November 2015 subject reports

MATHEMATICS SL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 48	49 - 59	60 - 71	72 - 83	84 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 17	18 - 20

The range and suitability of the work submitted

Overall the samples submitted were at the appropriate level and diverse in nature. It is encouraging to note that some schools are really encouraging students to find unique and personally exciting topics. However other schools still tend to push students in one mathematical direction (e.g. use of regression modelling) and this detracts from the spirit of the exploration and produces explorations that are homogenous. While many of the regular and standard explorations are still seen (lottery, probability in medicine, golden ratio, Monty Hall problem and so on), it was evident that schools have understood that these can be extended beyond the standard approach. This usually leads to improved performance in many of the criteria. However, some explorations still remain just a summary of common facts and/or general history of a topic. There were also candidates who produced work that reads like a common textbook problem or example. There were also many other fascinating and diverse topics that had not been covered previously. Students are obviously choosing topics of interest to them and then finding the mathematics within these to be explored. There were, therefore, numerous explorations centred around music, sport and computer games, for example. When a student is genuinely interested in a topic, this will invariably lead to a more successful outcome as they are more able to authentically relate these explorations to their own experiences. This results in better aims, rationales, obvious personal engagement and more relevant reflections. The majority of the work submitted had mathematics commensurate with the Standard Level course, but there were some students who attempted to explore mathematics which was well beyond the course, for example, graph theory, and had difficulty demonstrating thorough knowledge when it was clear that they did not fully understand what they were writing.

Candidate performance against each criterion

Criterion A

Most students appear familiar with the assessment criteria and make a concerted effort to provide the elements of a good mathematical paper. Introductions, rationales and aims are generally dealt with in specific terms and language (e.g. "I aim to ..."). A notable issue, however, is a lack of clarity and coherence in explanations. For instance, there were quite a few candidates who provided page after page of repetitive calculations which affected the conciseness and flow of the paper. Students need only provide one or two sample calculations and can then summarise the rest in a table. Another point is that vague aims make it hard to ensure that the exploration is complete .Students, aware of authenticity issues, are increasingly including inline citations, however there are still far too many who only have a bibliography and do not cite sources of ideas and images in the text where they occurred. This is something that teachers should monitor and require students to correct between the initial and final drafts of the paper.

Criterion B

This criterion was understood well by the teachers. Most students provided clear and well annotated graphs, tables, diagrams and used good notation. Computer technology was common especially for graphing, and equation editors were well used to write formulas. Some issues to highlight are poorly labelled graphs and diagrams, and terms used without clear definition. A few candidates still used inappropriate calculator/computer notations, like * for multiplication and “E” for power of ten. There is also a concern that the same variable is written in a variety of fonts and written in both uppercase and lowercase. Subscript is often also not used consistently. Fractions are often written both as 2/3 and with a horizontal divider. These types of inconsistencies may be penalised.

Criterion C

There is some confusion as to how much ‘personal interest’ contributes to ‘personal engagement’. Some teachers weighed this aspect heavily and awarded high marks for work that basically included comments about how much the student enjoyed the topic or the enthusiasm they demonstrated. However personal engagement was seldom evident in these explorations, and so should not be awarded high levels. There were also samples where the students got the highest possible level just because the students had presented interesting ideas. Good explorations should include many examples of "wondering" about the topic from a mathematical point of view and self-developed sample cases designed to further understanding of the topic at hand.

Criterion D

Reflection of some kind was common. However critical reflection that addressed the mathematical results and their impact on understanding of the topic were rarer. A number of teachers awarded the top level for simply summarizing the results or commenting on outcomes, without truly reflecting on the processes used and considering the limitations and implications of these results; anecdotal comments alone are rarely critical in this sense. The weaker explorations still have very little reflection in the body of the work. In these cases, students often leave their reflection for the conclusion and then make only superficial comments regarding how hard they found the topic.

