Mathematics SL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 14	15 – 30	31 – 42	43 – 55	56 – 67	68 – 80	81 – 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 2	3 – 5	6 – 8	9 – 11	12 – 14	15 – 17	18 – 20

The range and suitability of the work submitted

There was a good variety of topics as a whole and overall the quality of explorations is improving. There were also a few novel and surprising new ideas. However, sometimes, the topics explored still seem to have been a little too prescriptive in either format or topic suggesting that teachers are over-advising students on what to explore. For instance, apart from the usual typical textbook topics, like the Golden ratio, Monty Hall Problem etc., there were a number of statistical explorations that followed the same format and in some schools this was all that was produced. There were also a lot of physics based explorations which proved difficult to obtain top levels on demonstrating understanding in mathematics. Similarly explorations involving regression techniques were common, with many candidates not demonstrating much understanding and simply using technology to generate functions/models. There were also quite a few attempts on the SIR model in one form or another, and many of these did not manage to achieve good marks as data was often hard to come by. Explorations based on the history of mathematics can run the risk of neglecting mathematical content. Finally, explorations of the modelling type continue to be very popular and often scored well.

In summary, a good exploration requires students to extend pre-existing ideas from their original manner and apply their own creative twists.

Candidate performance against each criterion

Criterion A

The vast majority of student reached at least a level 2, although obtaining a level 4 remains difficult. For a higher mark, students should be encouraged to state their aim clearly or even in a separate paragraph so they know what they are heading to and can refer back to it throughout and at the end of the exploration.

Criterion B

Success here seems to vary considerably by school. In some cases, notations and representations were correct and appropriate. In others students were still not defining terms properly, using computer notation, which were incorrectly condoned by their teachers, and using inappropriate equal signs for approximate values. These are very common mistakes as are inconsistencies in the degree of accuracy used in results. The mathematical presentation must further the aim of the paper if it is to be appropriate. If possible and necessary, a number of different forms should be shown – just having one simple graph or one table may not achieve the higher levels, particularly if it contains errors. However, including graphs that are completely irrelevant does not raise the score in this criterion either.

Criterion C

This remains the area which caused the most difficulty to both students and teachers − students are still struggling to show evidence of their personal engagement and teachers in general are quite varied in their expectation and level awarding for this criterion. Students need to avoid the common textbook topics unless they can develop an interesting extension or perspective to it. There are still some teachers who are equating personal interest in the topic with personal engagement in the mathematics; they continued to mark according to how interested the student is and awarded the top level of 4. This should be discouraged and the teachers should read the descriptor or see examples of good quality work.

Criterion D

Students generally find it difficult to score well on this criteria − very few of them were able to critically reflect on their results, their data collection, their models, and so on. Again, students who reflect at each stage do better than those who leave all of the reflection for the end. In doing so, they use the reflection to drive the exploration and not just describe the results. Simply stating results without consideration of error, validity, and limitations is still common across samples.

Criterion E

Only a few students reach level 6 here. This is usually because of errors in the mathematics but also there are places where the students do not demonstrate clear understanding even when they may have that understanding. For instance, a number of students regurgitated or copied advanced mathematics from the web or a text book without showing their clear understanding by applying or using them with their own examples. It is imperative for teachers to check and show clearly on the work that the mathematics is correct or not. It is also advisable to clarify that the student really does understand what they have written. There is still an issue with regression analysis being conducted using technology alone (and no analytical methods or understanding of the appropriateness of a chosen regression model). There are also many candidates using linear regression for data that is clearly not linear.

Recommendations for the teaching of future candidates

Teachers should introduce the exploration in stages by starting, in the students' first year of IB DP, with "mini" explorations and assessing these on, for example, two criteria. This will help students to become more familiar with the criteria and what is required for each.

Teachers should also encourage students to try to bring a personal and original approach to their mathematics. This is the original aim of doing an exploration and would certainly help with their personal engagement.

Students should be aware that pasting a URL into a citation generator might not actually give an adequate bibliographic entry; they need to go to find the authors' names and similar information themselves.

There is not always in-text referencing, particularly images not being cited at the point in the paper where they appear, (although sometimes the sources are listed in the bibliography). A significant minority of students are still providing only a list of URLs in place of an actual bibliography. Teachers should instruct students in referencing.

Similarly, teachers should explicitly instruct their students in the use of mathematical typesetting software.

Finally, it is important to maintain the standard of quality work that is expected from the mathematics SL course, choose a topic that is of interest and allow appropriate mathematics to be developed. Learning new mathematics should not be considered better or showing more personal interest than developing mathematics that they have seen from the SL course.

