November 2017 subject reports

Mathematics SL

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0–15	16–32	33–44	45–57	58–71	72–85	86–100

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–2	3–5	6–8	9–11	12–14	15–17	18–20

The range and suitability of the work submitted

The vast majority of the work was suitable and there was a wide range of worthy explorations submitted. The more common topics that often lead to lower attainment were being explored less often and so it appeared candidates might be getting better guidance on avoiding these trivial explorations. However, still some of the work was lacking in-depth analysis being either a historical report or on a typical textbook problem that is not taken any further.

There were samples from some schools in which the candidates all submitted explorations of a very similar style; usually a modelling style using the same regression analysis. This can suggest unhelpful guidance from teachers by not allowing candidates free range to explore the topics that interest them. Many explorations involved linear or other regression models. Most of these were done using technology which in itself is not a problem when the reasons for the choice of model are discussed in detail and the candidate does not just blindly try to find the best fit without justification. In many cases the candidates were able to show their understanding of the process involved through some calculation or analysis of the results as well as applying it to the real-life situation under investigation.

Some candidates attempted advanced mathematics above the scope of the course and were often, therefore, unable to demonstrate their knowledge and understanding. Their work, in many of these cases, was simply a duplication of external sources. They might think that extra marks could be obtained by doing more difficult mathematics however this harder material often resulted in the opposite effect with candidates losing marks not only in Criterion E but also in A and B as well.

Candidate performance against each criterion

Criterion A: The key to writing a good exploration is to have a clear aim. This allows all the other criteria to follow. Candidates should therefore try to avoid vague aims e.g. 'I want to look into this topic'. Otherwise this criterion was high scoring with at least some organization and coherence and all the necessary elements in place. Candidates should be advised not to make their introductions too lengthy with unnecessary back stories and contrived rationales. Candidates should also avoid pages of repetitive calculations as this obviously affects the conciseness of the piece.

Candidates mostly include inline citations however there are still some candidates who only have a bibliography and did not cite sources of ideas and images in the text where those things occurred. Teachers are advised to ensure candidates correct this between the initial and final drafts of the work.

Criterion B: This criterion is well understood by teachers and candidates. Candidates were using a wide variety of mathematical presentation tools, with some very professional looking explorations. Still, there were a few cases in which inappropriate and/or inconsistent notations and undefined symbols and variables were evident. There was also many poorly scaled or labelled graphs. Issues of accuracy and use of appropriate approximation sign is still a concern.

Criterion C: Candidates continue to struggle in this criterion. Often "I have always been interested in..." is the only personal engagement shown. Although this provides some evidence of personal engagement this is only minor and does not justify a mark greater than 1 out of 4.

Criterion D: Reflection needs to be more than just descriptive to reach the higher levels. Simply stating results or commenting on results does not constitute a critical analysis. In general, reflections need to be more in depth. It was uncommon to find any reflections which discussed the validity, limitations and implications of the results and mathematical processes, and so very few candidates achieved a level 3 in this criterion.

Criterion E: **Demonstrating** understanding is key to receiving the highest levels. Work that relies heavily on the use of GDC by entering data and writing down results with no explanation given would not be sufficient since it is not just the answer but also the **reasoning** and **explanation** that are essential for the top levels. The majority of candidates do choose to use relevant mathematics commensurate with the level of the course. There were many attempts at using maths above Mathematics SL level with varying degrees of success. Problems occurred when candidates tried to connect maths with art or social sciences where the maths was above their level. Regression models were popular. As mentioned before this is not a problem unless candidates do nothing more than let the technology do the work for them without showing any understanding themselves or justifying their choice of model.

Recommendations for the teaching of future candidates

Schools should encourage a variety of exploration types rather than guiding candidates in a single direction.

More explicit teaching of personal engagement and reflection strategies should be provided by spending more time on criterion C and D with examples, explanations and discussions. The higher levels are hard to earn, mostly because the candidates do not really know what to do to earn them. Hence, the exploration process should be started earlier so there is time for reflection, peer reviewing, and input from the teacher can be built in to activities earlier in the course.