Criterion E

Most of the work contained mathematics commensurate with the level of the course and in cases where it did not, the teachers had usually noted that and marked accordingly. In particular, some students carefully chose topics that reflected their Mathematics SL learning and did well in presenting the mathematics with good understanding. Others took risks with topics that were either unsuited to mathematics at the appropriate level (i.e. too simple), or where the mathematics were at such a level that good understanding was difficult to demonstrate (i.e. too difficult). For example, these students may simply end up substituting different values into a scientific or complicated mathematical formula sourced from the internet with no explanation and therefore demonstrating very little understanding of the mathematics used. The explorations mentioned in the first section which were often factual or historical in nature will invariably include very little accompanying mathematics and so it is difficult to achieve well in this criterion.

Recommendations for the teaching of future candidates

- It is obvious perhaps that prior discussion about the suitability of a chosen topic can save the student from hours of unproductive or unrewarding work. This would be particularly helpful for the average or below average candidates.
- Schools that performed well appeared to have explained the assessment criteria clearly to their students. This is an important consideration for teachers who wish to improve the performance of their students. Simple things such as how to organize a paper, how to present clear and useful mathematical representations, how to ‘wonder about’ the topic, how to consider and reflect on results, and how to present mathematical understanding can lead to better explorations.
- Teachers should develop a timeline for completion with multiple opportunities for reflection and informal feedback to ensure that there is every opportunity for the student to have success in their exploration. More importantly, teachers must provide criteria specific feedback within the sample and background information on the 5/EXCS form.
- Clear referencing needs to be emphasized. Specifically, students must understand the requirement to cite their sources within the exploration itself. They should also be aware of sources when they do research to generate statistical data or to support their exploration.
- Teachers should take time to look at the samples provided on the OCC and to share these with their students. This will also help to broaden the types of work they consider.
- Students should mention why they chose a certain accuracy and use the approximate sign when given rounded values.

Further comments

- There were a number of statistically based and data oriented explorations. Although the mathematics used was at the appropriate level, such as regression and Chi squared tests, the quality of the work produced was not of the standard required for an exploration.
- A few schools are expecting an unnecessarily high level of mathematics. Despite some very good work being produced by the students there is an expectation by some that the mathematics in the explorations must go beyond the level of the SL course. This is not true - the mathematics need be commensurate with the level of the course.
- When marking the explorations teachers are encouraged to annotate the student work, and are required to check correctness and accuracy of the mathematics used and comment fully on the 5/EXCS form. It is important for the moderation process that levels are fully justified and that the criteria descriptors are not merely regurgitated.
- Teachers from schools where several teachers mark the student's work should ensure that there is internal standardisation between the various teachers involved in the marking to ensure consistency across the whole sample.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 49	50 - 60	61 - 70	71 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Binomial expansion and finding binomial coefficient without a GDC
- Interpreting features of a function f, given the graph of a derivative f'
- Understanding the relationship between positive area and negative integral
- Finding derivative of a general function using chain rule
- Basic vector geometry
- Interpreting multiple transformations of a function
- Composite functions
- Cosine value of an obtuse angle
- Using multiple rules of logarithms

The areas of the programme and examination in which candidates appeared well prepared

- Interpreting a box and whisker plot
- Working with vector between two points and vector equations of lines
- Integration of a polynomial function and finding "c" using a given boundary condition

- Finding amplitude and period from a given trigonometric function
- Inverse functions
- Working with a quadratic function and its graph

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: box and whisker plot

This question was answered correctly by a large majority of candidates. Although a few candidates seemed not to be familiar with box and whisker plots, most of the errors seen were arithmetic in nature.

Question 2: vector diagram

While many candidates answered all parts of this question correctly, there were a number of common errors. Some candidates did not recognize that in order for vectors to be equal, they must be both parallel and in the same direction. Others left their answers in unfinished form, such as $p-q+p$.