Further comments

With the new upload of explorations, teacher and candidate anonymity is important. Schools should therefore inform candidates to not include these details on cover pages or in headers, and teachers should not use candidates' names, or the teacher's, in their marking justifications. Schools should ensure they are familiar with the information and advice given on uploading work.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 26	27 – 35	36 – 46	47 – 57	58 – 68	69 – 90

The areas of the programme and examination which appeared difficult for the candidates

- Vector geometry
- Applying probability properties involving complements and independence
- Integration using substitution
- Solving inequalities
- Manipulating algebraic expressions and equations
- Proportional reasoning
- Reasoning involving patterns
- Factorials
- Using correct mathematical notation

The areas of the programme and examination in which candidates appeared well prepared

- Quadratic functions
- Trigonometric identities
- Binomial theorem
- Simple operations with vectors
- Using formulas for sequences and series
- Properties of logarithms

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Quadratic function

This question was answered correctly by the majority of candidates, although many gave the axis of symmetry as simply 2, rather than writing the equation $x = 2$, as required. Some candidates mistakenly attempted to factorise the quadratic in part (a).

Question 2: Trigonometric identities

Most candidates performed well on this question, apart from some occasional calculation errors. In part (a), some candidates drew a right-angled triangle and used the Pythagorean theorem to find the missing side length of the triangle, while others used the Pythagorean identity and did not need to draw a triangle. In part (b), candidates performed well with the double angle identity for cosine, no matter which of the three forms they chose to use.

Question 3: Pascal's triangle and binomial theorem

Although this question was generally well done, it was surprising to note that many candidates seemed unfamiliar with Pascal's triangle, a topic which appears in the Mathematics SL guide. In addition, even the majority of candidates who were familiar with the triangle, and who answered part (a) correctly, tended to use the more cumbersome $\binom{n}{r}$ formula to find the value of the binomial coefficient in part (b), not recognizing the relationship between the two parts of the question. In part (b), common errors were to write the coefficient rather than the entire term, and to leave the brackets off the $(2x)^3$, leading to an incorrect value for the coefficient.

Question 4: Vector equation of a line, perpendicular line and vector

The majority of candidates were able to earn some, if not all the marks in part (a) of the question. There were a number of candidates who found the correct vector between points P and Q but stopped before finding the equation of the line. There were also notation issues such as beginning the equation with "$L =$", and using some incorrect mixture of column vectors and the unit vectors $\boldsymbol{i}, \boldsymbol{j}$, and $\boldsymbol{k}$. In part (b), most candidates attempted to set some scalar product equal to zero, though many did not chose the direction vector from their line, and many used an incorrect column vector $\begin{pmatrix} 2 \\ n \\ 0 \end{pmatrix}$ rather than $\begin{pmatrix} 2 \\ 0 \\ n \end{pmatrix}$. As is typical, there were a number of candidates who seemed not to have been exposed to vectors during the course.

Question 5: Probability with combined and complementary events, independent events

Candidates were less successful than would be expected on this question. Many candidates drew a correct Venn diagram, but many did not know how to read their diagram to find the correct value of $\mathrm{P}(B)$. Many tried unsuccessfully to find $\mathrm{P}(A)$, not considering the independence of the two events. Many candidates wrongly assumed $\mathrm{P}(A)+\mathrm{P}(B)=1$, and a large number used the incorrect $\mathrm{P}(A \cup B)=\mathrm{P}(A)+\mathrm{P}(B)$, rather than the formula for the union of two events given in the formula booklet.

Question 6: Integration using substitution or inspection

Nearly all candidates knew to integrate $f'(x)$, though the large majority integrated incorrectly. Even those candidates who attempted u-substitution were rarely successful. However, most candidates did attempt to substitute the initial conditions to solve for C, and many knew the correct value for $\sin\frac{\pi}{2}$.

Question 7: Discriminant

This proved to be the most challenging question in Section A for the large majority of candidates. Many candidates did seem to realize that the question had something to do with the discriminant, and most were able to set up the correct equation $m-\frac{1}{x}=x-m$. However, a large number of candidates were unable to manipulate this equation into the necessary quadratic form. Among those who were able to find the correct expression for the discriminant, most were unable to correctly solve the resulting inequality.

Question 8: Vectors and trigonometry

Candidates were generally successful answering parts (a) and (b) of this question, although some tried to work backwards from the given result in part (b) and therefore did not earn the mark in that part. In part (c), many candidates answered both parts correctly, although some did not give an answer in terms of θ, as required. In part (d), most candidates began with a correct approach, setting up a ratio with the areas for the two triangles, but the majority were unable to earn full marks because they did not recognize that $\sin(180-\theta)=\sin\theta$, or because they were unable to show the necessary reasoning to correctly complete the question. Part (e) was left blank by many candidates, some started but did not finish the question, and only a small number were able to find the correct coordinates.