More care needs to be taken in presenting tables and graphs and make sure that the notation is correct and variables are defined. Perhaps spending time looking at Mathematics based articles would help here.

Candidates need to better understand what meaningful and critical reflections are. Opportunities should be provided for them to write what they think, what they have learnt, what they could do to improve their work, to consider limitations of their approaches etc.

Teachers should remind candidates that the exploration is a piece of mathematical **writing** and should read smoothly and clearly throughout.

Care must be taken in helping candidates to choose a suitable area of study that is within their mathematical grasp. High levels in criterion E are not awarded just because the mathematics is hard.

Further comments

Some teachers used bullets at the beginning of the exploration explaining how different levels were awarded and these were helpful, but difficult to read because the examiner had to keep referring back. It is more helpful for teachers to add notes in the margin, alongside the candidates' work. Many of the explorations were still not marked by teachers. They showed no annotations/comments on the candidate's work and some errors had not been noted.

Teachers should take care when uploading the explorations – for example diagrams may need to show colour if this is mentioned in the text. There were others that were quite illegible, difficult to read, pages in the incorrect order etc. Comments written in pencil may not scan well and should be checked.

Teachers from schools where several teachers mark the candidate's work should ensure that there is internal moderation between the various teachers involved in the marking to ensure consistency across the whole sample.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–14	15–28	29–37	38–50	51–63	64–76	77–90

The areas of the programme and examination which appeared difficult for the candidates

- Range of a function
- Inverse functions
- Finding the length of a semicircle
- Working with limits, behaviour of exponential functions
- Optimization
- Working with a combination of topics in a question, e.g. discriminant and logarithms
- Finding the area between two functions
- Recognizing patterns involving geometric progressions

The areas of the programme and examination in which candidates appeared well prepared

- Probability of successive events and tree diagrams
- Arithmetic sequences
- Application of cosine rule
- Composite functions
- Derivatives and integrals involving polynomials
- Basic working with vectors – finding a vector between two points; the vector equation of a line

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Tree diagram, probability

Most candidates were able to answer both parts of the question correctly. There were a few arithmetic errors seen, and some candidates confused “exactly one” green ball with “at least one” green ball.

Question 2: Arithmetic sequence

Nearly all candidates answered this straightforward question with no problems. Candidates correctly selected and used the appropriate formulas from the booklet. Again, there were some arithmetic errors. Some candidates incorrectly found a positive common difference in part (a), but went on to use this value correctly, earning follow-through marks in parts (b) and (c).

Question 3: Range and inverse of a function

While a good number of candidates were able to earn full marks on this question, there were many who struggled with it. In part (a), while many candidates seemed to recognize the range of the function, some expressed the values using x rather than y, and some were not able to express this range using notation that included the end values of 0 and 7. Some candidates seemed unfamiliar with the notation in part (b), especially part (b)(ii), though many were able to determine the correct values using the given graph. In part (c), a number of candidates incorrectly reflected the given graph across the x-axis or y-axis, rather than reversing the known coordinates and reflecting the graph across the $y = x$ line.

Question 4: Trigonometry

In part (a), nearly all candidates recognized that the cosine rule was required and substituted correctly, although some were unable to use $\cos\frac{\pi}{3} = \frac{1}{2}$ to show the required result. In part (b), it was quite surprising that many candidates were unable to find the correct length of a semicircle, even with the correct diameter given in part (a) of the question. Among those who were able to find the correct perimeter of the shape, there were some who did not give the answer in exact form, as specified in the question.

Question 5: Composite functions and limits

In part (a), virtually all candidates had an appropriate method for the composite function, although a small number of candidates gave the function for $(f \circ g)$, rather than $(g \circ f)$. Unfortunately, most candidates were not successful in part (b) of this question. Many did not apply the limit at all, and among those who did, many gave the $\lim\limits_{x \to \infty} \mathrm{e}^{-x}$ as 1, rather than 0, and others simply substituted e^{0}, ignoring the ∞ altogether. It was pleasing to see that some candidates were successful using a graphical approach, translating the horizontal asymptote of the parent function $y = \mathrm{e}^{x}$.