Question 3: antidifferentiation with a boundary condition

It is pleasing to note that most candidates earned full marks on this question. There were a few candidates who neglected to consider the constant after integrating the given function, but who were still able to earn some of the marks for this question.

Question 4: sinusoidal function

Parts (a) and (b) were usually well done, with nearly all candidates identifying the correct amplitude and most able to find the correct period of the given function. Candidates were not as successful graphing the function in part (c), even those who had answered the first parts of the question correctly. Common errors in part (c) included sketching a cosine function (with its maximum at x=0), graphs with incorrect periods, and graphs drawn outside the given domain.

Question 5: inverse and composite functions

Candidates were generally more successful with part (a) of this question, finding the inverse of a given function. Most candidates recognized the need to swap x and y, and most were able to find the correct inverse. There were many candidates, however, who tried to expand the binomial rather than simply using the cube root, and were unsuccessful working with the resulting cubic polynomial.

In part (b), a significant number of candidates were unable to form the necessary composite, and were therefore unable to find $g(x)$. A common error was to neglect the $g(x)$ altogether, with many candidates beginning with $(x-5)^3=8x^6$, therefore having no $g(x)$ to find. There were also some candidates who left their final answer unfinished, writing $\sqrt[3]{8x^6}+5$, rather than $2x^2+5$.

Question 6: binomial expansion

This proved to be the most challenging question in Section A for the majority of candidates. While most candidates seemed to realize that the question had something to do with binomial expansion, many had trouble identifying the correct term, and nearly all were unable to work with the binomial coefficient using the $\binom{n}{r}$ formula, which is given in the formula booklet. For the small number of candidates who were able to correctly apply the $\binom{n}{r}$ formula, the resulting algebraic equation was relatively simple and nearly always led to these candidates finding the correct answer.

Question 7: arithmetic sequence and rules of logarithms

Most candidates were able to earn at least some of the marks in this question, and a good number earned full marks. Nearly all candidates were able to set up the correct expression for the 13th term of the sequence and began with a correct equation. This question required using multiple rules of logarithms, and numerous approaches were used to obtain the correct answer. Many candidates earned partial marks for applying some, but not all, of the rules correctly, and some candidates forfeited the final mark by leaving their final answer written in unfinished form as 3^4.

Question 8: quadratic functions

Candidates were very successful answering parts (a) and (b) of this question, using the vertex and y-intercept of the function. In part (c), nearly all candidates incorrectly applied the reflection, or neglected the reflection altogether, which led to them finding an incorrect value of q. In part (d), most candidates began with a correct approach, setting up a correct equation with $f(x) = g(x)$. While many were successful in earning full marks here, there were a number of algebraic errors which kept some candidates from finding the correct x-values..

Question 9: vectors

The majority of candidates answered part (a) correctly, though there many who lost one mark for writing their equation using "$L_1 =$", not realizing that L_1 is the name of the line, and not a vector to be used in the equation. Part (b) was generally well done, however, some common errors included using the same parameter for both lines, and working backwards from the given coordinates, which is not a valid approach to a "show that" question.

In part (c), most candidates recognized the need to find the vector $\overrightarrow{CA}$ to form the angle, although some used $\overrightarrow{AC}$ or other incorrect vectors. While a number of candidates correctly found $\cos A\hat{C}D = -\frac{1}{2}$, many did not recognize that this would lead to an obtuse angle.

Question 10: calculus

As expected, this question proved to be the most challenging on the paper for most candidates.

In part (a), while most candidates recognized that the local minimum occurs where $f'(5)=0$, many were unable to give a complete explanation as to why this was a local minimum, rather than a local maximum.

Part (b) was answered correctly by only a few candidates. Most did not recognize that the graph of f is concave down where the graph of f' has a negative gradient (in other words, where $f''(x)<0$). Some candidates who seemed to have the right idea gave vague answers as to the range of values. It is best to write answers using correct mathematical notation, such as $2<x<4$.