Question 9: Sequences and series, logarithms

Most candidates were able to earn at least some of the marks in this question, and a good number earned full marks. Candidates were generally successful in parts (a) and (b) of the question, although a few answered part (a) with $r=2$ rather than $r=\frac{1}{2}$, which prevented them from showing the required result in part (b). In part (c) most candidates correctly attempted to subtract the terms, although this was followed in some cases by incorrect expressions such as $\frac{\log_2\left(\frac{x}{2}\right)}{\log_2(x)}$. There were some candidates who correctly applied the properties of logarithms, but then did not recognize that the resulting logarithmic expression was equivalent to -1, which was necessary for part (d). Part (e) was a bit more challenging, though many were able to begin with the correct equation due to the "show that" nature of the previous parts of the question.

Question 10: Patterns in derivatives

It is pleasing to note that many candidates were able to earn at least some of the marks in this final question of the paper. Part (a) was generally well done, including recognizing the pattern to find the 19th derivative. Part (b)(i) was understood by most candidates, though poor notation such as missing brackets in the second derivative caused some to find an incorrect third derivative. In part (b)(ii), many candidates wrote the correct value of p, but very few were able to recognize the pattern and show the required reasoning required to earn full marks here. In part (c)(i), a good number knew to use the product rule, but some were unable to carry this through completely. In (c)(ii), most candidates earned a mark for substituting π into their derivative, but in many cases, a previous error prevented them from showing the given result.

Recommendations and guidance for the teaching of future candidates

It is important that teachers communicate to their students that final answers should be given using proper mathematical notation, and that this proper notation should be used throughout their working. Of particular concern in this paper are the number of candidates who failed to write brackets in their working, which led them to incorrect answers, especially in questions 3, 8, and 10.

It is helpful for students to practise working with past IB examinations, in order to be familiar with the style of the questions. It is also important to note that in a long question which asks a candidate to "show" a particular result in one part, it is possible for candidates to move forward in the question using the given result in later parts of the question, even if they did not know how to answer the "show that" style question.

As always, there were questions on this paper which required candidates to use and show their reasoning. Throughout the course, candidates should be working with questions and situations that require more of them than simply substituting into known formulas.

Finally, it is necessary for both teachers and students to be familiar with all of the Mathematics SL guide, including the notation list, and it is important for students to be exposed to the entire syllabus. For example, on this paper, it was clear that many candidates have had little or no practice working with topics such as vectors or integration using substitution.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 15	16 – 30	31 – 42	43 – 53	54 – 63	64 – 74	75 – 90

General comments

The paper was accessible to a great number of candidates who were well prepared. There were significant opportunities to solve problems using graphic display calculators and those that took that opportunity generally performed very well. The last parts of section A and section B were appropriately challenging and served as good discriminators between a level 6 and a level 7 candidate.

The areas of the programme and examination which appeared difficult for the candidates

- Recognizing conditional probability
- Displacement vs distance travelled
- Transforming data
- Graphical interpretation of derivatives
- Differentiation and integration of trigonometric functions
- Understanding the concept of the derivative as a rate of change

The areas of the programme and examination in which candidates appeared well prepared

- Area between curves
- Graphs of functions
- Normal distribution
- Volumes of revolution
- Cumulative frequency diagrams
- Modelling with trigonometric functions

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Composite functions

This question was done well by most candidates. A few composed the functions in reverse order in part (b) and some left their answer unfinished, losing the final mark. Most candidates used their GDC effectively to find the solutions in part (c) while some candidates preferred the less efficient analytical approach using the quadratic formula.

Question 2: Graphing functions and tangents

The majority of candidates found the coordinates of A using their GDC. While most did so successfully, a few used the trace feature, which resulted in an inaccurate answer being obtained. A significant number attempted to find the minimum using differentiation. While this approach was valid, it was not the most time efficient, particularly as use of the GDC was required in (b). Graphs were generally well drawn. However, few candidates paid close attention to the domain. It is disappointing that candidates continue to lose marks over this, as it is a regularly assessed skill. In most cases, a horizontal tangent was seen although some interpreted “tangent” as “derivative” and attempted to draw the graph of a degree two polynomial.

Question 3: Area of a sector

For the most part, question 3 was very well done. As part (a) asked for an exact answer, candidates lost a mark for reporting a three significant figure answer, even if they had initially given the correct exact answer. Some worked in degrees, before attempting to convert their answer to radians. Candidates are encouraged to read the question carefully. Many candidates interpreted part (c) as arc length AB which indicated that they were not familiar with the IB notation for the length of a line segment.