Question 6: Optimization

This question was very poorly done by the majority of candidates. Very few were able to find a function for the area of the rectangle in terms of x, although those who did were nearly all able to earn full marks here. There were a multitude of incorrect approaches to this question which earned no marks, including simply integrating $f(x)$, attempts to find the roots of the function, and assuming that the rectangle was a square.

Question 7: Discriminant with logarithmic equation

A good number of candidates were able to earn a mark for correctly rewriting the equation in the form $k^2 = 6x - 3x^2$, and many recognized the need to use the discriminant, but some were unsuccessful beyond this point. The majority of candidates who set one side of their quadratic equal to zero and set the discriminant equal to zero were able to earn full marks.

Question 8: Calculus

In part (a), nearly all candidates were able to find the correct derivative of $f(x)$ and use this to show that $f'(1) = 1$. Parts (b) and (c) were also answered successfully by a large number of candidates, with most recognizing that the gradient of the normal line was the negative reciprocal of $f'(1)$. However, a few candidates substituted the coordinates of P in the wrong order. Surprisingly, part (d) was not as well done. Although most candidates earned a few marks for recognizing the need to integrate, many made errors by overcomplicating the problem by breaking the area into many parts, rather than using the simpler method of subtracting the functions and integrating from $x = -1$ to $x = 1$.

Question 9: Vectors

Nearly every candidate correctly found $\overrightarrow{\mathrm{AB}}$, and most were able to find a correct equation for the line. In part (a)(ii), a typical error was to write the equation using the form $L =$, rather than in correct vector form. Although part (b) was not as well done as part (a), it was pleasing to note that candidates were able to find the value of p using a variety of valid methods. A smaller number of candidates were able to earn full marks in part (c). While most recognized the need to use the scalar product of vectors, many failed to find the vector $\overrightarrow{\mathrm{DC}}$ or $\overrightarrow{\mathrm{CD}}$, and instead used $\overrightarrow{\mathrm{OD}}$ in their scalar product.

Question 10: Infinite geometric series

Many candidates were able to recognize the geometric pattern in part (a), with the correct common ratio of p. While a large number of candidates earned full marks in this part, there were some who worked backwards with the given value of $\frac{2}{3}$ for p, and therefore did not earn all the available marks for this "Show that" question. Although a good number of candidates also earned full marks in part (b), this part of the question proved to be more difficult for most. Many candidates failed to recognize that there were two geometric series here, one for the sum of the squares and one for the length of the line segment. Poor notation also kept some candidates from earning marks here, as it was often unclear which series they were attempting to work with.

Recommendations and guidance for the teaching of future candidates

More emphasis should be placed on writing a coherent solution using correct mathematical notation. Candidates often seemed confused by their own working, and poor notation often kept them from communicating their thinking clearly. When practicing examinations in class, it is also important to remind candidates about the importance of reading a question carefully. This includes paying attention to things like command terms and restrictions on variables.

As always, it is imperative that teachers and candidates be familiar with the entire Mathematics SL syllabus. In this paper, it is clear that certain topics and techniques, including informal treatment of limits, optimization problems, and finding areas between functions are among topics that are not being covered well by some schools.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–16	17–32	33–43	44–54	55–66	67–77	78–90

The areas of the programme and examination which appeared difficult for the candidates

- Graphing a function in a given domain
- Conditional probability
- Volume of revolution
- Finding the coefficient of a term in a binomial expansion
- Properties of symmetry of the normal distribution
- Recognizing and applying the binomial distribution
- Relationship between acceleration, velocity and distance travelled

The areas of the programme and examination in which candidates appeared well prepared

- Sine rule and area of a triangle
- Analysing key features of the graph of a function.
- Finding magnitude and angle between two vectors
- Linear regression and using the regression equation to make a prediction
- Integration of a polynomial

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Sine rule, area of a triangle

Most candidates were able to demonstrate a good knowledge of these topics, and were able to answer both parts of this question correctly. A few candidates had their calculator set to radians but did not first convert the angles from degrees, which resulted in an incorrect, and inappropriate negative value. A minority attempted to answer one, or both, parts using only right-angle trigonometry. This approach was generally unsuccessful.