In part (c), a good number of candidates recognized that there was a relationship between area and integrals, though most failed to account for the fact that a positive area below the x-axis gives a negative integral. Many candidates were able to earn at least some marks on this part, especially those who recognized $\int_a^b f'(x)dx = f(b)-f(a)$. In part (d), although a few candidates were successful using the chain rule or the product rule to find the derivative of $g(x)$, the overwhelming majority were not. The most common error was to write $g'(x)=2f(x)$. Many candidates failed to use $g(6)$ and $g'(6)$ in their linear equation, substituting $f(6)$ and $f'(6)$ instead. In addition, many candidates who answered part (c) incorrectly found their follow-through calculations to be quite cumbersome.

Recommendations and guidance for the teaching of future candidates

It is important that teachers communicate to their students that final answers should be given using good mathematical form. For example, values which lead to integers, such as 3^4, must be simplified. Further information has been provided about this on the OCC, and also recently in the September 2015 coordinators notes.

Candidates should be reminded to show their working in a neat and organized manner. Explanations should be clear and complete. Candidates must cross out any working they do not wish to have marked.

While it is important to be familiar with algorithms and concrete examples, it is also necessary for candidates to understand the concepts being used. This is apparent in question 10, for example, where candidates struggled to apply calculus concepts when the function was not given.

It is essential that both teachers and students are familiar with the complete Mathematics SL guide, especially the syllabus content, command terms and notation list.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 36	37 - 50	51 - 60	61 - 69	70 - 79	80 - 90

The areas of the programme and examination which appeared difficult for the candidates

The vast majority of candidates attempted to answer all the questions, although in some centres there appeared to be some areas of the syllabus which proved difficult for the candidates:

- Conditional probability and concept of independence.
- Finding all the solutions to a trigonometric equation.
- Deciding which questions should be solved with a Graphic Display Calculator (GDC) and which cannot or should not be solved using analytical approaches.
- Interpreting answers in the context of the problems.
- Prematurely rounding values, leading to inaccurate answers.
- Finding a volume of revolution using the GDC.
- Solving complex equations and inequalities using the GDC.

The areas of the programme and examination in which candidates appeared well prepared

The following topics were well understood by a significant number of candidates:

- Geometric sequences.
- Trigonometry in non-right angled triangles.
- Linear regression with the use of the GDC.
- Arc length and area of a sector.
- Basic trigonometric equations.
- Discrete probability distributions.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Arc length, area of a sector

Most candidates were successful at answering this question. Only a few candidates showed incomplete answers for part (b) by finding the area of the unshaded sector.

Question 2: Probability distributions

Part (a) was well done by the great majority of candidates, who recognized that the sum of the probabilities is 1. Part (b) was generally solved efficiently, though a few candidates seemed to be unaware of what the expected value meant. A small number of candidates wrote $E(X) = np$ and did not know how to continue. They seem to relate expected value only to the mean of a Binomial distribution.

Question 3: Logarithmic function, asymptote, *x*-intercept and volume of revolution.

Part (a): Many candidates found the equation of the asymptote correctly, although some left this part of the question blank, or did not write the answer as an equation.

Part (b): Most candidates could answer this question correctly.

Part (c): Many candidates did not set up the integral correctly: they either wrote incorrect limits or did not square the function. Many of the candidates who set up the correct integral had difficulties in evaluating it using their GDC or tried to find the integral analytically. Some candidates simply did not multiply by the factor of π when evaluating in the calculator.

Question 4: Geometric sequences

Parts (a) and (b) were accessible to most candidates, although a few found u_6 instead of S_6.

In part (c), most of them wrote the correct inequality but made errors by trying to use an analytical approach instead of making efficient use of the GDC. Also, many treated the inequality as an equation. This may be a correct approach if they later use their answer to solve the inequality. Some of the candidates did not consider that the value of *n* had to be an integer, losing the final mark. Another common error was not to round up the resulting value of *n*.