Question 4: Area between curves

This question was very well done by most candidates who used their GDCs effectively and did not struggle to obtain the correct answer using an analytical approach. Some candidates set up the subtraction in the wrong order while others integrated each function independently. These methods normally resulted in an error at some stage. The most common wrong answer seen of 0.583 came from candidates finding only the area under f.

Question 5: Normal distribution

Part (a) of this question was generally well done with candidates making good use of their GDCs. Not many approaches involving standardizing the variable were seen. However, part (b) proved challenging for most of the candidates, not recognizing that conditional probability was needed here. Many of them found $\mathrm{P}(X \le 2.15)$, and did not recognize that the intersection was $\mathrm{P}(2.15 \le x < w)$.

Question 6: Volumes of solids

This question was generally well done, with most managing to obtain full marks in both parts. In general, the GDC was used well throughout, although a fair number of candidates attempted inefficient and time consuming analytical approaches. Sadly, one of the most common errors was candidates not substituting into the correct formula for the volume of the solid which is given in their formula booklet. Follow through answers were often obtained from errors in part (a).

Question 7: Binomial distribution

Most candidates had little difficulty with part (a). Some good progress was made in part (b) by a significant number of candidates but once a correct expression was set up using the formula for variance, many failed at using their GDC correctly to solve this inequality. A common error was for the number of trials to be taken as five, rather than the four stated in the question and a large number of candidates were unable to correctly interpret their answer as they did not follow through with the inequality symbol in their working.

Question 8: Transforming data

Most students did very well with this question, gaining most of the marks in parts (a), (b) and (d). In (c)(i), many worked with the mean rather than the median. A common mistake was finding 5% of the mean or the sum of the scores (252), instead of using the median. In part (ii), they understood that the variance was the square of the standard deviation but could not proceed beyond this point.

Question 9: Linear motion and kinematics

Many candidates found this question difficult, and struggled to demonstrate a strong understanding of kinematics, particularly the differences between distance and displacement, and speed and velocity.

Most candidates answered parts (a), (b) and (c)(i) successfully. In (c)(ii), most were able to calculate the velocity, but only a few understood that speed is always positive. Similarly in (d), while the majority in (d)(i) were able to find the displacement between $t=1$ and $t=p$, few understood that the distance is also always positive. In (d) (ii), many students attempted tedious algebraic manipulation instead of using their GDC to find the area under the curve. While some did find the distance travelled between $t=0$ and $t=1$, they used the same value of the distance between 1 and their value of p (instead of using $4.45\ldots$). Many had by this stage forgotten that the velocity function had two branches and did not separate the calculation of the displacement.

Question 10: Transformations of trigonometric functions and rate of change

Part (a) was generally well done as most students could find the vertical shift and correctly identified the period as 12. Although candidates recognized that the value of a could be found from the amplitude, few correctly observed the reflection of the curve in the x-axis and simply wrote that $a=6$. In part (b), the most common error was correct placement of the brackets to describe the horizontal translation which most had found correctly in (b)(i).

Part (c) was not well done, with few candidates making any progress with it. In general, those who were successful found the point of inflection and the maximum positive rate of change on f, before drawing conclusions about those points of interest on g. Many struggled to find the correct first and second derivative of either f or g (not required), with many algebraic mistakes being made.

Recommendations and guidance for the teaching of future candidates

There were a number of questions in this paper where candidates were poorly prepared in the proper use of their GDC. Candidates should be aware of when an analytical approach is necessary and when one using their GDC will suffice. In general in Paper 2, once an equation has been set up, there is little reason why its solution should not come directly from the GDC. Teachers are encouraged to spend time with their students going through the solution using technology, so that students are aware of what is required to demonstrate full understanding and to gain all the available marks.

Cover all aspects of the syllabus including the subtle variations contained there-in, e.g the difference between scalar and vector quantities, particularly in the kinematics section.

When candidates are giving answers from their GDC, it is not considered adequate working to state “found on the GDC”. They need to be trained to sketch the output of their graphing screen, showing the key features which leads to the answer. Candidates should also be clear in their sketches if they are using the function or one of its derivatives.

Candidates should be encouraged to read questions carefully and note important words for their answer such as “exact” or “domain”. They should also continue to work on rounding to the correct number of significant figures and follow through in their work using more than 3 significant figures to avoid inaccurate answers.

Teachers must ensure that the IB syllabus for Mathematics SL is fully covered and that candidates are familiar with the whole guide. This includes terminology and notation.