Question 2: Functions

Parts (a) and (b) of this question were answered reasonably well with many candidates able to earn the majority of the marks. While most were able to do so through effective use of their GDC, some had difficulty rounding values correctly, or giving their answers to three significant figures.

In part (a), many of those who attempted an analytical approach were often unsuccessful. The most common incorrect answer seen was ± 0.816, where the given domain was not considered. A few found the y-intercept rather than the x-intercept.

In part (c), it was clear that the majority of candidates were able to accurately enter the function into their GDC. However, few candidates considered carefully the domain when sketching the function, the exact location of the maximum turning point, and/or did not clearly show a change of concavity on the graph of the function between the maximum turning point and $x = 7$. This was disappointing as similar questions have appeared regularly in past examinations.

Question 3: Angle between two vectors

This question proved very straightforward for the majority of candidates. In part (a), the most common error resulted from incorrect arithmetic, where a few candidates evaluated their correct expression incorrectly. In part (b), most were able to find $\left|\overrightarrow{\mathrm{AC}}\right|$ and $\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{AC}}$. When finding the angle, there were a few candidates who either substituted incorrectly into the scalar product formula, or attempted to use the cosine rule. Most were able to obtain the correct answer in degrees; few worked in radians.

Question 4: Probability distribution of a discrete random variable, conditional probability

In part (a), while most candidates knew to equate the sum of probabilities to 1, a significant proportion equated an expression for the expected value of the distribution to 1. Many tried to solve their equation analytically, which often led to algebraic errors. Although many candidates rejected the value -0.2625, some forgot about the context of the question and gave both solutions to the quadratic equation. Most candidates were able to do part (b).

Many struggled with the conditional probability in part (c) with few answering this part correctly. The greatest difficulty was with interpreting $\mathrm{P}(X > 0 \cap X = 2)$; many assumed the events to be independent. Consequently $P(X > 0 \cap X = 2) = P(X > 0) \times P(X = 2)$ was the most common error seen.

Question 5: Volume of revolution

In part (a), most candidates were able to substitute the point into the function and form a correct equation. While some used their GDC to solve the equation, many attempted an algebraic approach. As with other questions where an equation was to be solved, this was often unsuccessful, and was a far less efficient approach to take. The most common incorrect answer seen was ± 2.32.

In part (b), few were able to correctly substitute into the volume of revolution formula, with many either forgetting to square the function before integrating, or squaring only part of the function. Of those who substituted correctly, it was surprising how many did not multiply their answer by π.

Question 6: Binomial theorem

While successful and concise responses were seen, this question proved challenging for the majority of candidates, with many either leaving the question blank or making little progress with it. A common error was to either use ax^2 in the binomial term rather than $(ax)^2$, or to rewrite $(ax)^2$ as ax^2 when attempting to simplify their term. Many forgot to take into account the multiplying factor ax^3. Few solved their equation in terms of a using their GDC, and instead attempted an algebraic approach which often resulted in an arithmetic error.

Question 7: Normal distribution

A significant number of candidates were able to find the standard deviation, which earned the first three marks. However, few were able to make any further progress with this question. The candidates who were successful in finding the value of h, frequently did so with the aid of diagrams. Those that scored well, also often showed an in-depth understanding of the concepts involved, such as the symmetry of areas under the normal curve, while using precise notation.

Many candidates attempted a trial and error approach involving different values of h. However, few obtained all the marks as their solution lacked sufficient rigour. When attempting a trial and error approach, it is important that the candidate communicates how they know their answer is correct. In this question, at least two values for $192 - h$ were required, one which gave $P(192 - h < X < 192) < 0.8$ the other $P(192 - h < X < 192) > 0.8$. Most candidates gave only one value, and stated their final value for h to two significant figures.

Question 8: Linear correlation, cumulative frequency curve, binomial distribution

The majority of candidates were successful in parts (a) and (b). A few attempted to find the equation of a line between two of the given points, believing that to be the equation of the regression line. Many of these candidates earned follow through marks in part (b). Some incorrect answers were seen in part (c), but the majority were able to give the correct answer of 40 hives.

