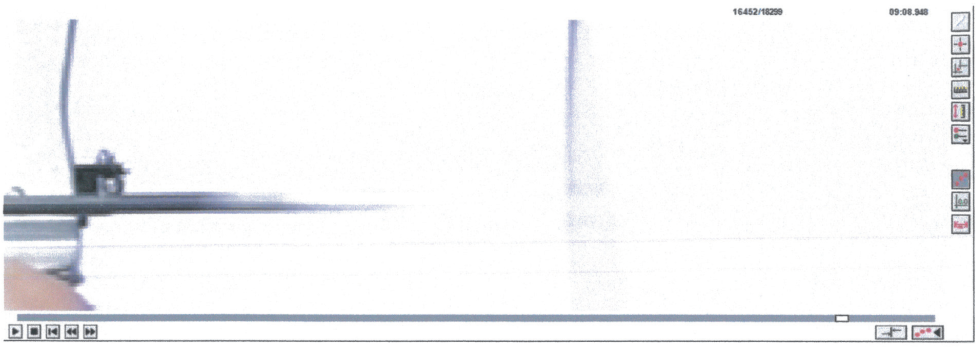
11. Appendix Three

Sample frame in Logger Pro. This was taken at a drawlength of 50 cm. The images have been scaled down. The white, blurred area is the nock of the arrow. This is the worst the blur can get; it gets more when the speed of the arrow increases. It measures about 1 centimetre.

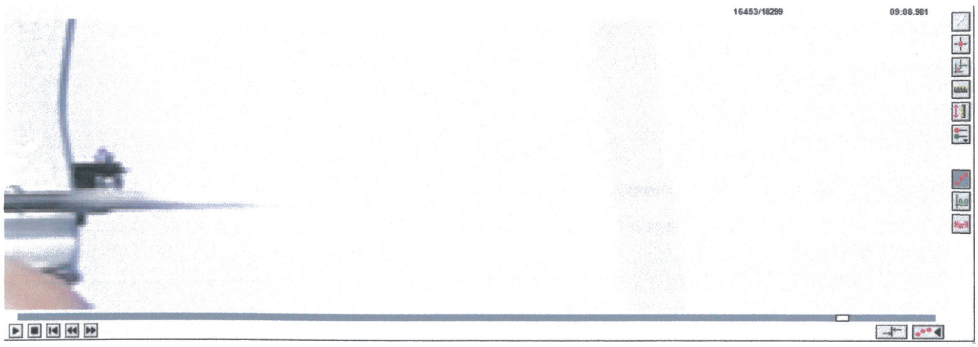
Frame one



Frame two



Frame three



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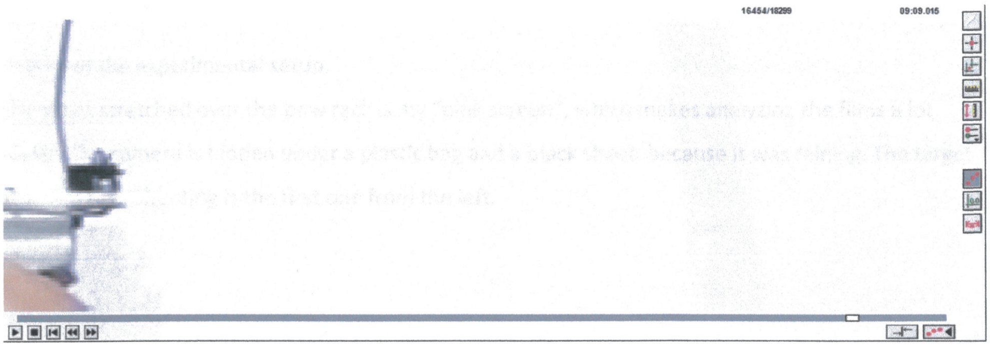
Frame four



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12. Appendix Four

Picture of the experimental setup.

The sheet stretched over the bow rack is my "pink screen", which makes analyzing the films a lot easier. The camera is hidden under a plastic bag and a black sheet, because it was raining. The target at which I am shooting is the first one from the left.

13. Bibliography

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