Question 5: Probability: independent events and conditional probability.

Parts (a) and (b) of this question have been attempted in a satisfactory manner. Even the weakest candidates were able to get some of the available marks, showing clear working. However, part (c) proved difficult for many candidates as they could not find the probability of the intersection C' and D, while they did not encounter any problems in finding $P(C \cap D)$. The great majority seemed unaware that $P(C'|D) = P(C')$, which would have led them to the correct answer in an efficient way.

Question 6: Kinematics

Part (a): The great majority of the candidates understood that when a particle is at rest the velocity is equal to zero, though some of them were unable to solve the equation using their GDC.

Part (b): Many candidates did not answer this question or did it incorrectly. Many recognized that the acceleration had to be zero but could not move past that. There were errors in trying to find the derivative of the velocity by applying the power rule to an exponential function. Many tried to use fruitless analytical approaches to find the value of *t* instead of using their GDC.

Question 7: Derivative of a logarithmic function and application

Part (a): Most candidates applied the chain rule properly to find the derivative of $\ln\left(x^2\right)$. Many candidates struggled with part (b), and a great number left this question blank. Those who attempted this question recognized that the gradient had to be $\frac{2}{d}$ and substituted the point (1, -3) into the equation of the tangent but many were unable to progress any further.

Some candidates changed the original expression from $\ln\left(x^2\right)$ to $2\ln\left(x\right)$, which is not a correct relationship for the domain given, and did not lead to the correct answer in part (b)

Question 8: Trigonometry, sine and cosine rules, area of a triangle

Parts (a) and (b) were solved in a satisfactory manner by the great majority of candidates. In part (c), a common error was working only with 3 sf for half the area of triangle ABC, thus getting an inappropriate value for the sine of theta (more than 1) and being unable to find the two possible values of the angle. Some candidates rounded the value of the sine to 1, getting incorrect angles. The use of the cosine rule was generally recognized in part (d), although some candidates did not use an obtuse angle.

A few candidates had their GDC in radian mode, which made them lose marks.

Question 9: Linear regression and modelling

Part (a) was solved correctly by most candidates. Some candidates, however, attempted to find the equation of a line by using two of the given points. Others seem to have entered the *y*-values as frequencies and therefore found incorrect coefficients.

The most common difficulty in part (b) was realizing that the answer had to be truncated in the context of the problem.

In part (c), the majority of candidates substituted *t* by 0, and found the answer correctly. On rare occasions, *t* = 1995 was used instead of 0.

Part (d): although most candidates recognized that the point (5, 64) satisfied their equation, many had problems solving it. Analytical approaches of various kinds were seen instead of using their GDC.

(e) Most candidates were able to set up the correct equation, but most struggled with solving the equation. Many who did solve it correctly did not continue to report the correct year.

Question 10: Normal probability

Part (a): Many candidates attempted to standardize the value and found the *z*-score.

Some of the candidates who correctly found the *z*-score only used this value correct to 3 significant figures, getting the inaccurate answer of 3.66.

In part (b) some candidates worked with the left tail instead of the one to the right, obtaining an incorrect value for *w*.

In part (c) many candidates recognized the need to restricting the population to 0.95 and answered the question with no difficulties.

Part (d) was not attempted by many candidates, but those who did generally did it incorrectly.

Recommendations and guidance for the teaching of future candidates

- Candidates should be reminded to carry through more than 4 significant figures in their working and thus avoid premature rounding of answers in intermediate steps.
- This paper has clearly revealed the need for a better training on the use of the GDC. This should be done explicitly and more attention should be given to solving equations and inequalities. Teachers should promote discussions in class as to when the GDC is needed to solve a problem, when it is convenient to use it and when analytical approaches are needed.
- Teachers should emphasize the need to present work clearly and to label the sub part of each question.
- Teachers must ensure the maths SL syllabus is fully covered.