While many candidates were successful in part (d), this part did cause quite a few problems. Common difficulties were with reading values from the graph, and with correctly interpreting the scale on each axis. In part (e), few candidates recognized the binomial distribution. Those who did were generally successful.

Question 9: Kinematics

Most candidates were able to answer part (a) successfully.

In part (b), the majority of the candidates understood that when the velocity of a particle is decreasing, acceleration is negative, and consequently were able to find the correct interval. However, a considerable number of candidates appeared not to know this condition. A common error was to state the interval for which the acceleration decreases.

In part (c), the majority were able to correctly find the velocity function and obtained full marks. A few candidates did not attempt to find the value of the constant of integration. However, few candidates appeared to have considered the graph of v. Doing so would have been helpful, not only in part (d), but also in previous parts with the checking of work.

Few candidates were successful in part (d), with most either not recognizing the times at which the velocity was increasing, or confusing distance and displacement. A common error was with the absolute value missing in the integral, or being used incorrectly eg $\left|\int v\,\mathrm{d}t\right|$ rather than $\int |v|\,\mathrm{d}t$.

Question 10: Trigonometric equations and their applications

Part (a) appeared to be the easiest part for candidates in this question, with many successfully showing that $f(2\pi) = 2\pi$. However, a significant number of candidates were not able to make any further progress after substituting 2π into the function. This was surprising as $\sin\left(2\pi - \frac{\pi}{2}\right)$ is easily evaluated on the calculator. A few candidates appeared to make up a value for a.

Many correct answers were seen in part (b)(i). However, although the question was relatively straightforward, a significant number were unable to obtain the correct coordinates of P_0 and P_1. Of those who did find coordinates in (b)(i), most were able to find the equation of the line. Some candidates used an incorrect notation such as $L = x$.

Part (c) was poorly done. The most common response was one where candidates considered the specific case $\mathrm{P}_1 - \mathrm{P}_0$, rather than the general case $\mathrm{P}_{k+1} - \mathrm{P}_k$. This did not answer the question and was not awarded any marks.

Part (d) also proved quite difficult for many candidates. Whether through insufficient time or a lack of understanding, many candidates simply found $\frac{300}{2\pi}$.

Recommendations and guidance for the teaching of future candidates

It is essential that both teachers and candidates are familiar with the Mathematics SL guide, especially the syllabus content (including prior knowledge), command terms and notation list, so that candidates are adequately prepared for this examination. While candidates have a formula booklet in the examination, they will only be supported by it if they are familiar with its contents. There is little reason for the formulae for volume of revolution and distance travelled to be stated incorrectly by the candidate.

Teachers are encouraged to teach for a deeper understanding of concepts, so that candidates will better remember, for example, when one integrates f and when one integrates f^2, how to recognize a binomial distribution, and knowing when to use the cumulative distribution on their GDC. Most of the difficulties encountered in this paper were with the problem-solving questions (6, 7, 8e, 9d and 10c). Candidates should frequently be given the opportunity to explore, discuss and reflect upon unfamiliar problems in a group setting.

Candidates must have access to a GDC at all times during the course and be given proper instruction on its correct use. There were a number of questions in this paper where candidates were poorly prepared in the use of their GDC. Candidates should be aware of when an analytical approach is necessary and when one using their GDC will suffice. In general, for Paper 2, once an equation has been set up, there is little reason why its solution should not come directly from the GDC. Failure to make use of the GDC when appropriate, could result in candidates having insufficient time to complete the paper. Candidates should be reminded to consider the reasonableness of their final answer before progressing onto subsequent parts. For example, checking that values found are consistent with the information provided e.g. length, areas and probabilities should always be positive values. Candidates need more practice reproducing graphs form their GDCs and graphing over the given domain.

Teachers should emphasize that in general, to ensure a good score, steps indicating the method used must be given. Candidates should be given regular feedback on how they present their solutions, encouraged to show their working, and reminded to clearly indicate to which part of a question a given solution belongs. However, it was encouraging that many candidates, particularly in some of the more challenging questions, communicated their solutions very clearly and with precision.

All teachers should read the subject reports after each session, which continue to repeat recommendations regarding skills that are absolutely essential for Mathematics SL but are still not well understood or applied.